







Acquisition Time and Probabilities of Detection and False Alarm in Direct Sequence Code Division Multiple Access Systems

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Abstract— Multiple access interference (MAI) is one of the most important issues to consider in the design, implementation and operation of 5G systems using code division multiple access (CDMA) technique based on spread spectrum that offers a more flexible solution, compared to other techniques. Especially, it permits a much higher user bit rate, due to the bandwidth allocated to the emitted signal; thus, allowing very attractive multimedia services. In this paper, we develop the expressions of the detection probability, false alarm probability and the average acquisition time - in a direct sequence CDMA (DS-SS) transmission system - using a serial search and taking into consideration MAI. The evolution of these parameters according to the signal-to-noise ratio (SNR) and the decision threshold leads to study performances of the system in terms of detection and average acquisition time. The obtained results reveal that the acquisition time decreases when the SNR increases. Additionally, it is found that the probability of detection increases when the threshold decreases, and increasing SNR it reaches a constant value for an SNR/chip around 0 dB for all thresholds.

Keywords— Direct sequence code division multiple access; Multiple access interference; Signal-to-noise-plus-interference ratio; Detection probability; False alarm probability; Acquisition time.

1. INTRODUCTION

The performance of the code division multiple access (CDMA) technique depends on the correct time synchronization between the transmitter and the receiver [1]. A good synchronization allows reducing enormously the multiple access interference (MAI). Indeed, for correctly despreading, it is necessary that the pseudo noise code (PN-code) which presents good properties of auto-correlation (which allows the synchronization during an inter-correlation) is in phase with the received signal. This will minimize the correlation error and therefore, maximize the processing gain. The synchronization of the PN-code is therefore a crucial point at the receiver level; it takes place in two steps:

- a) The acquisition or initial synchronization: it aligns the code identical to the transmission and locally generated with the received signal through the detection of a peak of auto-correlation.
- b) The tracking or fine synchronization: it allows maintaining a fine alignment of the received data.

The most used search strategies are the serial, the parallel and the hybrid ones [2, 3]. The serial search strategy involves testing each possible code phase or cell in the uncertainty region one at a time using a simple hardware and large acquisition times, while in the

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parallel search strategy, the use of massive parallel detectors is required to test all the cells simultaneously at a time which improves the acquisition time, however, the system requires very complex hardware. The hybrid approach was suggested by Zhuang [4], it is a compromise between hardware complexity and acquisition speed. To make a decision whether a cell corresponds to the synchro position or not, one can either compare the output detector to a threshold value or use maximum selection. The threshold could be set fixed for all the cells or adaptive.

The simultaneity of transmissions enabled by CDMA causes increased detection difficulty determined by the importance of MAI. This is of course a function of the number of active users, and the evaluation of the performance of a system (through the calculation or measurement of the error rate) according to this parameter is an essential feature. Many acquisition systems were proposed, typically attempting to develop an interference cancelation to enhance acquisition process.

In previous work, we presented an exact, simple and more realistic method to model the statistics of MAI for a Direct Sequence CDMA (DS-CDMA) system using pseudo noise (PN) sequences and direct sequence binary phase shift keying (DS-BPSK) scheme through an additive white gaussian noise (AWGN) channel [5]. Using this model, we estimated the signal to noise plus interference ratio (SNIR) in the system and investigate the performance of the system where results were presented in terms of SNIR and bit error rate (BER) and where the impact on the system performances of relevant parameters such the number of active users, the spreading sequence length and the signal to noise ratio SNR is studied in synchronous and asynchronous cases [6].

This work concerns the performance study of the DS-CDMA system, where we present the mathematical developments leading to the expressions of detection probability, false alarm probability and average acquisition time for a cell-by-cell detection. Theoretical expressions are illustrated and validated by simulations and used to evaluate performances of the system.

2. ACQUISITION TIME

The acquisition time directly determines the response time of the complete system. In conventional acquisition systems, only the structure of the acquisition systems changes to gain better execution in terms of acquisition time. Among these systems, the serial search acquisition is the most used one since it achieves hardware simplicity of the receiver [7, 8].

2.1. Serial Search Acquisition

The serial acquisition system is a correlation acquisition technique where we multiply the local PN code (generated at the receiver) by the received PN code (generated at the transmitter), to produce the autocorrelation measure (see Fig. 1). This is why these structures are based on the use of one or more correlators. The system gradually shifts the local sequence of the code by a fixed step ΔT_c ($\Delta^{-1} = 1, 2, 4$) and tests all possible phases in a serial way until a phase alignment is detected [9, 10].

In communication systems, the problem of detection consists of observing the received signal and making a decision about the presence or absence of the desired signal. This situation can be described in terms of statistical hypothesis testing [11].

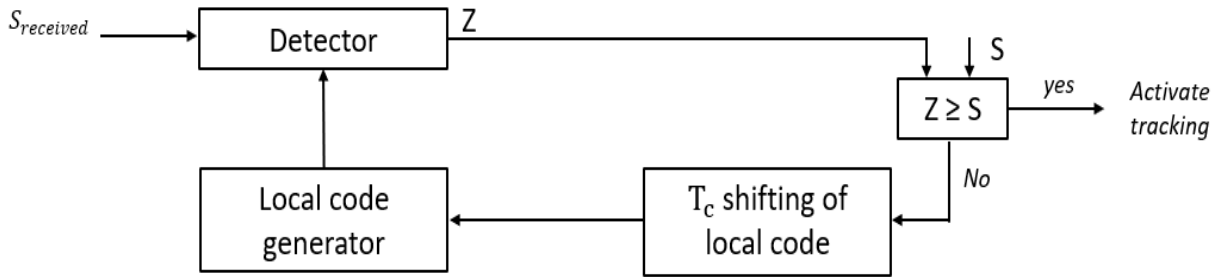


Fig. 1. Serial search acquisition.

The main purpose of code acquisition is to achieve synchronization between the transmitter and the transmitted signal. This is achieved by multiplying the received signal by shifted versions of the local code. Each relative position between the codes (of the transmitter and the receiver) is called a “cell”. The total number of cells needed to verify the acquisition is called the “region of uncertainty”. The position in which the codes are in phase (synchronized) is called “cell H_1 ” and the position for which the codes are not synchronized is called “cell H_0 ” (see Fig. 2).

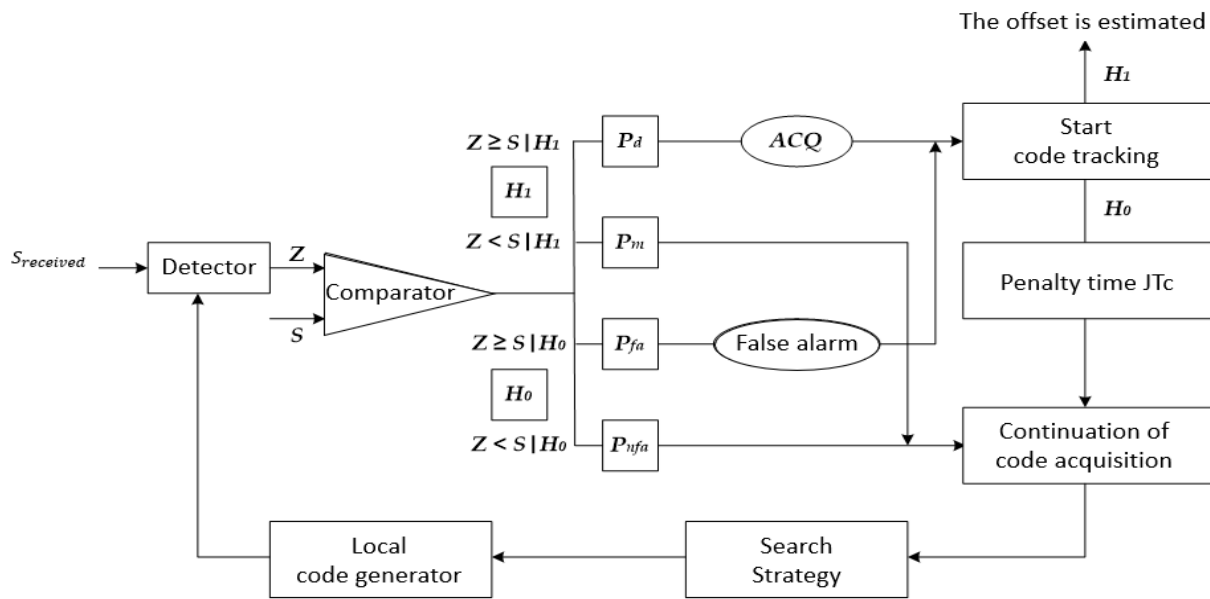


Fig. 2. Principle of code acquisition.

Regarding to the two hypothesis H_0 and H_1 , we have four situations:

- a) The detector decides H_0 when H_0 is true; it is a correct decision with a non-false alarm probability P_{nfa} .
- b) The detector decides H_0 when H_1 is true; it is a wrong decision with a miss probability P_m .
- c) The detector decides H_1 when H_0 is true; it is a wrong decision corresponding to a false alarm with a false alarm probability P_{fa} .

d) The detector decides H_1 when H_1 is true; it is a correct decision corresponding to a detection with a detection probability P_d .

As shown in Fig. 2, code acquisition is a closed-loop process controlled by the search strategy block. In this process, the cell associated with each relative position, between the codes, is tested by the detector. If the synchronization between the transmitter and the receiver is achieved, the receiver starts the code tracking operation. If this is not the case, we correct the estimate and we try again with another relative position. The decision variable Z is compared to a threshold S . If Z represents a synchronization position and it is greater than S , the detector declares that the codes are possibly in phase (hypothesis H_1) and the cell H_1 will be detected with a probability of detection P_d :

$$P_d = \Pr\{Z \geq S|H_1\} \quad (1)$$

Otherwise, cell H_1 will be missed with a miss probability P_m .

$$P_m = \Pr\{Z < S|H_1\} \quad (2)$$

For each position, the synchronization may be declared incorrectly, with a probability of false alarm P_{fa} , or the desynchronization is detected correctly with a probability P_{nfa} .

$$P_{fa} = \Pr\{Z \geq S|H_0\} \quad (3)$$

$$P_{nfa} = \Pr\{Z < S|H_0\} \quad (4)$$

In a previous work, we have shown that the locally generated PN code is synchronized with the received PN sequence if the decision variable Z is greater than the threshold S ; otherwise, the phase of the local PN code is then updated, i.e., the local PN code is delayed by a multiple or submultiple of the chip time T_c [5]. S is obtained from the value of the desired false alarm P_{fa} to obtain a high probability of detection P_d [7, 10].

2.2. Average Acquisition Time

In general, a false alarm increases the acquisition time. Indeed, the code tracking will then be activated, but the system will quickly realize that it is a false acquisition. In this case, it gives control back to the acquisition process to resume the search after a certain time called "penalty time".

The search for the correct phase is carried out in a region of uncertainty. This region is decomposed into N finite elements (N phases), where N is the period of the PN sequence. We assume that the search area contains only one phase that matches the correct phase.

Considering the following notation:

- T_i is the integration time, i.e., the time required to examine a cell.
- T_{fa} is the false alarm time, it is the time necessary to realize that we have taken the wrong cell.

We examine all possible phases from 1 to N in the region of uncertainty which is equal to the period of the PN code. If there is no detection of the correct phase, we will start the cycle again until the correct phase is detected. For this, a penalty time T_{fa} is associated to the probability of false alarm. This time must be very large compared to the integration time and therefore, $T_{fa} = K_{fa} T_i \gg T_i$ and $K_{fa} \gg 1$ is generally random in several applications but according to [12] it can be considered constant. In addition, there is another penalty time which is associated with the probability of non-detection and is equal to $N.T_i$. If we encounter a non-detection case, the correct phase cannot be detected until the next cycle.

The average acquisition time is calculated by examining the different possible scenarios for locking the correct cell. The acquisition time is a random variable for this we use its average [10, 13].

We developed the expression of the average acquisition time [see Eq. (20) in the Appendix], we found:

$$\bar{T}_{acq} = (N - 1)(T_i + P_{fa} \cdot T_{fa}) \left(\frac{1}{P_d} - \frac{1}{2} \right) + \frac{T_i}{P_d} \quad (5)$$

$T_{fa} = K_{fa} \cdot T_i$ is the penalty time related to the probability of false alarm.

T_i is the time needed to reach a decision, if we perform full autocorrelation over the entire period of the sequence which is equal to N then $T_i = N \cdot T_C$. Eq. (5) becomes:

$$\bar{T}_{acq} = N \cdot T_C \left[(N - 1)(1 + P_{fa} \cdot K_{fa}) \left(\frac{1}{P_d} - \frac{1}{2} \right) + \frac{1}{P_d} \right] \quad (6)$$

Fig. 3 illustrates the variation of the acquisition time as a function of the SNR per chip for $N = 7, 63$ and 255 , considering only the chip duration $T_C = 1 \mu s$ and the penalty time $K_{fa} = 1000$.

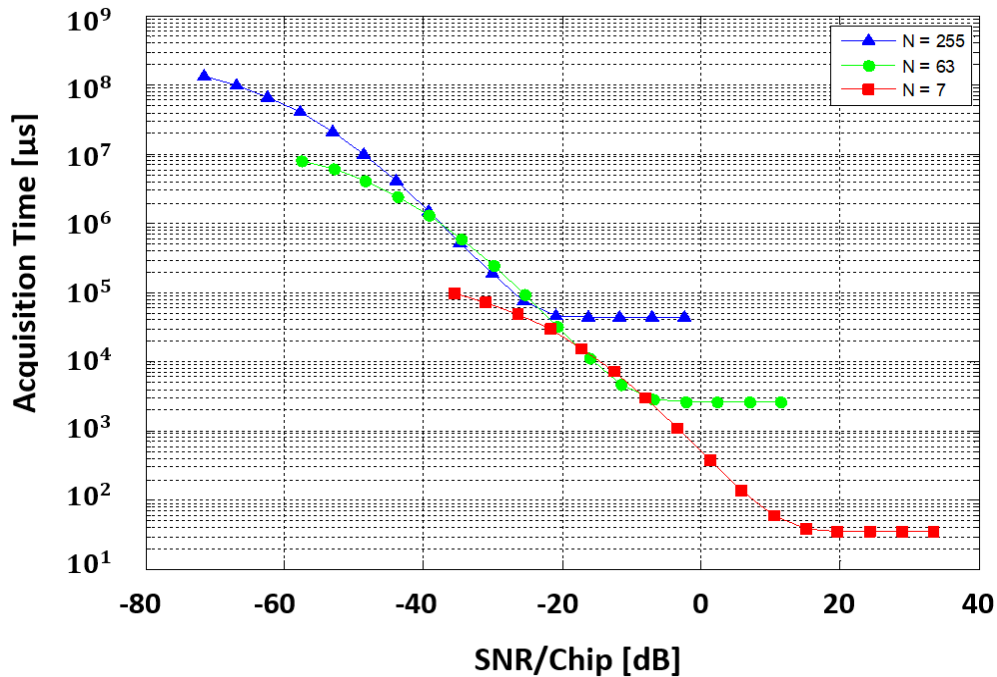


Fig. 3. Acquisition time vs SNR/chip for different values of N .

When N increases, the average acquisition time also increases. This is explained by the fact that when N is large, the search time for the correct phase becomes long, which implies a longer average acquisition time. On the other hand, for a given code length, the average acquisition time decreases when the SNR increases and becomes constant from a limit value, this limit value decreases when N increases.

3. DETECTION AND FALSE ALARM PROBABILITIES

The false alarm probability and the detection probability can be determined from the probability density function of the signal envelope plus the noise. The noise includes both additive gaussian noise due to channel and the MAI. We showed in a previous work [3] that

the MAI is an additive noise with a gaussian probability density function (PDF) for an equal-received-energy. The PDFs are given by [14]:

$$P(\text{noise}) = r \exp\left(\frac{-r^2}{2}\right) \quad (7)$$

$$P(\text{signal} + \text{noise}) = r \exp\left(-\frac{r^2 + \frac{E}{N_0}}{2}\right) \cdot I_0\left(\sqrt{\frac{2E}{N_0}} r\right) \quad (8)$$

r is the energy of the received signal, E the energy of the carrier, $\frac{N_0}{2}$ the bilateral power spectral density of the noise and $I_0(x)$ is the 0-order modified Bessel function. We deduce the probabilities of detection P_d and false alarm P_{fa} :

$$P_d = \int_T^\infty P(\text{signal} + \text{noise}) dr = Q\left(\sqrt{\frac{2E}{N_0}}, S\right) \quad (9)$$

$$P_{fa} = \int_T^\infty P(\text{noise}) dr = Q(0, S) \quad (10)$$

where $Q(a, b)$ is Marcum's function [15].

Fig. 4 illustrates the variation of the false alarm probability as a function of the decision threshold S .

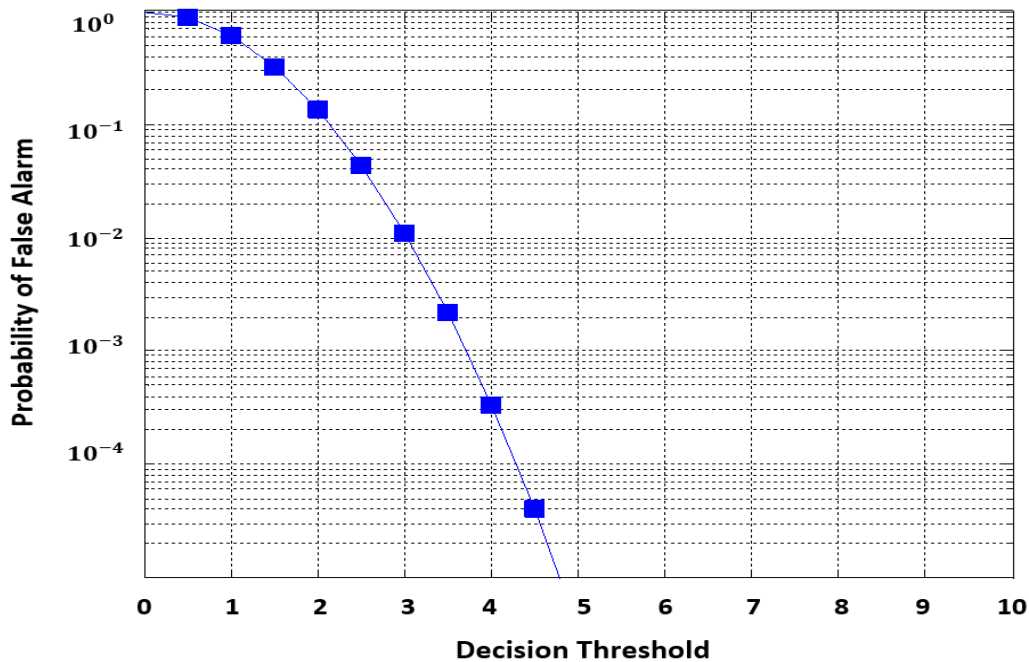


Fig. 4. Probability of false alarm vs the decision threshold.

The decision threshold should be chosen based on the lowest desired false alarm probability value. For example, for a chosen false alarm probability of 10^{-4} corresponds approximatively to a decision threshold of 4 (4 times the average energy per chip time).

However, the performance of the system is also evaluated in terms of probability of detection and average acquisition time. To evaluate these two parameters, we consider the following realistic hypotheses:

- A PN code of length $N = 63$.

- A duration of the chip $T_C = 1 \mu s$, which gives for $\Delta^{-1} = 1$ an update of the search process of $T_C = 1 \mu s$ and 63 iterations required to explore all the region of uncertainty.
- A penalty time equal to $K_{fa} \cdot N \cdot T_C$ s, with a penalty constant $K_{fa} = 10^3$.
- A false alarm rate P_{fa} of 10^{-4} .

We can then plot the variations of the probability of detection and the average acquisition time as a function of the SNR. The results are represented in Figs. 5 and 6.

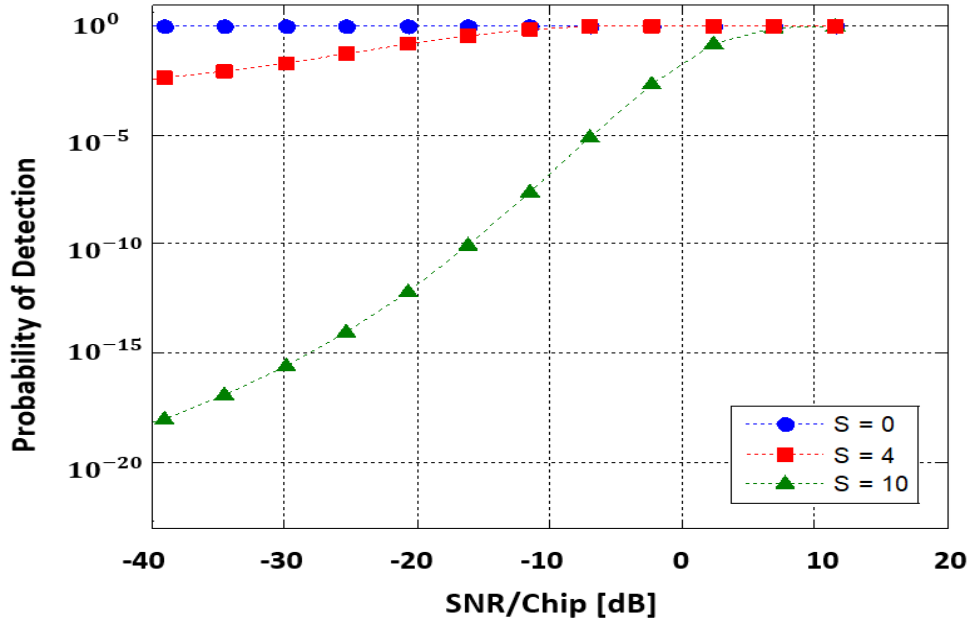


Fig. 5. Probability of detection vs SNR/chip for different values of the decision threshold.

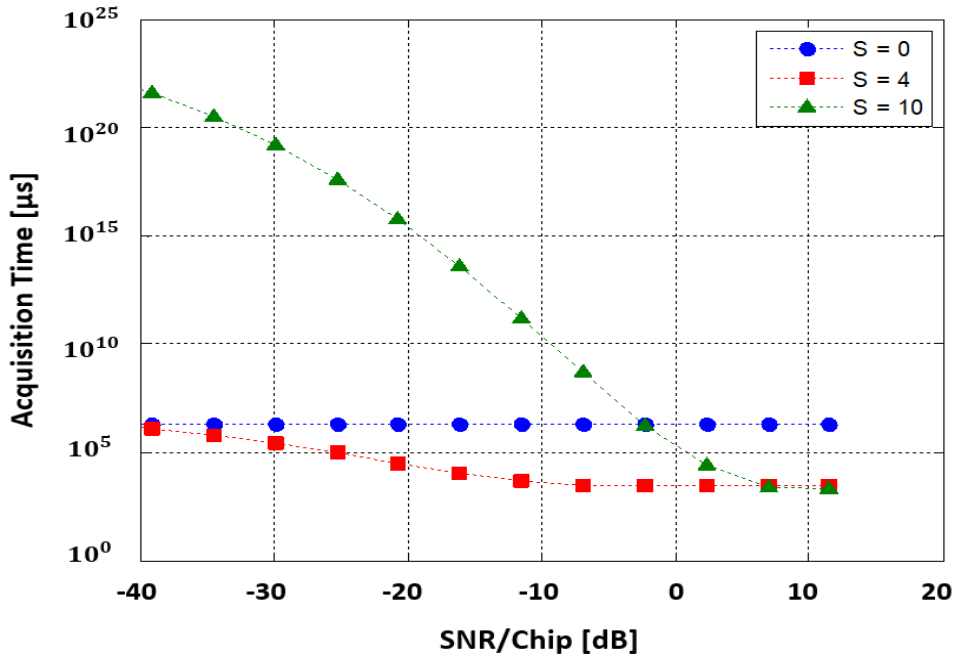


Fig. 6. Average acquisition time vs SNR/chip for different values of the decision threshold.

The probability of detection increases when the SNR increases, then the time of the correct phase becomes short, which implies a shorter average acquisition time. Moreover, the probability of detection decreases when S increases while the acquisition time has a

minimum value for a decision threshold value equal to 4, and this for all the values of the SNR. This threshold value corresponds to the value of the false alarm probability 10^{-4} often used. In fact, for low values of the SNR, it offers an acquisition time equivalent to that given by the optimal threshold $S=0$, and for large values of the SNR, it offers an acquisition time equivalent to that offered by the threshold $S=10$ for which the false alarm probability is the lowest and the detection probability is equal to that obtained by adopting the optimal threshold $S=0$.

4. CONCLUSIONS

In this work, the expressions for the average acquisition time, the probability of detection and of false alarm determined in a DS-CDMA system using PN-codes were analyzed for a channel with delay and Gaussian white noise. We took into account the MAI as an additive noise with a Gaussian probability density function. The acquisition time decreases when the SNR increases, and the probability of detection increases when the threshold decreases and increasing SNR, it reaches a constant value for an SNR/chip around 0 dB for all thresholds.

Results show relatively the same performance in terms of acquisition time as for a joint two-cell detection system, a more complex than our system based on cell-by-cell detection [16], and in terms of probability of detection compared to a complex system based on an adaptively varying threshold scheme that uses a cell-averaging constant false alarm rate (CFAR) algorithm to maintain a constant false alarm rate [4].

It is difficult to produce a high-quality communication service under varying states of the propagation channel. Further, the received signals are also subject to fading and noise addition. Thus, since the signal level is not stable (unknown) and still the environment - where the signal propagates - contains interferences, the correct acquisition cannot be achieved using a fixed threshold. These facts lead to adaptive signal processing techniques using an adaptive detection threshold. Several CFAR techniques have to be employed in DS-CDMA code acquisition to combat variability and instability in detection probability and false alarm probability.

Appendix

The average acquisition time is calculated by examining the different possible scenarios for locking the correct cell. Each scenario is characterized by n the index of the cell corresponding to the synchronization, j the number of missed "miss" detections, and k the number of false alarms.

The detection time corresponding to this scenario is given by:

$$T_{acq}(n, j, k) = nT_i + jNT_i + kT_{fa} = (n + jN)T_i + kT_{fa} \quad (A-1)$$

where $n + jN$ is the number of phases examined, $j + 1$ the number of correct phases found, and $K = (n + jN) - (j + 1)$ the number of incorrect phases found. The probability of this scenario is: $P(n, j, k) = P_r(\text{The correct phase position } n, j \text{ miss, } k \text{ false alarm})$

According to Bayes's identity:

$$P(n, j, k) = P(k/n, j) P(j/n) P(n) \quad (A-2)$$

where:

$$\begin{aligned}
 P(n) &= P_r(\text{The position of the correct phase} = n) = \frac{1}{N} \text{ (uniform law)} \\
 P(j/n) &= P_r(j \text{ miss}/n) = (1 - P_d)^j P_d \text{ (with } P_d \text{ detection probability)} \\
 P(k/n, j) &= P_r(k \text{ false alarm}/n, j) = P_r(k \text{ false alarm among } K \text{ positions)} \\
 &= \binom{K}{k} P_{fa}^k (1 - P_{fa})^{K-k}
 \end{aligned}$$

In this case, Eq. (12) is written as:

$$P(n, j, k) = \frac{1}{N} C_k^K P_d (1 - P_d)^j P_{fa}^k (1 - P_{fa})^{K-k} \tag{A-3}$$

C_k^K is the number of combinations of k elements among K .

The expectation of synchronization time is:

$$\begin{aligned}
 \bar{T}_{acq} &= \sum_{n,j,k} T_{acq}(n, j, k) P(n, j, k) \\
 \bar{T}_{acq} &= \sum_{n=1}^N \sum_{j=0}^{\infty} \sum_{k=0}^K \frac{1}{N} C_k^K P_d (1 - P_d)^j P_{fa}^k (1 - P_{fa})^{K-k} (nT_i + jNT_i + kT_{fa}) \\
 \bar{T}_{acq} &= \sum_{n=1}^N \sum_{j=0}^{\infty} \frac{1}{N} P_d (1 - P_d)^j \sum_{k=0}^K C_k^K P_{fa}^k (1 - P_{fa})^{K-k} (nT_i + jNT_i + kT_{fa})
 \end{aligned} \tag{A-4}$$

Considering the following identities of the binomial distribution

$$\begin{aligned}
 \sum_{k=0}^K C_k^K P_{fa}^k (1 - P_{fa})^{K-k} &= 1 \\
 \sum_{k=0}^K K C_k^K P_{fa}^k (1 - P_{fa})^{K-k} &= K P_{fa}
 \end{aligned}$$

Then Eq. (14) becomes:

$$\bar{T}_{acq} = \sum_{n=1}^N \sum_{j=0}^{\infty} \frac{1}{N} P_d (1 - P_d)^j (nT_i + jNT_i + K P_{fa} T_{fa}) \tag{A-5}$$

We replace K by its value: $n + jN - j - 1$, then:

$$\begin{aligned}
 nT_i + jNT_i + K P_{fa} T_{fa} &= nT_i + jNT_i + (n + jN - j - 1) P_{fa} T_{fa} \\
 nT_i + jNT_i + K P_{fa} T_{fa} &= n(T_i + P_{fa} T_{fa}) - P_{fa} T_{fa} + j(NT_i + (N - 1)P_{fa} T_{fa})
 \end{aligned} \tag{A-6}$$

Substituting Eq. (16) into Eq. (15) gives:

$$\bar{T}_{acq} = \sum_{n=1}^N \sum_{j=0}^{\infty} \frac{1}{N} P_d (1 - P_d)^j (n(T_i + P_{fa} T_{fa}) - P_{fa} T_{fa} + j(NT_i + (N - 1)P_{fa} T_{fa})) \tag{A-7}$$

Using the following identities:

$$\begin{aligned}
 \sum_{j=0}^{\infty} P_d (1 - P_d)^j &= 1 \\
 \sum_{j=0}^{\infty} j P_d (1 - P_d)^j &= \frac{1 - P_d}{P_d}
 \end{aligned}$$

Eq. (17) becomes:

$$\begin{aligned}\bar{T}_{acq} &= \sum_{n=1}^N \frac{1}{N} \left[n(T_i + P_{fa}T_{fa}) - P_{fa}T_{fa} + \left(\frac{1-P_d}{P_d} \right) (NT_i + (N-1)P_{fa}T_{fa}) \right] \\ \bar{T}_{acq} &= \frac{1}{N} \left[(T_i + P_{fa}T_{fa}) \sum_{n=1}^N n - \sum_{n=1}^N \left[P_{fa}T_{fa} + \left(\frac{1-P_d}{P_d} \right) (NT_i + (N-1)P_{fa}T_{fa}) \right] \right]\end{aligned}\quad (A-8)$$

Knowing that

$$\sum_{n=1}^N n = \frac{(N + N^2)}{2}$$

and

$$\sum_{n=1}^N (C) = N(C), \text{ where } (C) \text{ is a constant}$$

Then Eq. (18) becomes:

$$\begin{aligned}\bar{T}_{acq} &= \left[(T_i + P_{fa}T_{fa}) \frac{N+1}{2} - P_{fa}T_{fa} + \left(\frac{1-P_d}{P_d} \right) (NT_i + (N-1)P_{fa}T_{fa}) \right] \\ \bar{T}_{acq} &= \left[P_{fa}T_{fa} \left[\frac{N+1}{2} + (N-1) \left(\frac{1-P_d}{P_d} \right) \right] + T_i \left[\frac{N+1}{2} + N \left(\frac{1-P_d}{P_d} \right) \right] \right] \\ \bar{T}_{acq} &= \left[(N-1)P_{fa}T_{fa} \left(\frac{2-P_d}{2P_d} \right) + T_i \left(\frac{2N + (-P_d)(N-1)}{2P_d} \right) \right] \\ \bar{T}_{acq} &= \left[(N-1)P_{fa}T_{fa} \left(\frac{2-P_d}{2P_d} \right) + T_i \left(\frac{2N + (2-2-P_d)(N-1)}{2P_d} \right) \right]\end{aligned}\quad (A-9)$$

Finally:

$$\bar{T}_{acq} = (N-1)(T_i + P_{fa}T_{fa}) \left(\frac{1}{P_d} - \frac{1}{2} \right) + \frac{T_i}{P_d}\quad (A-10)$$

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