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Stability Analysis of Model-Free Adaptive Fuzzy Logic Control System Applied for Liquid Level Control in Soda Production

Snejana Yordanova[1], Milen Slavov[2*]

[1, 2] Department of Industrial Automation, Faculty of Automation, Technical University of Sofia, Sofia, Bulgaria
E-mail: milen.slavov@solvay.com

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Abstract— Efficient control of liquid level - in a carbonization column (CCl) of soda ash production plants - is a difficult task because the plants are nonlinear, subjected to disturbances and lack a reliable mathematical model. To attain such efficient control, model-free fuzzy logic controllers (FLC) based on empirical knowledge are successfully developed and implemented, and adaptation mechanisms are added to aid the FLC tuning and compensate for plant changes. However, the stability analysis - of the adaptive FLC (AFLC) systems - is a critical issue that needs addressing. For this reason, the current investigation is devoted to the development of a method for analyzing AFLC system stability using robust stability and robust performance criteria. The suggested method is employed for the stability analysis of a designed PID AFLC utilized for liquid level control in an industrial CCl. The obtained results reveal that the AFLC preserves stability and high system performance in the whole range of adaptation and considered changes of the plant and the operation conditions. Moreover, the results unveil that the developed method can also be applied for the design of a robust FLC system that competes with adaptive counterparts.

Keywords— Adaptive system; Fuzzy logic; Level control; Soda production; Stability analysis; System robustness.

Abbreviations

2ISO/SISO	- two-/single-input single-output
N, Z, P, G, S	- Negative, Zero, Positive, Great, Small
(A)FLC	- (adaptive) fuzzy logic control(ler)
CCl	- carbonization column
FU	- fuzzy unit
GA	- genetic algorithms
MF	- membership function
PDC	- parallel distributed compensation
P(I)(D)	- Proportional (Integral)(Derivative)
PLC	- programmable logic controller
TSK	- Takagi-Sugeno-Kang model
ZN	- Ziegler-Nichols model

Designations

$\Psi(e)$, μ	- nonlinear function, degree of matching
$P(s)$, $C(s)$, $S(s)$, $W_f(s)$, $W_d(s)$, $\Phi(s)$ $W_1(s)$, $W_2(s)$	- transfer functions of plant, controller, system sensitivity, disturbance filter, differentiator, closed loop system, pre- and post-processing
H, H_r, y	- level, its reference, plant output
e, de, dy	- system error, derivatives of e and of y
o, u or U	- FU output, control action
ds	- signed distance used as SISO FU input
k, T, τ	- ZN plant model gain, time constant, delay
K_d, T_d	- differentiator gain and time constant
K_e, K_{ds}	- scaling factors for e and SISO FU input ds
K_i, T_i	- integrator gain and time constant
T_f	- input exponential filter time constant
K_a, K_{a1}	- SISO and 2ISO FLC denormalisation gains
a	- relation gain $K_{a1}= a\, K_a$
r, K	- gains of sector lines that bound control curves in 2ISO control surface projection
δ	- disk diameter for violated sector bounding
D_ω, Δ	- significant frequency range, variation
$l(s)$	- plant model multiplicative uncertainty
^{o}, ^{n}	- superscripts for nominal, normalized
$_{au}$, $_{s}$, $_{o}$, $_{max}$, $_{m}$, $_{lin}$, $_{opt}$, $_{rob}$	- subscripts for augmented, stabilised, GA optimised, maximal, modified, linearized, optimal, robust

* Corresponding author

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1. INTRODUCTION

The control of the liquid level in a carbonization column (CCl) is important for the quality of the soda ash produced. It is, however, a challenging task since the plant is nonlinear, subjected to random disturbances and hence difficult to be modelled [1]. The chemical reaction between the ammonia brine solution and the carbon dioxide gases in counter flow is reversible and exothermic. Some of the resulting soda crystals stick on the cooling surface, which requires an alternation of operation mode with washing mode. The parallel operation of several columns makes the plant multivariable and sensitive to changes in the pressure in the common supply with the pre-carbonated solution. Besides, the reference for the level depends on the pre-carbonated solution produced and its distribution among the parallel CCl in operation mode.

The level is one of the most often controlled variables in industry. It is also among the first to experiment with developing a stable and robust control without plant model based on fuzzy logic and genetic algorithms (GA); thus, gaining experience in the successful implementation of intelligent techniques [2-8]. Model-free fuzzy logic controllers (FLC) for level for different applications are designed using simulations [9] and real-time control of plant models in a laboratory environment [10-13] with little regard to the impact of industrial noise and disturbance. Only a few FLC are designed and tested in a close-to-industry environment using programmable logic controllers (PLC) or low-cost microcontrollers [10, 11]. In order to consider different factors that cause changes in the plant, more sophisticated adaptive FLC (AFLC) for level are suggested [14-16]. The common problems of the AFLC include limited to simulations testing, the lack of procedures for ensuring system stability, the complicated adaptation or auto-tuning mechanism for industrial implementation and the brisk parameter changes in adaptation.

The experience accumulated helps various model-free FLC to be suggested for the control of the liquid level in the CCl in the "Solvay Sodi" in the town of Devnya, Bulgaria in [17, 18] using empirical knowledge about the control of the plant. The FLC is programmed in the existing industrial general purpose PLC after transforming the fuzzy logic into ordinary logic conditions [19]. Then the PLC-FLC is used in real-time level control in industrial CCl. The system shows an increased dynamic accuracy and a reduced control variance which leads to saving the lifetime of the expensive final control elements.

In order to more precisely reflect the plant nonlinearity and changes and to compensate for the subjectivity in the expert-based tuning of the FLC parameters, a fuzzy online auto-tuning is suggested in [20]. The AFLC is built on the principle of parallel distributed compensation (PDC) [21, 22] which assumes that the plant operates in a small number of overlapping linearization zones where it has different properties. In each zone, the corresponding FLC parameter is tuned to the best basic gain for the zone reflecting the local linear plant model. The final value for each tuning parameter is computed via the weighted average of all its basic gains depending on the degree of belonging of the current operation point to all plant linearization zones. The PDC principle is implemented by a Sugeno model designed with the measured level or its reference as input to represent the current operation point and expert-defined input membership functions (MF) to describe the linearisation zones. The Sugeno model performs a soft blending of the tuned basic gains for each FLC parameter. This adaptive approach is applied to a PID FLC using a Sugeno PD FLC and a parallel to it linear integrator of the system error developed in [20]. Two scenarios are suggested: an adaptation only of the PD FLC post-processing gain or only of the integrator gain via the soft blending of three basic values for each gain. The adaptive PID FLC is programmed in the PLC for the two scenarios, and the basic values are determined from experiments in real-time control of the level in the industrial CCl. The approach is limited to online auto-tuning of only one of the two FLC parameters because the determination of the basic gains for two or more tuning parameters via experiments by trial and error and in real-time and industrial environment is difficult, not very precise and time consuming. The PDC AFLC is based on a simple and effective auto-tuning mechanism for smooth parameter adaptation which is tested in the industry. It is also easy to be designed and implemented in the engineering practice.

The problem with the tuning of more PID AFLC basic gains used in simultaneous adaptation of both parameters can be solved objectively via GA optimisation and simulations in a similar way it is performed for a PDC tuning in [23]. The plant changes accounted for are reflected in the experimental data used. However, this approach requires the derivation and validation of a nonlinear plant model from experimental data.

The present investigation is motivated by the importance of the stability analysis of model-free adaptive FLC systems and the lack of simple means for it. The aim is to develop an approach for stability analysis of a model-free adaptive FLC system based on robust stability and robust performance criteria. The robustness related approach is intended to help the fast and broad implementation of FLC in industry for significant improvement of the system performance. Its development and testing are based on the developed model-free PID AFLC for level in a CCl for soda ash production in [20].

The investigation uses MATLAB™ and its Fuzzy Logic toolbox [24] and data from the industrial implementation of the PLC-AFLC in the real-time control of the liquid level in a CCL of the plant "Solvay Sodi" in the town of Devnya.

The paper is organized as follows. In section 2, the theoretical background of the present research based on Popov stability criterion and the idea of Morari robustness is presented. In section 3, the robust stability and robust performance criteria for a model-free AFLC system are derived from the example of the level control in a CCl. Section 4 is devoted to the development of a methodology for the design of a stable AFLC system and its application for the PID AFLC of the liquid level in a CCl. The simulation investigations of the designed stable model-free 2ISO PID adaptive FLC and its 2ISO PID robust FLC equivalent systems for level are presented in section 5 where the simulated step responses are compared. Section 6 contains the conclusion and the vision for future research.

2. THEORETICAL BACKGROUND

The stability and robustness of a nonlinear system are preconditions for its successful industrial implementation. In a wide range of operations almost all plants to be controlled exhibit nonlinear properties. A simple and reliable nonlinear plant model is difficult to be derived. Besides, the plant changes with time, with the operation mode, and as a response to the impact of the industrial environment. The nonlinear FLC also adds to the complexity of the problem. Then, an FLC should be designed to ensure the closed-loop nonlinear system stability and robustness against all model uncertainties and disturbances that accompany the industrial operation.

The nonlinear system stability can be studied using the Popov criterion for absolute stability of the equilibrium state in large, i.e. for unlimited violation of the initial equilibrium [25]. The nonlinear time-invariant system is presented as a unity feedback system which comprises a static nonlinearity bounded within a sector $0<\Psi(e)/e<K$ with input the system error e and output $o=\Psi(e)$, and a stable linear dynamic part, described by the transfer function $P_s(s)$. The nonlinear system is stable according to the Popov criterion if the modified Nyquist plot of the dynamic part $P_m(j\omega)=\text{Real}P_s(j\omega)+j\omega.\text{Imag}P_s(j\omega)$ is located below and on the right of the Popov line through the point $(-1/K, j0)$ with an arbitrary slope [25].

This stability criterion can be applied to a single-input single-output (SISO) Mamdani or Sugeno FLC system with known linear plant model $P(s)$ [26]. In this system, the SISO fuzzy unit (FU) performs a static nonlinear mapping $o=\Psi(e)$ of the input e into the output o expressed in a sector-bounded control curve. The dynamic pre-processing $W_1(s)$ and post-processing $W_2(s)$ to the FU that shape the FLC algorithm are united after the FU to make with the plant transfer function an augmented plant $P_{au}(s)=W_1(s).W_2(s).P(s)$. If the augmented plant is stable, then $P_s(s)=P_{au}(s)$ but if it is not stable, then it can be stabilised by local feedback with gain r as $P_s(s)=P_{au}(s)[1+r.P_{au}(s)]^{-1}$. The equivalent compensation of the local feedback is a parallel gain r to the FU which makes narrower the bounding sector for the location of the FU control curve, i.e. $r<\Psi(e)/e<K$ [26].

The application of the Popov criterion to the FLC system stability analysis meets two basic problems related with the nonlinear plant and with the use of two-input single-output (2ISO) FLC such as the widely spread PID-based FLC.

The nonlinear plant with an unknown plant model can be represented by an expert-defined nominal linear plant model $P^o(s)$ and a plant model multiplicative uncertainty $l(s)=\Delta P(s)/P^o(s)$, $\Delta P(s)= P(s)-P^o(s)$. Then the Popov criterion is applied to a whole family of plant models, defined by **F**:$[P^o(s), l(s)]$ which is expressed as **F**:$[P:|P(j\omega)-P^o(j\omega)|/|P^o(j\omega)|<l_m]$, $l_m=\max_\omega|l(j\omega)|$ in the significant frequency range D_ω. The nominal linear plant model $P^o(s)$ can be an approximate model for the most often used operation point. The plant model multiplicative uncertainty can be computed by accounting for the difference in the time or frequency domain characteristics of the nominal plant model and of the "worst" varied plant model with respect to the closed-loop system stability. Thus, the Popov criterion for nonlinear system stability is integrated with the criteria for robust stability and robust performance defined for a linear system [27] yielding a solution to the first problem.

According to [27], the robust stability and the robust performance of a linear system are expressed in the fulfilment of the following conditions respectively:

$$|\Phi^o(j\omega).l(j\omega)|<1, \text{for } \forall\omega\in D_\omega \quad (1)$$

$$|S^o(j\omega).W_f(j\omega)|+|\Phi^o(j\omega).l(j\omega)|<1 \ , \text{for } \forall\omega\in D_\omega, \quad (2)$$

where $\Phi^o(s)=P^o(s).C(s).[1+P^o(s).C(s)]^{-1}$ is the closed–loop system transfer function, called also complementary sensitivity, for a nominal plant model $P^o(s)$ and a linear controller $C(s)$, $S^o(s)=1-\Phi^o(s)$ is the system sensitivity and $W_f(s)$ is a disturbance shaping filter with $|W_f(j\omega)|$=0.3-0.9 for most industrial plants [27].

In case of a nonlinear FLC system the linear nominal plant model $P^o(s)$ in Eqs. (1) and (2) is substituted by the linear nominal stabilised dynamic part $P_s^o(s)$ and the linear controller

$C(s)$ is replaced by the linearized nonlinear SISO FU of the FLC $o=\Psi(e)\approx(K-r)e$ which makes a P controller with a gain $(K-r)$. So, the robust stability and robust performance conditions in Eqs. (1) and (2) are modified for the linearized FLC system in the following way respectively:

$$|\Phi_{\text{lin}}^{\text{o}}(j\omega).\,l_{\text{s}}(j\omega)| < 1, \text{for } \forall\omega\in D_{\omega} \quad (3)$$

$$|S_{\text{lin}}^{\text{o}}(j\omega).\,W_{\text{f}}(j\omega)| + |\Phi_{\text{lin}}^{\text{o}}(j\omega).\,l_{\text{s}}(j\omega)| < 1, \text{ for } \forall\omega\in D_{\omega}, \quad (4)$$

where

$$\Phi_{\text{lin}}^{\text{o}}(s) = P_{\text{s}}^{\text{o}}(s).\,(K-r).\,[1+P_{\text{s}}^{\text{o}}(s).\,(K-r)]^{-1} \quad (5)$$

is the closed-loop system transfer function for the linearized FLC and the stabilised dynamic part with nominal plant model:

$$P_{\text{s}}^{\text{o}}(s) = W_1(s).\,W_2(s).\,P^{\text{o}}(s).\,[1+r.\,W_1(s).\,W_2(s).\,P^{\text{o}}(s)]^{-1} \quad (6)$$

$S_{\text{lin}}^{\text{o}}(s) = 1-\Phi_{\text{lin}}^{\text{o}}(s)$ is the system sensitivity and $l_{\text{s}}(s) = [P_{\text{s}}(s) - P_{\text{s}}^{\text{o}}(s)]/P_{\text{s}}^{\text{o}}(s)$ is the stabilised dynamic part multiplicative model uncertainty.

According to Popov stability criterion the modified Nyquist characteristics of the stabilised dynamic part for all plant models of the defined family should be located below and on the right of the Popov line through the point $(-1/(K-r), j0)$. This ensures robust stability of the FLC system and modifies condition in Eq. (3) by replacing $P_{\text{s}}^{\text{o}}(j\omega)$ and $l_{\text{s}}(j\omega)$ with $P_{\text{m}}^{\text{o}}(j\omega)$ and $l_{\text{m}}(j\omega)$, respectively.

The PID-based 2ISO FLC are most widely used due to the clear algorithm for the derivation of the fuzzy rules. The two inputs to the FU are the normalised system error e^{n}, and the computed in the pre-processing derivative or rate either of the error de^{n} or of the smoother changing at step reference plant output dy^{n}. A 2ISO FU together with the pre-processing makes a PD FLC. With integral post-processing the FLC performs a nonlinear PI algorithm and with PI post-processing–nonlinear PID.

An approach for application of the linearization technique and the Popov-Morari criteria to the 2ISO FLC closed-loop system stability and robustness is suggested in [26] yielding a solution to the second problem. It is based on the o^{n}-e^{n} projection of the control surface which is sector bounded with an exception of a small area around the origin of the coordinate system approximated by a disk. The gain K of the upper sector line is assumed to confine an equivalent SISO FLC control curve and is used in the SISO FLC design from the criteria for system robust stability as in Eq. (3) or the stronger – robust performance as in Eq. (4). The 2ISO FLC parameters are the same as the computed for the SISO FLC with the exception of the gain in the post-processing which is $K_{\text{a1}}=a.K_{\text{a}}$. The factor a for correction of the SISO FLC gain K_{a} is computed from the requirement of desired shrinking of the disk area where the conditions for sector bounded projection are violated. Thus, the design of a 2ISO FLC from robust stability or robust performance criteria passes through the design of an equivalent SISO FLC for which these criteria are derived.

3. ROBUST STABILITY AND ROBUST PERFORMANCE CRITERIA FOR MODEL-FREE ADAPTIVE FLC

The robust stability and robust performance criteria of a model-free adaptive FLC system are derived on the basis of the developed adaptive Sugeno PID FLC in [20, 23] put in regular operation for PLC real-time control of the liquid level in a carbonization column in "Solvay Sodi"-Devnya. All AFLC system parameters are shown in Table 1.

Table 1. Parameters of PDC adaptive PID FLC system.

Parameters	From initial data		From robustness criteria	From GA optimisation
Signals	$\lvert\Delta e\rvert_{max}$=20[%]	D_ω=[10^{-4}÷5]$\left(\frac{2\pi}{T^o}\right)$ $[\frac{rad}{s}]$	-	-
FLC	Pre-processing K_d=4, T_d=30[s], K_e=0.05, K_{ds}=0.01, T_f=12[s]	FU K=6.5, δ=0.5 , δ_{max}=2, a=0.075	-	-
PDC	Ranges for Nominal K_{a1}^o=(40÷60) [%], K_a^o=(40/a÷60/a) [%], K_i^o=(1/300÷1/50)[s^{-1}]	Greatest variations ΔK_{a1}=30 [%], ΔK_a=30/a [%], ΔK_i=1/100 [s^{-1}]	K_{a1}^o=40 [%], K_i^o=1/300 [s^{-1}]	K_{a1}=[51 32 89][%], K_i=[1/34 1/374 1/179] [s^{-1}]
Plant model	Nominal k^o=0.5, T^o=150[s], τ^o=20[s]	Varied k=1, T=120[s], τ=50[s]	-	-

The block diagram of the adaptive system is presented in Fig. 1. The block diagram includes an exponential measurement noise filter $W_f(s)=(T_f s+1)^{-1}$ with time constant T_f, a 2ISO PD Sugeno FLC with denormalisation gain K_{a1} in the post-processing and a parallel to it linear integrator of the system error $e=H_r-H$ with a time constant T_i. A first order differentiator with transfer function $W_d(s)=K_d T_d s\ (T_d s+1)^{-1}$ with a time constant T_d and a gain K_d approximates the PLC module for numerical computation and smoothing of the derivative of error de. The gain K_e normalises the error and its derivative in the range [-1, 1] accounting for a maximal change of error $\lvert\Delta e\rvert_{max}$.

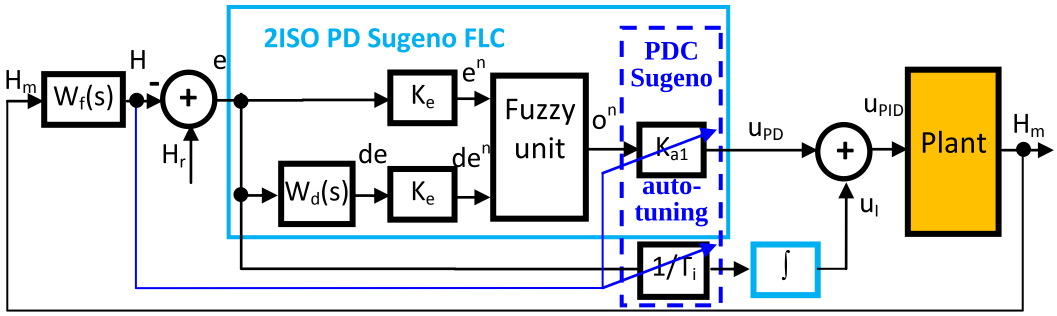
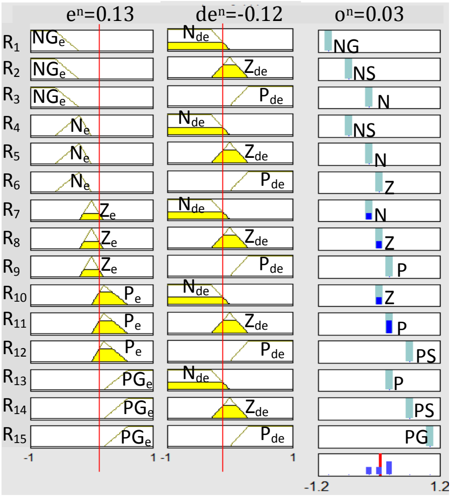


Fig. 1. Block diagram of the adaptive PID FLC.

The FU of the PD Sugeno FLC is built of standard triangle and trapezoidal on both ends orthogonal MF. In the example, five input MF are accepted for the normalised error e^n with labels Le=[NG$_e$ N$_e$ Z$_e$ P$_e$ PG$_e$] and three for de^n with labels Lde=[N$_{de}$ Z$_{de}$ P$_{de}$]. Seven output singletons labelled Lo=[NG=-1 NS=-0.6 N=-0.2 Z=0 P=0.2 PS=0.6 PG=1] ease the defuzzyfication and hence, the PLC implementation. The meanings of the labels are: "N" for negative; "P" – for positive; "S" – for small; "G" – for great; "Z" – for zero. The fuzzy rules are soft, i.e. medium (soft) terms such as NS, N, P, PS dominate over the big or great (hard) terms in the conclusion. The MF and the fuzzy rules are shown in Fig. 2.



e^n	de^n		
	N_{de} (-1 -1 -0.3 0)	Z_{de} (-0.3 0 0.3)	P_{de} (0. 0.3 1 1)
NG_e (-1 -1 -0.6 -0.2)	R_1 : NG=-1	R_2 : NS	R_3.: N
N_e (-0.6 -0.2 0)	R_4 : NS=-0.6	R_5 : N	R_6.: Z
Z_e (-0.2 0 0.2)	R_7 : N=-0.2	R_8 : Z	R_9.: P
P_e (0 0.2 0.6)	R_{10}: Z=0	R_{11}: P	R_{12}: PS
PG_e (0.2 0.6 1 1)	R_{13}: P=0.2	R_{14}: PS=0.6	R_{15}: PG=1

Fig. 2. FU MF and fuzzy rules, e.g. R_3: IF e^n is NG_e AND de^n is P_{de} THEN o^n is N.

The PDC auto-tuning is obtained using a Sugeno model with the measured level H as an input. It performs continuous smooth online simultaneous adaptation of the two tuning PID FLC parameters - the PD FLC post-processing gain K_{a1} and the integrator time constant T_i or the integrator gain $K_i=1/T_i$, as function of H, i.e. of the specific nonlinearity. The plant nonlinearity is approximated by three overlapping linearization zones described by expert-suggested input MF Low=[0 0 35 50], Norm=[35 50 65] and High=[50 65 100 100] around the main operation points $H=H_r$=[40 50 60] [%]. For each zone, a basic gain for the corresponding tuning parameter is determined either empirically [20] or by optimisation [23]. So, there are three basic gains for K_{a1} and three - for K_i. Three outputs map the corresponding degrees of matching μ_j, $j=1\div3$, of the current measured level to the MF that define the linearization zones. They are used in the soft blending of the basic gains of the zones for the two tuning parameters:

$$K_{a1}=[\sum_{j=1}^{3}\mu_j(H).K_{a1j}]/[\sum_{j=1}^{3}\mu_j(H)]\,,T_i=[\sum_{j=1}^{3}\mu_j(H).T_{ij}]/[\sum_{j=1}^{3}\mu_j(H)], \quad (7)$$

where $\sum_{j=1}^{3}\mu_j(H)=1$ for orthogonal MF.

The task is to derive a robust stability and a robust performance criterion of an adaptive FLC system based on the suggested adaptive PID FLC in [20] in order to study its stability and robustness. For that purpose, the Popov-Morari approach from [26] is applied.

First, the nonlinear plant with unknown model is represented by a family $F_P=[P^o(s), \Delta P(s)]$ of linear models defined by a nominal linear plant model $P^o(s)$ and a given plant model uncertainty $\Delta P(s)$. For the example of the liquid level control in a CCl a Ziegler-Nichols (ZN), nominal linear plant model is accepted $P^o(s)=k^o.e^{-\tau^o s}(T^o s+1)^{-1}$. The additive uncertainty is computed as $\Delta P(s)=P(s)-P^o(s)$ on the basis of the "worst" varied plant model $P(s)$ with parameters $k=k^o+\Delta k$, $T=T^o+\Delta T$, $\tau=\tau^o+\Delta\tau$ where each parameter change tends to violate the closed-loop system stability, i.e. $\Delta k>0$, $\Delta T<0$, and $\Delta\tau>0$.

In the same manner, each of the tuning parameters K_{a1} and K_i, which changes with the level H in the course of adaptation, can be represented by a family - $F_{Ka1}=(K_{a1}^o, \Delta K_{a1})$ and $F_{Ki}=(K_i^o, \Delta K_i)$, determined by corresponding nominal parameter values K_{a1}^o, K_i^o and parameter uncertainties ΔK_{a1}, ΔK_i. The parameter uncertainties are expert-assessed as the greatest possible variations during adaptation by soft blending of the local for the linearization zones basic gains (values). All variables in the PLC implementation of the PID AFLC change in the range [0, 100] [%], which helps the expert assessment of the ranges for K_{a1}^{o} and K_{i}^{o} and also of ΔK_{a1} and ΔK_{i} given in Table 1.

The controller in Fig. 1 is built of two parallel branches – a 2ISO PD FLC and a linear integrator. The control surface of the 2ISO FU of the PD FLC is shown in Fig. 3 (left). Accepting the first input e^n as a single input to the FU, the o^n-e^n projection of the control surface is computed. It comprises a number of control curves as depicted in Fig. 3 (right). Each control curve is associated with a SISO FU and is obtained for a different value for the second input de^n. All control curves in the projection are located in a sector between the abscissa and the line with gain K. The sector bounded condition is violated in a small area around the origin approximated by a disk with a diameter δ.

The gain K of the upper sector line confines all control curves through the origin. Therefore, it determines the necessary for the stability analysis virtual equivalent SISO FU.

In order to obtain the linear dynamic part of the SISO FU, first the pre-processing part of the initial 2ISO PD AFLC in Fig. 1 is transformed as depicted in Fig. 4. The 2ISO FU inputs are reduced from two to one by using the signed distance $ds = e + de$ as a single input to the equivalent SISO FU and then the new pre-processing $W_1(s)$ is moved after the SISO FU. The SISO FU in Fig. 4 (right) preserves only the error input from the 2ISO FU and the relevant five of the rules, each for a different MF. Its control curve is shown in a bold line in the o^n-e^n projection in Fig. 3 (right).

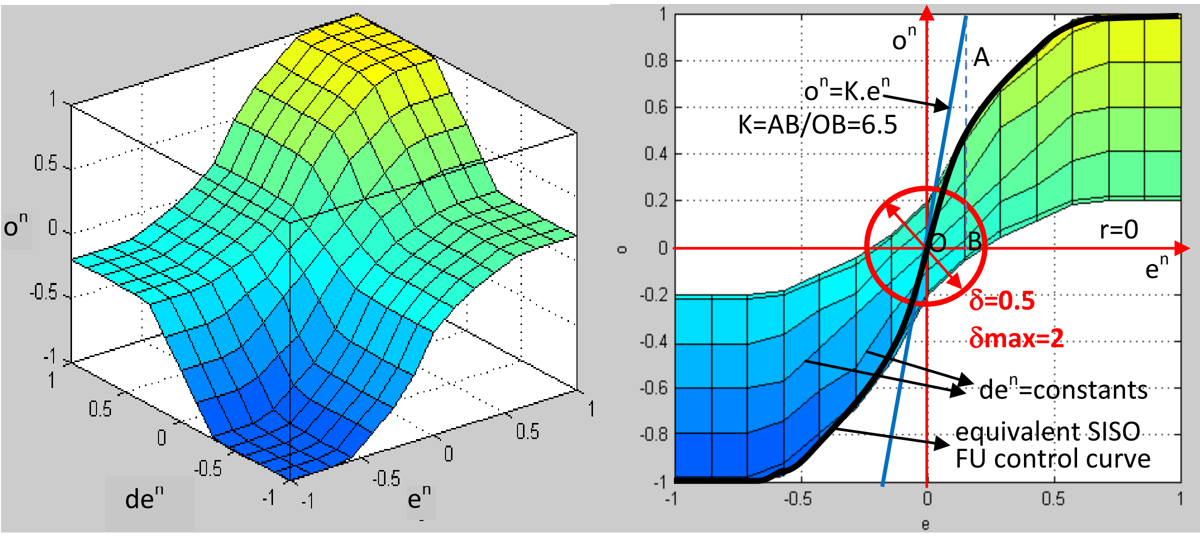


Fig. 3. 2ISO FU control surface (left) and its o^n-e^n projection (right).

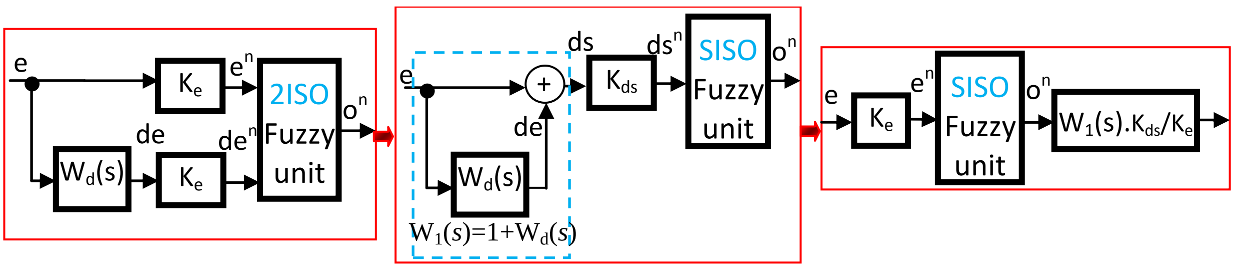


Fig. 4. PD FLC transformations from 2ISO FU to SISO FU.

The block diagram of the equivalent SISO PID FLC closed-loop system becomes as shown in Fig. 5, where the operators and the parameters that change are presented as families, each determined by the nominal operator or parameter and its “worst” with respect to system stability variation.

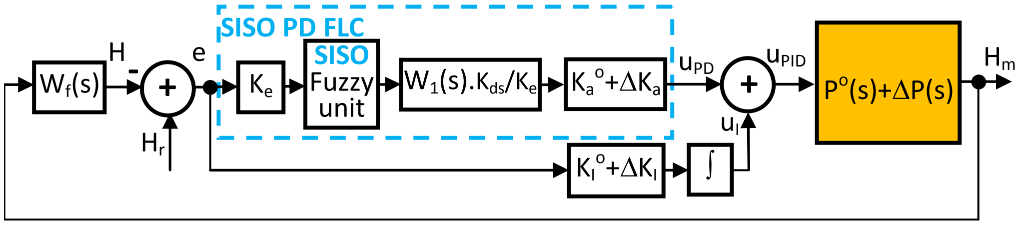


Fig. 5. Equivalent SISO PID FLC closed-loop system.

The gain K_a of the equivalent SISO PD FLC in Fig. 5 is obtained from the relationship with the gain K_{a1} of the initial 2ISO PD AFLC in Fig. 1 $K_{a1}=a.K_a$ [26] as $K_a^o=K_{a1}^o/a[\%]$ and $\Delta K_a=\Delta K_{a1}/a$ [%]. The scaling factor a is computed to shrink to acceptably small size the area in the o^n-e^n projection, where the condition for sector bounded control curves for some constant values for de is violated.

The SISO FU in Fig. 5 is linearized to become a proportional controller with gain K. Thus, the transfer function of the linearized SISO PID FLC becomes:

$$C_{lin}(s)=K.K_{ds}.K_a.[1+W_d(s)]+K_i/s. \quad (8)$$

As a result the closed-loop system with the linearized SISO PID FLC has the following transfer function for nominal plant model and adaptation parameters:

$$\Phi_{lin}^o(s) = P_s^o(s).[1+P_s^o(s)]^{-1}, \quad (9)$$

where the nominal transfer function of the open-loop linear system is:

$$P_s^o(s) = C_{lin}^o(s).P^o(s) \quad (10)$$

with $C_{lin}^o(s) = K.K_{ds}.K_a^o[1+W_d(s)]+K_i^o/s$.

The multiplicative model uncertainty is computed from $l_s(s)=[P_s(s)-P_s^o(s)]/P_s^o(s)$, where:

$$P_s(s) = C_{lin}(s).P(s) = \{K.K_{ds}.(K_a^o+\Delta K_a)[1+W_d(s)]+(K_i^o+\Delta K_i)/s\}[P^o(s)+\Delta P(s)]. \quad (11)$$

The robust stability and robust performance criteria for the SISO FLC are expressed in Eqs. (3) and (4), respectively, where $\Phi_{lin}^o(j\omega)$ is determined for the AFLC from Eq. (9), $l_s(j\omega)$ is computed for $P_s^o(s)$ from Eq. (10), and $P_s(s)$ is obtained from Eq. (11). The significant frequency range D_ω for which Eqs. (3) or (4) has to be satisfied is related with the cutting frequency of the open-loop system. The approximate assessment of D_ω is based on the time constant T^o of the nominal plant model $D_\omega\approx\left[(10^{-4},10^{-1}).\left(\frac{2\pi}{T^o}\right),(10^{-1},10^{3}).\left(\frac{2\pi}{T^o}\right)\right]$ [rad/s].

The satisfaction of the robust stability and robust performance conditions in Eqs. (3) and (4) for the FLC with linearized SISO FU for the family of plant models that represents the nonlinear plant and the expected ranges in adaptation of the FLC parameters ensures stability of the initial 2ISO AFLC.

The robust stability and robust performance conditions in Eqs. (3) and (4) can be applied to AFLC with SISO or 2ISO FU and various structure and adaptation principles accounting for the defined nominal and varied parameters and the derived in a similar way new $\Phi_{lin}^o(s)$, $l_s(s)$, $P_s^o(s)$ and $P_s(s)$ used in Eqs. (3) and (4).

4. METHODOLOGY FOR STABILITY ANALYSIS OF AFLC SYSTEM AND ITS APPLICATION FOR PID AFLC OF LIQUID LEVEL IN CARBONIZATION COLUMN

The methodology for stability analysis of an AFLC closed-loop control system and its application on the PID AFLC for liquid level in a CCl are presented in the steps below.

The input data includes:

- a block diagram of the AFLC and its parameters that do not change,
- a designed FU-type of FU (Mamdani or Sugeno), input and output MF (number, shape and parameters), fuzzy rules (standard, hard or soft, specific) and aggregation, implication and defuzzyfication methods,
- an expert-defined nominal linear plant model $P^o(s)$ and plant model uncertainty $\Delta P(s)$ that represent the nonlinear plant or expert-defined $P^o(s)$ and "worst" with respect to system stability varied linear plant model $P(s)$ from which $\Delta P(s)$ can be computed,
- FLC parameters subjected to adaptation and their nominal values and maximal variations in the course of adaptation, and,
- significant frequency range D_ω and computed vector with discrete values for ω.

The input data for the example of the 2ISO PID AFLC system for the liquid level in a CCl designed empirically in [20] or via the GA optimisation methodology in [23] are the following:

- the block diagram of the Sugeno 2ISO PID AFLC system is shown in Fig. 1 with K_d, T_d, $|\Delta e|_{max}$ and $K_{ds}=1/[|\Delta e|_{max}.(K_d+1)]$ systemised in Table 1;
- the model-free 2ISO FU of the 2ISO PID AFLC for level is designed as a Sugeno model with MF and fuzzy rules depicted in Fig. 2. The aggregation of the fuzzy rules sub-conditions with AND connective is with MIN operator, the defuzzyfication is by "Weighted Average";
- the nonlinear plant is represented by a ZN model $P(s)=k.e^{-\tau s}(Ts+1)^{-1}$ – the nominal plant model has parameters (k^o, T^o, τ^o) and the "worst" with respect to system stability varied plant model has parameters (k, T, τ), all shown in Table 1;
- the adaptation parameters are K_{a1} and K_i and they accept nominal values and "worst" (highest) variations in the course of adaptation given in Table 1.

1. Transformation from a 2ISO AFLC to an equivalent SISO FLC when necessary.
   1.1. Reduction of the FLC FU from 2ISO to SISO:
   - Computation of the 2ISO FU control surface and its projection output-main input.
   - Assessment of K that determines the equivalent SISO FU.

The FU control surface and its o^n-e^n projection for the 2ISO PID AFLC for level are shown in Fig. 3 with a sector bounded control curve of a virtual equivalent SISO FU with K shown in Table 1.

1.2. Transformation of the pre-processing part of the 2ISO FU of the AFLC to a post-processing of the equivalent SISO FU.

The transformations of the pre-processing of the 2ISO FU of the PID FLC for level to a post-processing of the equivalent SISO FU is depicted in the block diagrams in Fig. 4.

1.3. Assessment of δ and δ_{max} from the projection of the control surface and computation of the scaling factor a=0.1 (1-δ/δ_{max}), of $K_a^o = K_{a1}^o/a$ and $\Delta K_a = \Delta K_{a1}/a$ of the equivalent SISO FLC.

The assessed δ, δ_{max} and computed a from the control surface projection of the 2ISO FU in level control in Fig. 3 (left) can be seen in Table 1.

2. Preparation of the SISO FLC system for the application of the robust stability in Eq. (3) and the robust performance in Eq. (4) conditions.

2.1. Linearization of the SISO FU and derivation of the transfer function of the linearized controller $C_{lin}(s)$.

The linearization model of the SISO FU is $o^n=Ke^n$ and the transfer function of the PID controller for level obtained after linearization of the SISO FU is expressed in Eq. (8).

2.2. Derivation of the open-loop system transfer function $P_s^o(s)$ for a nominal controller $C_{lin}^o(s)$ based on a linearized SISO FU and a nominal plant model $P^o(s)$.

$P_s^o(s)$ is obtained in Eq. (10) for a nominal $C_{lin}^o(s)$ with linearized SISO FU from Eq. (8) and a nominal plant model $P^o(s)$.

2.3. Derivation of the transfer function of the varied open-loop linearized system $P_s(s)$ based on varied adaptation parameters of the FLC and varied linear plant model.

The varied $P_s(s)$ for the linearized SISO FU is represented in Eq. (11). The impact of the plant model uncertainty and the AFLC parameter changes on the simulated step responses and the Nyquist plots of the open-loop dynamic system with PID for linearized SISO FU is shown in Fig. 6. The selection of the nominal value K_i^o has a small impact on the step response which is felt near the settling time. The variations of the plant model and AFLC adaptation parameters cause a significant increase of the step response and an appearance of a peak. The Nyquist plot expands and approaches the Nyquist critical stability point (-1, j0).

2.4. Computation of the multiplicative uncertainty $|l_s(j\omega)| = |P_s(j\omega) - P_s^o(j\omega)|/|P_s^o(j\omega)|$.

3. Check for fulfilment of the selected criterion in Eqs. (3) or (4) for all values of $\omega \in D_\omega$.

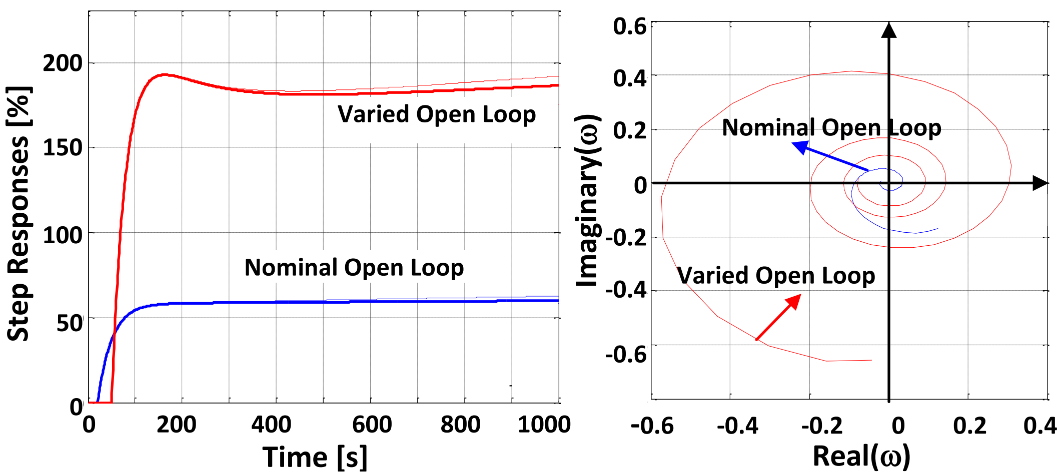


Fig. 6. Step responses (left) and Nyquist plot (right) of open-loop dynamic system with linearized PID FLC for nominal (K_{a1}^o=60[%], T_i^o=100[s], T_i^o=300[s] - thick line) and varied plant and adaptation parameters.

The observation of the conditions in Eqs. (3) or (4) ensures robustness of the linearized SISO FLC system, i.e. the system characteristics for the accepted plant model uncertainty and ranges of changes of the FLC parameters in the course of adaptation remain within a narrow envelope around their characteristics for nominal plant and adaptation parameters.

The satisfaction of the robust stability or the robust performance requirement for the linearized SISO FLC system ensures stability of the initial nonlinear 2ISO AFLC system. A MATLAB™ program is developed to check the fulfilment of both conditions in Eqs. (3) and (4). In case the selected robustness criterion is not satisfied, the procedure is repeated for changed initial data that determine a newly designed AFLC.

The graphs of the computed left side expressions in Eqs. (3) and (4) together with the magnitude characteristics of the multiplicative model uncertainty $|l_s(j\omega)|$ from Eq. (11) and of the nominal closed-loop system with the linearized SISO PID FLC $|\Phi^{o}_{\text{lin}}(j\omega)|$ from Eq. (5) are shown in Fig. 7 for different nominal values of the adaptation parameters. The robust stability and the robust performance conditions according to Eqs. (3) and (4), respectively are satisfied when the graphs of the left side expressions are below 1, i.e. below the unit line, for most frequencies of the defined significant frequencies range D_ω.

The following conclusions can be deduced from Fig. 7:

a) The adaptation gains that satisfy Eqs. (3) and (4) for a broader frequency range can be accepted for optimal ($K^{o}_{a1opt}[\%]$, $K^{o}_{iopt}[s^{-1}]$) and are shown in Table 1.
b) The optimal nominal values for the two subjected to adaptation tuning parameters are close to the middle local basic gains of the PDC adaptation mechanism of the AFLC, i.e. they correspond to the linearization zone accepted as "Norm" as seen from Table 1.
c) The increase of T_i^o makes the AFLC system robust in a wider frequency range. The greater T_i^o also slows down the nominal system step responses. So, a compromise between the robustness required and the performance of the nominal system is recommended.
d) For low frequencies when the magnitude of the closed-loop system $|\Phi_{\text{lin}}|$ is high the robust stability and robust performance curves have high peaks over the unit line, i.e. the system is not robust and is sensitive to disturbances and parameter changes.

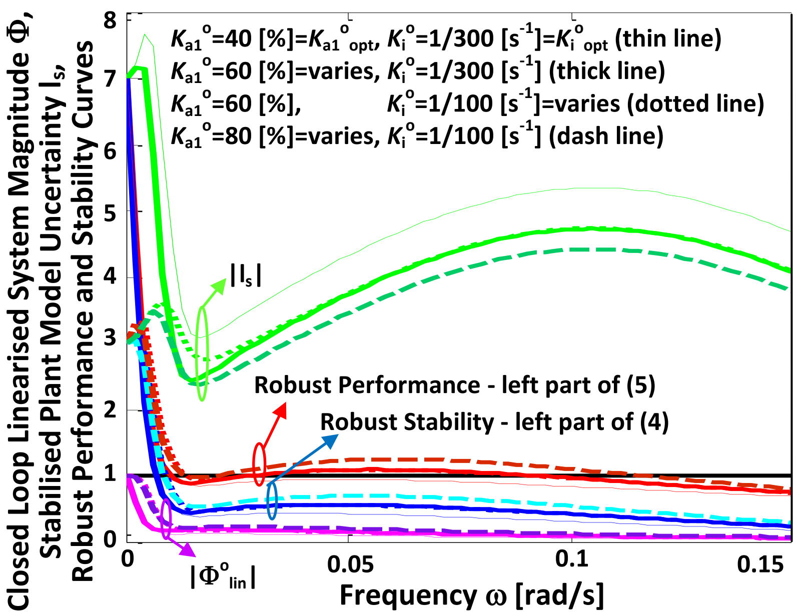


Fig. 7. Robust stability and robust performance analysis of PID AFLC system for different nominal parameters.

The flow chart of the methodology for AFLC system stability study is shown in Fig. 8.

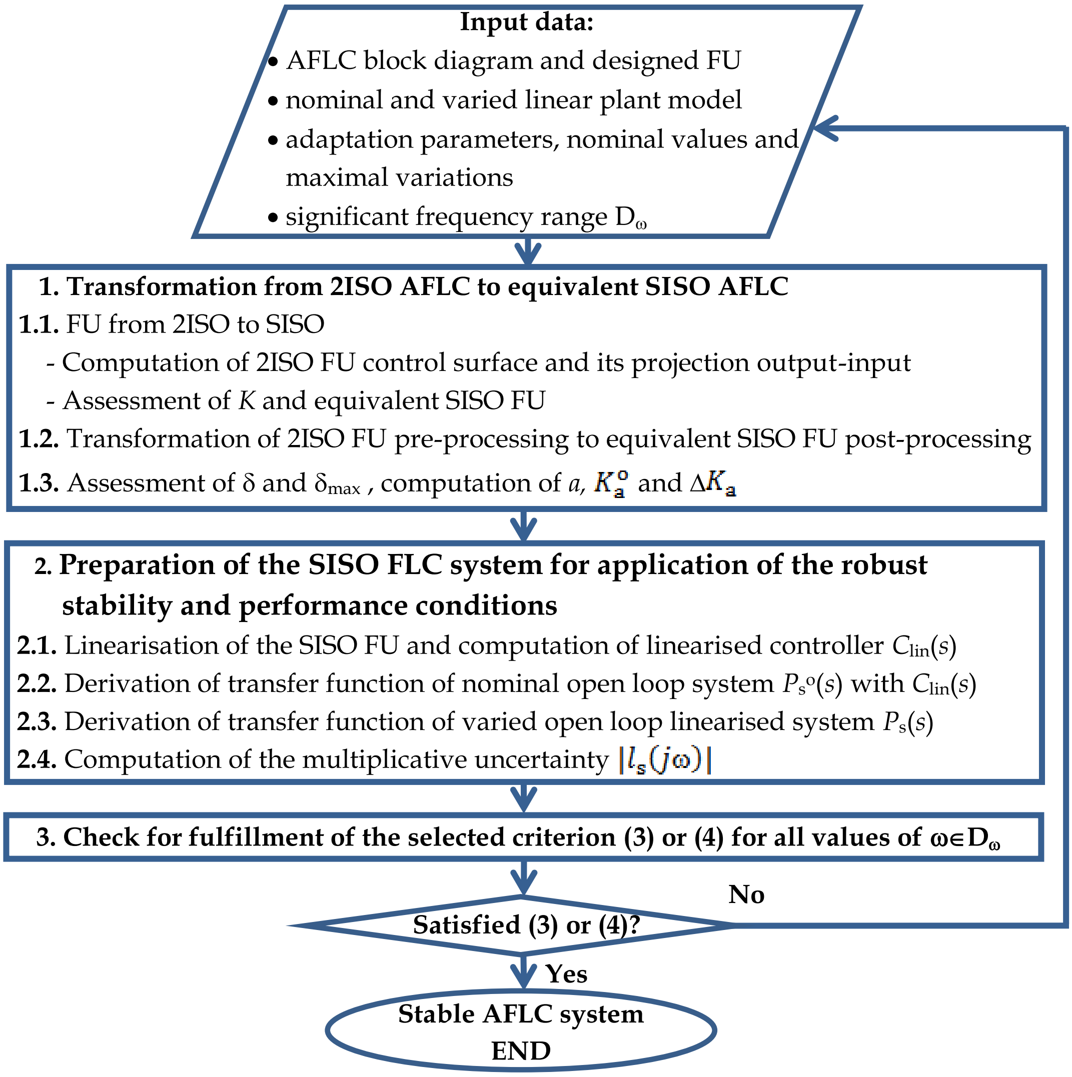


Fig. 8. Flow chart of the methodology for stability analysis of AFLC systems.

5. SIMULATION INVESTIGATIONS OF AFLC AND ROBUST SISO FLC SYSTEM FOR LIQUID LEVEL CONTROL IN CARBONIZATION COLUMN

The simulations are based on a derived and validated TSK plant model using experimental data and off-line GA parameter optimisation in [23]. It is built on the same Sugeno model used in the AFLC adaptation mechanism for recognition of the degree of matching of the current operation point, assessed by the measured level, to the defined by the MF three linearization zones. The nonlinear TSK plant model output is computed by soft blending of the outputs of the parallel operating local for each zone linear plant models, described by second order time-lags.

In order to reflect the impact of the system nonlinearity the reference for the level of the closed-loop systems changes stepwise from various operation points in the sequence H_r=50-60-50-40-50 [%]. The step responses of the investigated systems with respect to level $H(t)$ and control action $U(t)$ are simulated and compared. The adaptation of the two tuning parameters of the AFLC system is also simulated.

The designed 2ISO PID AFLC system stability is studied based on the derived robust stability and the robust performance conditions in Eqs. (3) and (4) for an equivalent SISO PID FLC system with constant nominal tuning parameters and linearized SISO FU. The nonlinear plant is represented by an expert-assessed nominal linear plant model and plant model uncertainty. The adaptation of the tuning parameters is reflected as variations from their nominal values. The nominal tuning parameters for which the condition in Eqs. (3) or (4) are satisfied for the equivalent SISO PID FLC, ensure also stability of the initial 2ISO PID AFLC system.

The simulation experiments pursue the following goals:

1. To study the effect of adaptation against the effect of robustness by comparing the simulated step responses, presented in Fig. 9 (left), of two 2ISO PID FLC systems:
   - System 1 (adaptive) with K_d=7/3 and a PDC adaptation mechanism with GA optimised basic gains K_{a1} and K_i according to [23] shown in Table 1. The step responses are denoted by subscript "o";
   - System 2 (robust) with K_d, K_{a1}^o and K_i^o from Table 1 that satisfy the robust performance condition in Eq. (4). The step responses are denoted by subscript "rob".

   The step responses with respect to H are close which means that the robust and the adaptive systems have close performance indicators. With respect to the control action U the AFLC system displays a smaller control span but a greater variation. So, the robust FLC system as simpler can equivalently substitute the AFLC system.
2. To verify the assumption of equivalency of the robust 2ISO PID FLC, System 2 with and the robust PID system with linearized SISO FU – System 3 with K_a^o=K_{a1}^o/a, [%] for which the robust stability and robust performance criteria in Eqs. (3) and (4) are derived. For that purpose the simulated step responses, presented in Fig. 9 (right), are compared. The step responses of System 3 are denoted by the subscript "lin".

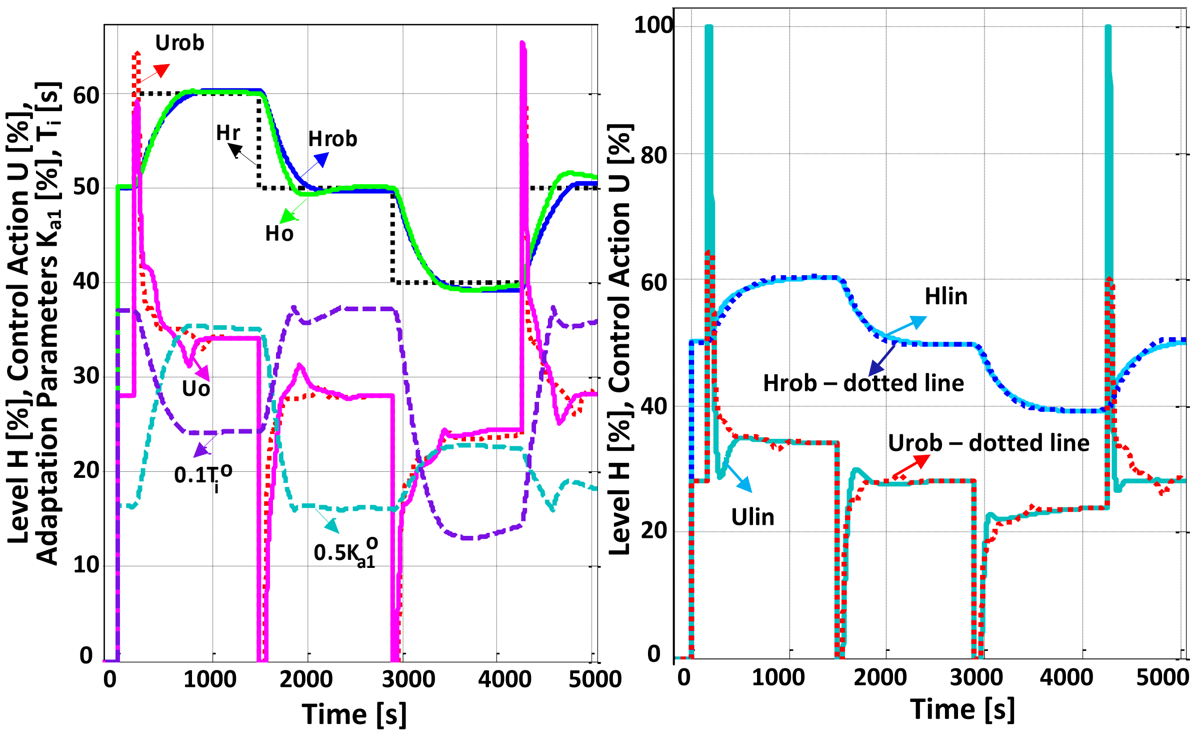


Fig. 9. Step responses of 2ISO PID FLC systems: adaptive GA optimised Ho, Uo; robust Hrob, Urob and equivalent SISO PID with linearized FLC – Hlin, Ulin.

The equivalency between the two systems is proven as the step responses in Fig. 9 (right) of the two systems are nearly identical. The span of *U* for two of the step responses of System 2 is smaller.

The simulation experiments 1 and 2 show that the satisfaction of the derived condition in Eqs. (3) or (4) by a PID system with linearized SISO FU is a proof for robust stability or robust performance of the 2ISO PID FLC and respectively of the initial adaptive 2ISO PID FLC.

The derived robust stability and robust performance conditions for an AFLC can be also used for the design of robust FLC systems.

6. CONCLUSIONS AND FUTURE WORK

In this paper, an approach for stability analysis is developed for adaptive FLC systems using a modification of the integrated Popov stability and Morari robustness criteria. This is based on derivation of robust stability and robust performance conditions for an equivalent linear system obtained from the AFLC system after a series of transformations. First, the nonlinear plant with unknown model is represented by an expert-assessed nominal plant model and model uncertainty and the FLC parameter changes in the course of adaptation - by nominal parameters and variations. Then, the 2ISO FU is reduced to SISO FU and the system block diagram is modified correspondingly to a unit feedback system of the SISO FU as a static nonlinear controller with sector bounded characteristic and a stable linear dynamic post-processing. Finally, the SISO FU is linearized.

A methodology for the application of the approach is suggested and applied to the stability analysis of the system with 2ISO PID AFLC for the control of the liquid level in a carbonization column for soda ash production in the town of Devnya, Bulgaria.

Simulation experiments show close step responses of the robust equivalent linear system, the robust 2ISO PID FLC and the 2ISO PID AFLC. This proves that the system transformations on which the approach lies lead to equivalent systems. Besides, it is possible to successfully substitute an adaptive FLC system by a simpler properly designed robust FLC system which preserves the system performance for the accepted changes of the plant and the AFLC parameters instead of compensating the plant changes by adaptation of the AFLC parameters.

The approach for stability analysis of a PID AFLC system with simultaneous PDC adaptation of the two parameters enables the estimation of the ranges of the parameters adaptation that ensure system stability. It is developed for SISO and 2ISO AFLC systems. The methodology can be also applied to other AFLC systems with various adaptation principles or for the design of competitive robust FLC systems. The adaptive PID FLC system for liquid level control in an industrial CCl will preserve stability and high performance for the whole range of adaptation and considered changes of the plant and the operation conditions.

The future work will focus on application of the robust 2ISO PID FLC for real-time PLC level control in industrial CCl and comparison with the designed PLC-2ISO PID AFLC.

REFERENCES

[1] C. Thieme, *Sodium Carbonates. - In: Ulmann's Encyclopedia of Industrial Chemistry*, Weinheim, Germany: Wiley-VCH Verlag GmbH & Co. KGaA, 2012.

[2] S. Ahmad, S. Ali, R. Tabasha, "The design and implementation of a fuzzy gain-scheduled PID controller for the Festo MPS PA compact workstation liquid level control," *Engineering Science and Technology*, vol. 23, pp. 307-315, 2020.

[3] N. Kanagasabai, N. Jaya, "Fuzzy gain scheduling of PID controller for a MIMO process," *International Journal of Computer Applications*, vol. 91, no. 10, pp. 13-20, 2014.

[4] S. Rajendran, S. Palani, "Design and implementation of fuzzy logic controller for dynamic system," *International Journal of Mechanical and Mechatronics Engineering*, vol. 14, no. 4, pp. 7-12, 2014.

[5] H. Shahid, S. Murawwat, I. Ahmed, S. Naseer, R. Fiaz, A. Afzaal, S. Rafiq, "Design of a fuzzy logic based controller for fluid level application," *World Journal of Engineering and Technology*, no. 4, pp. 469-476, 2016.

[6] S. Yordanova, "Intelligent approaches to real-time level control," *International Journal of Intelligent Systems Technologies and Applications*, vol. 7, no. 10, pp. 19-27, 2015.

[7] M. Fazzolari, R. Alcala, Y. Nojima, H. Ishibuchi, F. Herrera, "A review of the application of multiobjective evolutionary fuzzy systems: current status and further directions," *IEEE Transactions on Fuzzy Systems*, vol. 21, no. 1, pp. 45-65, 2013.

[8] B. Ataslar-Ayyildiz, O. Karahan, "Design of a MAGLEV system with PID based fuzzy control using CS algorithm," *Cybernetics and Information Technologies*, vol. 20, no. 5, pp. 5-19, 2020.

[9] H. Mousa, M. Koutb, S. El-Araby, H. Elsayed, "Design of optimal fuzzy controller for water level of U-tube steam generator in nuclear power station," *American Journal of Science*, vol. 7, no. 4, pp. 629–637, 2011.

[10] Z. Aydogmus, "A real-time robust fuzzy-based level control using programmable logic controller," *Elektronika ir Elektrotechnika*, vol. 21, no. 1, pp. 13-17, 2015.

[11] F. Chabni, R. Taleb, A. Benbouali, M. Bouthiba, "The application of fuzzy control in water tank level using Arduino," *International Journal of Advanced Computer Science and Applications*, vol. 7, no. 4, pp. 261-265, 2016.

[12] A. Shome, D. Ashok, "Fuzzy logic approach for boiler temperature and water level control," *International Journal of Scientific and Engineering Research*, vol. 3, no. 6, pp. 1-6, 2012.

[13] W. Tan, "Water level control for a nuclear steam generator," *Nuclear Engineering and Design*, vol. 241, no. 5, pp. 1873-188, 2011.

[14] S. Mahapatro, B. Subudhi, S. Ghosh, "Adaptive fuzzy PI controller design for coupled tank system: An experimental validation," *IFAC Proceedings Volumes*, Kampur, India, pp. 878-881, 2014.

[15] A. Venkataraman, "Design and implementation of adaptive PID and adaptive fuzzy controllers for a level process station," *Advances in Technology Innovation*, vol. 6, no. 2, pp. 90-105, 2021.

[16] A. Wincy Pon Annal, S. Kanthalakshmi, "An adaptive PID control algorithm for nonlinear process with uncertain dynamics," *International Journal of Automation and Control,* vol. 11, no. 3, pp. 262-273, 2017.

[17] S. Yordanova, B. Gueorguiev, M. Slavov, "Design and industrial implementation of fuzzy logic control of level in soda production," *Engineering Science and Technology, an International Journal*, vol. 23, no. 3, pp. 691-699, 2020.

[18] S. Yordanova, M. Slavov, G. Prokopiev, "Disturbance compensation in fuzzy logic control of level in carbonization column for soda production," *WSEAS Transactions on Systems and Control,* vol. 15, no. 8, pp. 64-72, 2020.

[19] Honeywell International Inc., *Experion Overview,* Release 300.1 May 5, 2006.

[20] M. Slavov, "Design and investigation of adaptive fuzzy level control system for carbonization column," *Proceedings of the Technical University of Sofia*, vol. 72, no. 2, pp. 66-72, 2022.

[21] G. Feng, *Analysis and Synthesis of Fuzzy Control Systems: A Model Based Approach*, Bosa Roca, US: CRC Press, Taylor & Francis, 2017.

[22] K. Tanaka, H. Wang, *Fuzzy Control Systems Design and Analysis: A Linear Matrix Inequality Approach*, New York: John Wiley and Sons, 2001.

[23] S. Yordanova, M. Slavov, B. Gueorguiev, "Optimisation of parallel distributed compensation for real-time control of level in carbonization column," *Automatic Control and Computer Sciences, Scientific Journal*, vol. 54, no. 5, pp. 379-390, 2020.

[24] The MathWorks Inc., *Fuzzy Logic Toolbox: User's Guide for Use with MATLAB*, Natick, MA: The MathWorks Inc., 1998.

[25] H. Khalil, *Nonlinear Systems*, Second Edition, New Jersey: Englewood Cliffs, Prentice-Hall, 1996.

[26] S. Yordanova, *Methods for Design of Fuzzy Logic Controllers for Robust Process Control*, Sofia: King, 2011.

[27] M. Morari, B. Zafiriou, *Robust Process Control*, New Jersey: Prentice Hall, 1989.