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A Hybrid-Analytical-Model for the Paraboloidal Reflector Antenna

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Abstract *–* In this paper, a computationally efficient method of moments (MoM)-physical optics (PO) formulation of the general paraboloidal reflector antenna problem is developed. The formulation hybridizes the integral equation approach characteristic of the conventional 'low-frequency' methods and the asymptotic solution of the wave-object interaction problem of PO. This formulation is based on an adaptation of an approach prescribed by Kerdemilidis - for a parametric description of the antenna's geometry - leading to a generalization in a relatively simplified format of an ordinarily analytically involved process. The obtained results show a good agreement with the reported results due to other approaches and, consequently, prove the validity of the proposed model.

Keywords *—* Hybrid-analytical model; Physical optics; Method-of-Moments; Paraboloidal reflector antenna.

1. INTRODUCTION

Evidence available from the open literature suggests that one of the earliest analytical investigations of the paraboloid reflector problem is that reported by Darbord [1] who - using geometrical optics theory or the "aperture method"- deduced expressions for the field reflected by the aperture. He did not, however, consider the antenna's radiation characteristics. As noted by Kerdemelidis [2] - although Morita [3] later utilized the same approach to compute the radiated fields - his use of the field at the surface of the aperture has since been shown to be incorrect. The contributions by Jones [4, 5], whose formulation has its basis in physical optics theory, are regarded as representing one of the earliest that evaluated both the radiating fields as well as cross-polarization features of the paraboloidal antenna. Jones focus was on a circular paraboloid with a diameter of 39.7λ, at the operating frequency, with the objective of elucidating the reflector's radiation and cross polarization characteristics through analysis. As may be expected of a pioneering contribution, the analysis was facilitated by a number of simplifying assumptions as well as certain approximations. The more prominent in this case is the use of polynomial approximations for the solution of the far-zone diffraction integrals. Several years later, Kerdemelidis [2] - using an approach similar to that developed by Jones - introduced an operator formulation to simplify the electric field expressions, derived from Maxwell's equations. He subsequently developed the far zone scattered electric field equations for the paraboloid with circular aperture in terms of dipole integrals. These integral equations were then solved through the use of series of approximations to provide certain illuminating details about the reflector's performance characteristics. Quite a few subsequent analytical investigations of the reflector antenna [6-8], utilized - essentially - the scattered field expression similar to that in [4] with

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various approaches to the treatment of the integrands representing the key difference between their contributions. As pointed out by Scott [9], rigor of the formulation is reduced - through this procedure - as an edge line integral effect is avoided at the expense of accuracy. In addition, the formulation absolutely ignores the contributions of near and Fresnel-zone integral equations [10].

It soon became apparent, as pointed out by Algar et al. [11], that the foregoing and a few other efforts, firmly established the fact that the physical optics (PO) radiation integrals provided reasonably accurate solutions to the far-zone field problem of the paraboloidal reflector. Because the purely PO approach unfortunately suffered from a few fundamental problems (including in particular, the fact that it takes current distribution as a given [2, 12]), alternative approaches capable of providing better results have emerged. These methods are generally classifiable as either the "aperture-field integral" or the "surface current integration" approaches, with which main lobe and side lobes characteristics are predicted. The former class includes the hybrid geometric theory of diffraction (GTD) and geometric optics (GO) methods, which predict singularities in the transition region adjacent to the GO incident and reflected fields' shadow boundaries. Variations of this approach including the unified asymptotic theory (UAT), unified theory of diffraction (UTD), and the equivalent current methods (ECM) - which provided significant improvements - did not completely resolve the problem, [8].

One possibility for addressing these issues is offered by the following considerations. First, it is noted that the surface current integration techniques (including physical optics) proceed by determining the scattered fields due to the illumination of the radiator's surface by some source fields. So, the problem is essentially that of determining the surface current distribution. Next, as noted in [13], the method of moments (MoM) is able to provide acceptably accurate current distributions but it is not well suited for use with electrically large structures like the high frequency methods. Therefore, the indication is that some hybrid of these two approaches can resolve the problems associated with the aperture-field integral and surface current integration methods. Thus, it is not surprising to find that a number of such hybrid methods, designed to combine the advantages of the MoM and the high frequency techniques, have been developed. In case of the hybrid MoM-PO techniques [13-26], the fundamental idea is to have PO extend the capabilities of the MoM to a larger class of problems than the electrically small structures that is most suited to treat. As remarked by Dong-Ho et al. [24], conventional hybrid methods routinely ignore higher order mutual interactions between subsystems because the structure includes a sub-reflector and a main reflector in addition to the feed. The authors of [24] consequently introduced the use of the following three different hybrid methods in an iterative scheme as a mean of accounting for the mutual interactions: i) finite-element method (FEM)/MoM + PO; ii) FEM/MoM + GTD and iii) FEM/MoM + PO + physical theory of diffraction (PTD). One important advantage claimed by that iterative scheme is that computational cost does not depend on either focal length or main reflector size. It is noteworthy that the MoM-PO method, utilized by Moumen and Ligthart [25], is able to account for blockage and mutual interactions due to struts and supports in the structure.

The hybridization is not exactly straightforward, i.e., the set of basis functions suitable for use with MoM is usually unsuitable when utilized for PO. For example, Taboada reported that the popular Rao-Wilton-Glisson (RWG) basis functions are inappropriate for PO because they tend to introduce fictitious cross-polar surface currents on account of an orthogonality requirement [23]. The authors negotiated this problem by utilizing a piecewise linear interpolation of a phase term to obtain an analytically suitable expression for the associated radiated field. In similar approach, Nazo [17] added a linear phase term to the RWG basis functions to enable the use of "large triangular mesh elements" in the PO region of the problem, and consequently enforcing the boundary condition $\left(\hat{a}_n \times \bar{H} = \bar{J}_s\right)$across the MoM-PO boundary. An interesting variation of the hybrid MoM-PO approach is described by Mei et al. [14], whose iterative time-domain method begins with a determination of MoM currents for the PO region. The fields excited by this current distribution are then utilized for the determination of current distribution for the MoM region to give a new PO current distribution. The fields excited by the updated PO current distribution, in turn, update the MoM current distribution and so on until a convergence is obtained. Given the fact that the MoM-PO method's usefulness essentially depends on the ability to prescribe the near and Fresnel zone integrals towards a rigorous treatment of total electric and magnetic fields, this presentation explores the possibility provided, in that connection, by Hansen and Bailin [10] in which integrals of interest to this rigorous treatment, are available. Also, a related development concerning the Fresnel zone fields of the reflector antenna is described by Galindo-Israel and Rahmat-Samii [6], where expressions were developed for the exact integral forms for the regions. Maclean [27] utilized what he referred to as "exact Maxwellian integrals" to evaluate both near and far zone re-radiated fields - in Cartesian coordinates - of a circular paraboloid. Rahmat-Samii [7] extended these concepts to the analysis of reflector antennas with elliptical apertures and involving the PO current whereas the paper's main objective was to compare the performances of the Jacobi-Bessel algorithm and an algorithm that uses the Jacobi-Bessel expansion over a circumscribed circular region. The interesting aspect, from his paper's point of view, concerns with the fact that the integration variables in the field expressions are defined in the projected aperture coordinates. By doing so, it became possible to derive the radiation field expressions valid for all regions. With the use of the "induced current method', Kerdemelidis [2] whose focus was on the solution of the reflector antenna problem to prescribe its polarization characteristics in a manner that avoids the limitations imposed by the approximations of GO and PO, introduced the use of paraboloidal coordinate variables for specifying both unit normal on the paraboloidal surface and element of surface area. Hence, he removed the limitation common to both PO and GO, of being unable to account for the influence of reflector antenna's curvature on surface current distribution. In this paper, a hybrid PO-MoM approach is developed, using an adaptation of the PO approach introduced by Kerdemelidis, [2], such that the approximate current available from the MoM treatment becomes applicable in the PO region.

2. PROBLEM FORMULATION

When the elliptic paraboloidal reflector antenna - whose geometry is described by Fig. 1 -is illuminated by an incident electric field, a scattered field exists at the aperture's surface such that the total electric field, which is the sum of the incident and scattered electric fields, has a tangential component that vanishes on the presumably perfectly conducting surface.

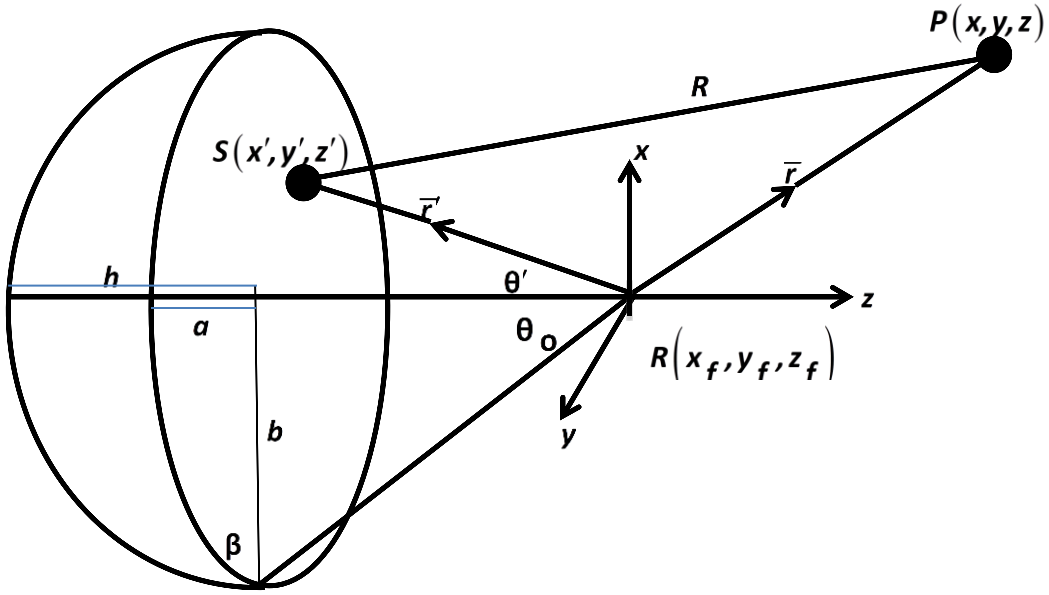

Fig. 1. Geometry of the elliptic paraboloidal antenna.

This boundary condition on the tangential component of the total field admits expression as:

$$\hat{a}_n \times \bar{E}_T = 0 \tag{1}$$

that is,

$$\bar{E}_i^{\tan} = -\bar{E}_s^{\tan} \tag{1a}$$

where $\hat{a}_n$ represents the outward unit normal vector, the subscripts T, i, and s stand for total, incident, and scattered, respectively. The superscript "tan"on the incident and scattered electric fields denotes that they are tangential field vectors to the aperture's surface. It should be remarked in this paper that the elliptical geometry of Fig. 1 reduces to a circular paraboloidal geometry when a is equal to b (that is eccentricity is zero). Following the approach described by Kerdemelidis [2], the space of the reflector can be divided into two regions: the "antenna region" which comprises the feed and paraboloidal reflector, and the region "external to the imaginary surface enclosing the antenna region" where both regions are connected through the surface. Hence, the problem of determining the paraboloid's electromagnetic response is then divided into two parts: i) determining the fields in the antenna region and ii) determining the fields in the region external to the surface. Maxwell's equations, governing the mutual coupling of the fields in these regions, are provided in [2].

As clearly formulated in [2], the "interior problem" (concerning a region internal to an imaginary bounding surface, separating the reflector from free space) may be resolved independent of the external problem; and the obtained solution extends to the external region through an application of the boundary conditions on the reflector's surface.

Using this approach, and for the "interior problem", the electromagnetic fields admit description in terms of vector's electric $\left(\bar{A}^e\right)$ and magnetic $\left(\bar{A}\right)$ potentials, according to [2, 28]:

$$\bar{E} = -j\omega\left[\bar{A} + \frac{1}{k^2}\nabla\left(\nabla\cdot\bar{A}\right)\right] - \frac{1}{\varepsilon}\left[\nabla\times\bar{A}^e\right]$$
$$\bar{H} = \frac{1}{\mu}\left[\nabla\times\bar{A}\right] - j\omega\left[\bar{A}^e + \frac{1}{k^2}\nabla\left(\nabla\cdot\bar{A}^e\right)\right] \quad (2)$$

provided that,

$$\bar{A} = \mu\iiint_{V'} \bar{J}\left(\bar{r}'\right)G\left(\left|\bar{r}-\bar{r}'\right|\right)dV'$$
$$\bar{A}^e = \varepsilon\iiint_{V'} \bar{J}^m\left(\bar{r}'\right)G\left(\left|\bar{r}-\bar{r}'\right|\right)dV' \quad (3)$$

with,

$$G\left(\left|\bar{r}-\bar{r}'\right|\right) = \frac{e^{-jk\left|\bar{r}-\bar{r}'\right|}}{4\pi\left|\bar{r}-\bar{r}'\right|} \quad (3a)$$

where μ and ε are the permeability and permittivity of the medium, and $\left|\bar{r}-\bar{r}'\right|$ stands for the distance between the source and field points. While $j=\sqrt{-1}$ represents the imaginary number, $k=\omega\sqrt{\mu\varepsilon}$ stands for the free space propagation constant. $\bar{J}\left(\bar{r}'\right)$ denotes the electric current distribution over the reflector's aperture and $\bar{J}^m\left(\bar{r}'\right)$ stands for magnetic current distribution over the aperture.

Following the application of the conventional "magnitude and phase approximations" [28], the radiation field becomes characterized by the electric field intensity vector given as [2]:

$$\bar{E} = \frac{j\omega\mu e^{-jkr}}{4\pi r}\hat{a}_r\times\left(\hat{a}_r\times\iint_{S'}\bar{J}_s\left(\bar{r}'\right)e^{jk\hat{a}_r\cdot\bar{r}'}dS'\right) \quad \textit{on the conducting surface,}$$
$$\textit{or} \quad (4)$$
$$\bar{E} = \frac{j\omega\mu e^{-jkr}}{4\pi r}\hat{a}_r\times\left(\iint_{S'}\left[\hat{a}_r\times\bar{J}\left(\bar{r}'\right)+\frac{1}{\eta}\bar{J}^m\left(\bar{r}'\right)\right]e^{jk\hat{a}_r\cdot\bar{r}'}dS'\right) \quad \textit{elsewhere}$$

whereas Kerdemelidis' approach may be described as purely 'physical optics', the proposal here is to hybridize the approach with MoM.

The starting point in the proposed solution is Kerdemilidis' approximation which prescribes the distribution of current on the perfectly conducting surface of the reflector according to:

$$\bar{J}_s = 2\left(\hat{a}_n\times\bar{H}\right) \quad (5)$$

After defining a radially directed unit vector (here denoted by $\hat{a}_r$) from the origin to a point on the reflector's surface, it was noted [2] (towards an approximate solution) that the electric and magnetic field intensities of the field incident on the surface of the reflector are related according to:

$$\bar{H}_i = \frac{1}{\eta}\left(\hat{a}_r\times\bar{E}_i\right) \quad (6)$$

and with the use of Eq. (6), Eq. (5) modifies to:

$$\bar{J}_s = \frac{2}{\eta}\left(\hat{a}_n \times \left(\hat{a}_r \times \bar{E}_i\right)\right) \tag{7}$$

so that by a vector identity, we have:

$$\bar{J}_s = \frac{2}{\eta}\left[\left(\hat{a}_n \cdot \bar{E}_i\right)\hat{a}_r - \left(\hat{a}_r \cdot \hat{a}_n\right)\bar{E}_i\right] \tag{8}$$

In this paper, imposing the boundary condition for the normal component of electric field intensity on the perfectly conducting reflector surface (which carries no charge density distribution), Eq. (8) reduces to:

$$\bar{J}_s = \frac{2\cos\left(\theta'/2\right)}{\eta}\bar{E}_i \tag{9}$$

since as noted in [2],

$$\hat{a}_r \cdot \hat{a}_n = -\cos\left(\theta'/2\right) \tag{9a}$$

With Eq. (9) which essentially prescribes the PO current in terms of the MoM current, we can then adapt Taylor's formulation [20] to suggest that:

$$\bar{E}_i^{\tan} = \left[\frac{j\omega\mu e^{-jkr}}{4\pi r}\hat{a}_r \times \left(\iint_{S'}\left[\hat{a}_r \times \bar{J}\left(\bar{r}'\right) + \frac{1}{\eta}\bar{J}^m\left(\bar{r}\right)\right]e^{jk\hat{a}_r\cdot\bar{r}'}dS'\right)\right]^{\tan} \tag{10}$$

which is an expression of the boundary condition on the total field over the reflector surface. In this region, the total field includes contributions from three sources: the field produced by the unknown currents, the field produced by the (known) PO current, and the incident field which is also a known quantity. The proposed hybrid MoM-PO approach consequently determines the desired (unknown) current distribution by suggesting that,

$$\bar{J} = \begin{cases} \bar{J}(\bar{r}') = \sum_n \alpha_n \bar{f}_n(\bar{r}'), \bar{r}' \in \text{internal region} \\ \bar{J}^m(\bar{r}') = \dfrac{2\cos(\theta'/2)}{\eta}\bar{E}_i, \bar{r}' \in \text{external region} \end{cases} \tag{11}$$

It then becomes possible, as remarked by Taylor [20] to develop a matrix equation in terms of the generalized network functions of the MoM [29] and in which the unknown current is separated from the known quantities according to the following expression:

$$\left\langle w_m, L\left(\bar{J}\right)\right\rangle = \left\langle w_m, \bar{E}_i\right\rangle - \left\langle w_m, L\left(\bar{J}^m\right)\right\rangle \tag{12}$$

In the foregoing inner product $\left(\langle\cdot,\cdot\rangle\right)$ expressions, α_n, w_m, *and* $\bar{f}_n$ represent unknown expansion coefficients, weighting functions, and known basis functions, respectively. We may expand the physical optics current $\left(\bar{J}^m\right)$ using the same basis functions as the unknown currents according to:

$$\bar{J}^{PO}\left(\bar{r}'\right) = \sum_{k=1}^{M}\alpha_k^{PO}\,\bar{f}_k\left(\bar{r}'\right) \tag{13}$$

in which as explained in [20], the α_k^{PO} represent the amplitudes of the physical optics currents. Accordingly, Eq. (12) then leads to the system of equations written in matrix notation as:

$$\sum_{n=1}^{N-M} Z_{mn}\alpha_n = V_m - \sum_{k=N-M+1}^{N} Z_{mk}\alpha_k^{PO} \tag{14}$$

with Eqs. (12) and (13), the problem essentially reduces to prescribing a suitable set of expansion coefficients (current amplitudes) for the PO currents and thus, the paper adapts a procedure elucidated in [2] to facilitate this prescription in a systematic manner.

First, the 'amplitudes of the physical optics current' can be written as [2, 22]:

$$\alpha_k^{PO} = \frac{2}{\eta}\cos\frac{\theta'}{2}\begin{cases}\hat{a}_{p_k}^{+}\cdot\bar{E}_i(\bar{r}'), & \bar{r}'\in T_k^{+}\\ \hat{a}_{p_k}^{-}\cdot\bar{E}_i(\bar{r}'), & \bar{r}'\in T_k^{-}\end{cases} \tag{15}$$

where the unit vectors symbolized by $\hat{a}_{p_k}^{\pm}$ are directed parallel to the edge 'k' and given by:

$$\hat{a}_{p_k}^{\pm} = \hat{a}_n^{\pm}\times\hat{a}_u^{\pm} \tag{15a}$$

provided that $\hat{a}_n^{\pm}$ and $\hat{a}_u^{\pm}$ represent normal to the triangular patch and the edge k, respectively. So, the cross product lies in the plane of the triangular facets, $T_k^{\pm}$.

The associated RWG [22] function, as illustrated in Fig. 2, is then given as:

$$\bar{f}_n(\bar{r}') = \begin{cases}\frac{l_n}{2A_n^{+}}\left(\bar{r}'-\bar{v}_n'^{+}\right), & \bar{r}'\in T_n^{+}\\ -\frac{l_n}{2A_n^{-}}\left(\bar{r}'-\bar{v}_n'^{-}\right), & \bar{r}'\in T_n^{-}\\ 0, & otherwise\end{cases} \tag{16}$$

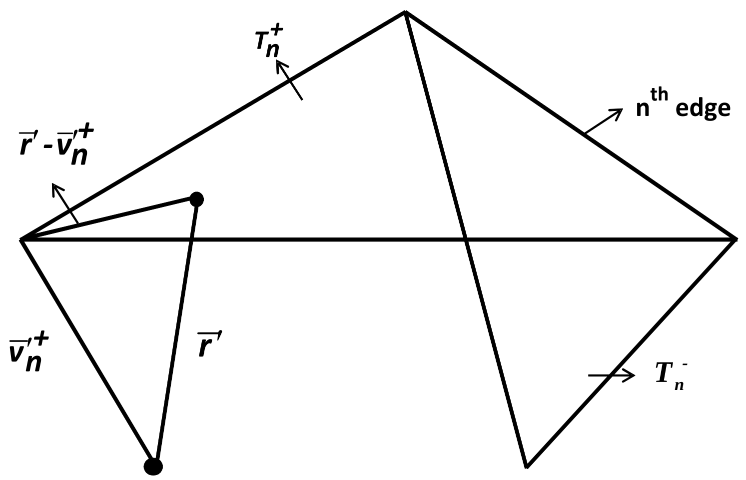

Fig. 2. Faceted 3-D scatterer with details of patch coordinates used in roof-top surface current expansion functions [22].

so that Eq. (12) modifies to:

$$\sum_{n=1}^{k}\alpha_n\left\langle w_m, \Im_1\left[\bar{f}_n(\bar{r}')\right]\right\rangle = \left\langle w_m, \bar{E}_i^{\tan}\right\rangle - \left\langle w_m, \Im_2\left[\bar{J}^{PO}(\bar{r}')\right]\right\rangle \tag{17}$$

where,

$$w_m = \bar{f}_m(\bar{r}) = \begin{cases} \dfrac{l_n}{2A_m^+}\left(\bar{r}-\bar{v}_m^+\right), & \bar{r}\in T_m^+ \\ -\dfrac{l_n}{2A_m^-}\left(\bar{r}-\bar{v}_m^-\right), & \bar{r}\in T_m^- \\ 0 & , \ otherwise \end{cases} \tag{17a}$$

$$\Im_1[*] = \frac{j\omega\mu e^{-jkr}}{4\pi r}\hat{a}_r \times \left(\hat{a}_r \times L^{MoM}(*)\right) \tag{17b}$$

and

$$\Im_2[*] = \frac{j\omega\mu e^{-jkr}}{4\pi r}\hat{a}_r \times L^{PO}(*) \tag{17c}$$

From Eqs. (17b) and (17c), the radiation integral admits the following representation:

$$L^{MoM}(*) = L^{PO}(*) = \iint_{S'} e^{jk\hat{a}_r\cdot\bar{r}'}[*]\,dS' \tag{18}$$

where,

$$\bar{r}' = \left(r'\sin\theta'\cos\varphi'\right)\hat{a}_x + \left(r'\sin\theta'\sin\varphi'\right)\hat{a}_y - r'\cos\theta'\hat{a}_z \tag{18a}$$

$$r' = f\sec^2\frac{\theta'}{2} \tag{18b}$$

$$\hat{a}_r = \sin\theta\cos\varphi\hat{a}_x + \sin\theta\sin\varphi\hat{a}_y + \cos\theta\hat{a}_z \tag{18c}$$

$$dS' = \left|\frac{d\bar{r}'}{d\theta'}\times\frac{d\bar{r}'}{d\varphi'}\right| \tag{18d}$$

3. COMPUTATIONAL RESULTS AND DISCUSSIONS

Computational results, obtained with the use of the analytical solution presented in the foregoing section, are presented and discussed in this section. The results serve the main purpose of validating the correctness and efficacy of the model through comparisons with published measurement results as well as simulation data provided by authors who utilized different approaches.

3.1. Model's Performance Evaluation

A comparative evaluation of computational results for radiation field patterns of a paraboloidal antenna, due to the model in this presentation, is undertaken in the following discussions, starting with published data available from Lashab et al. [30], as shown in the illustrations of Figs. 3 and 4.

3.1.1. Comparison of Numerical Results of E-plane Radiation Patterns with Published Data

The illustrations of Fig. 3 display superimpositions of E-plane radiation patterns for co-polarization (Co-Pol) generated with the use of wavelet-based-MoM, GRASP software, as well as the PO in [30] and the computational results obtained with the paper's model. These results concern a dipole-fed reflector antenna whose aperture diameter is given as 100 λ, focal length to aperture diameter ratio $f/D = 0.6$, and incident angle φ = 90º. As can be seen from the graphs, there are good agreements between the results due to the model developed here and the results reported in Fig. 4 of [30] for the dipole fed reflector, whose dimensions are as earlier described.

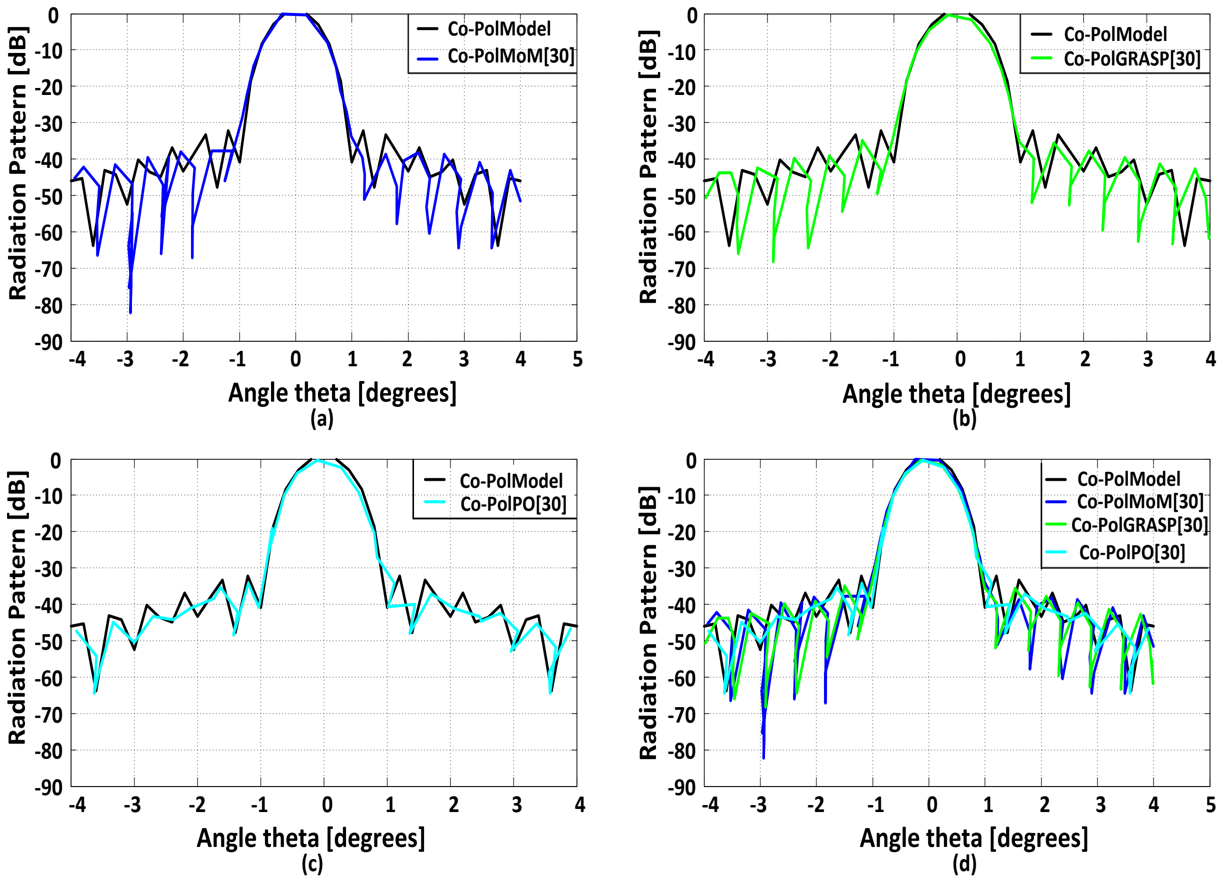

Fig. 3. Computational results for E-plane pattern of a paraboloidal reflector of the proposed model and the models of [30]: a) Co-Pol models; b) Co-Pol models using GRASP [30]; c) Co-Pol models using PO; d) Co-Pol models using MoM, GRASP, PO.

3.1.2. Further Comparison of Computational Results with Published Data

The illustrations of Fig. 4 are also for E-plane radiation patterns, but this time, concerning both co-polarization (Co-Pol) and cross-polarization (XPol), and with reference to Fig. 6 of [30]. In this case, the dipole-fed reflector antenna parameters are specified as aperture diameter = 20 λ, focal length to aperture diameter ratio $f/D = 0.8$, and incident angle φ = 45º. The comparison in Fig. 4(a) is between the model developed in this paper, and the wavelet-based MoM described in [30] and Fig. 4(b) is between the model developed in this paper and a pure Physical Optics approach described in [30]. In the case of Fig. 4(c), the comparison includes E-plane patterns due to the wavelet-based MoM and a pure PO approach as reported in [30]. It is clear from the graphs that the performance of the hybrid model developed in this work compares well with the wavelet-based MoM of [30], whose comparison with the PO approach has been discussed by Lashab and his associates [30].

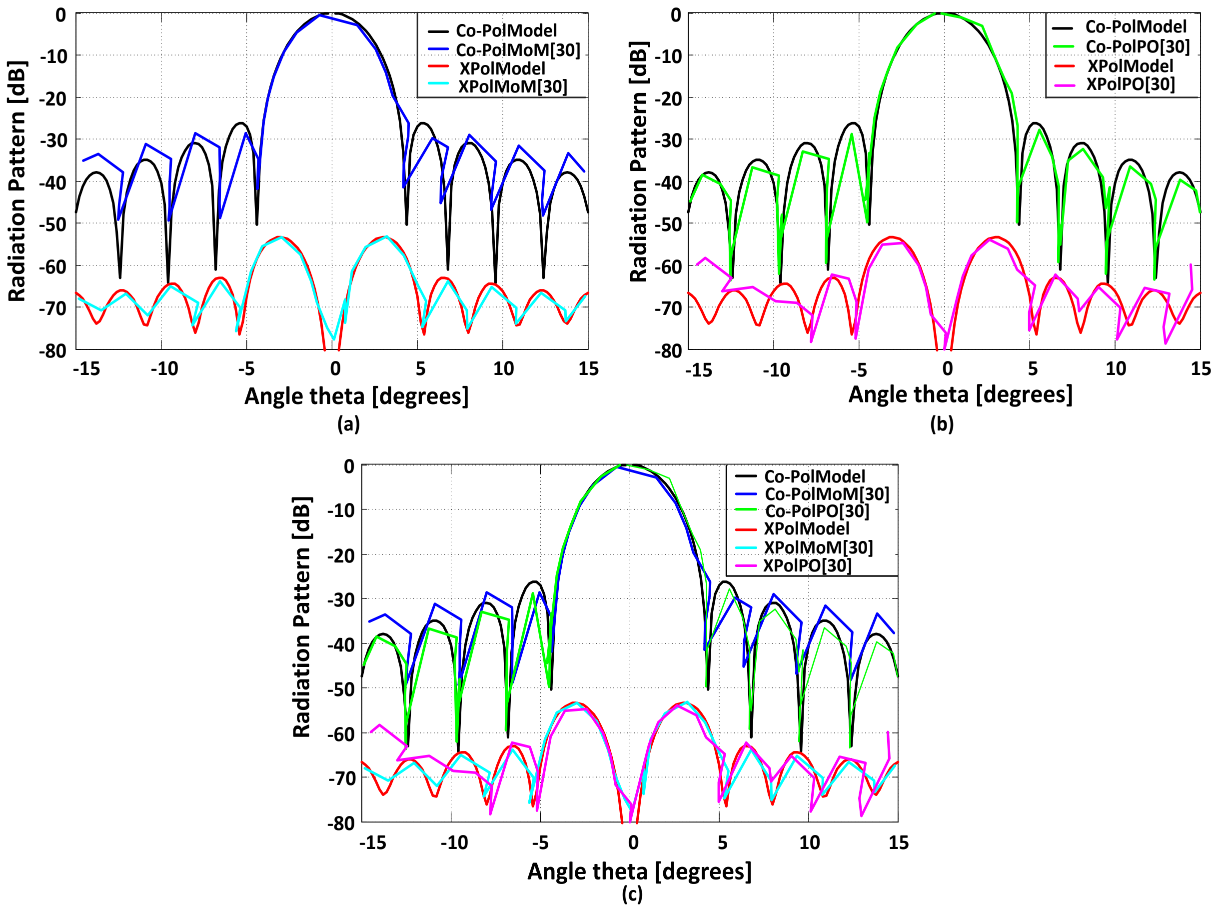

Fig. 4. E-plane radiation pattern obtained using the proposed model and the models demonstrated in [30]: a) paper's Co-Pol model vs Co-Pol model using MoM in [30] and paper's XPol model vs XPol model using MoM in [30]; b) paper's Co-Pol model vs Co-Pol model using PO in [30] and paper's XPol model vs XPol model using PO in [30]; c) Co-Pol and XPol models vs Co-Pol and XPol models using MoM and PO in [30].

A further evaluation of the hybrid model's performance is provided by the graphs of Fig. 5 in which antenna directivity characteristics for a 1-m diameter, horn-fed paraboloidal antenna operated at a frequency of 10 GHz, are compared. For this antenna, whose focus is said by Algar et al. [11] to be located in Cartesian coordinates at (0, 0, 0.4), the characteristics of the theta-component of directivity in the phi = 0° plane were obtained with the use of a combination of GTD and "master points method" [11], and compared with good agreements with results available from MoM approach.

It should be remarked that although no value was specified for reflector's f/D ratio by the authors of [11], the choice of 0.9 utilized in this paper for the computational results, proved suitable as suggested in Fig. 5. It is readily observed from the curves of Fig. 5 that the results match in most parts, the difference - in significant part – may be due to the differences in the f/D ratio utilized in this paper and that actually utilized but not stated by Algar et.al [11]. The CPU simulation time of 43 minutes for this paper's model compares favourably with those recorded in [11], which according to authors are: 7 hours and 14 minutes and 7 hours and 47 minutes for MoM and GTD-Master points, respectively.

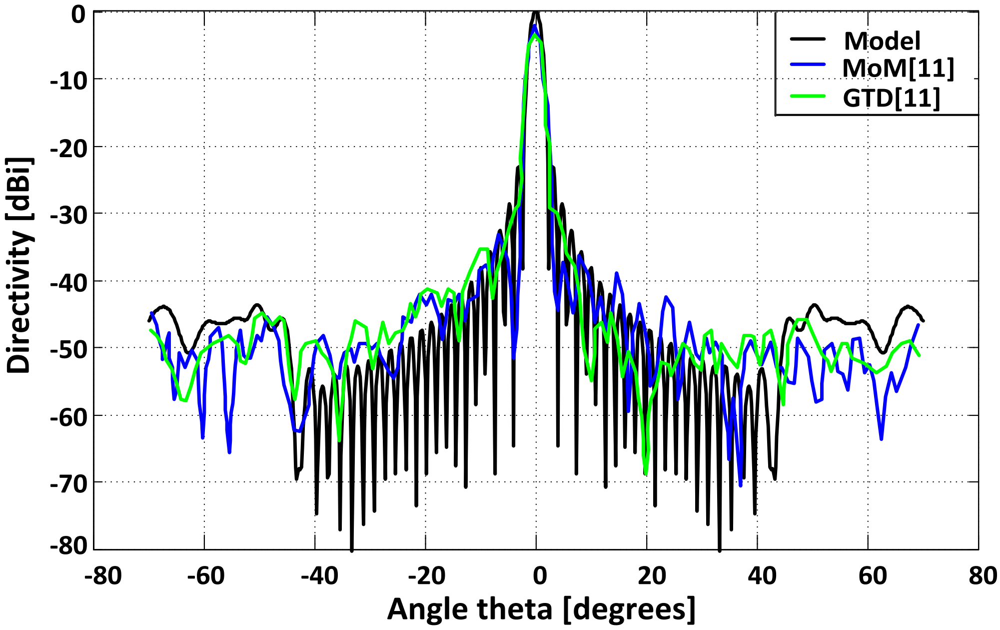

Fig. 5. Antenna directivity obtained with the proposed hybrid method and that reported in [11].

Figs. 6-8 graphically compare the paper's model performance against some published measurement results. In the case of Fig. 6, the comparison is with a set of measured and computational PO results described by Aubry et al. [31] for the on-axis "TARA antenna" operated at a center frequency of 3.297 GHz. An account for the blockage effects due to the struts and supporting feeds was provided in the computational results for this paper's model using the model described by Moumen and Ligthart [25].

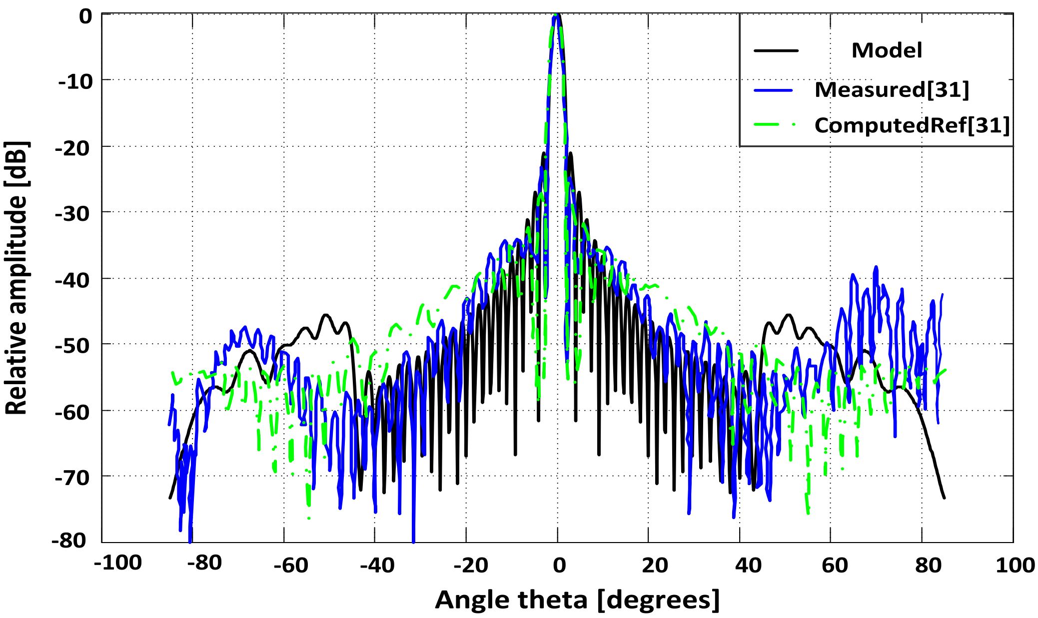

Fig. 6. Pattern of on-axis paraboloidal antenna with blockage effects.

The illustrations of Fig. 7 display comparisons of the computational results with the published results available in [26] for the far-zone field of the "TARA antenna". Aperture blockage in this case is minimal and accounted for using the method described in [26].

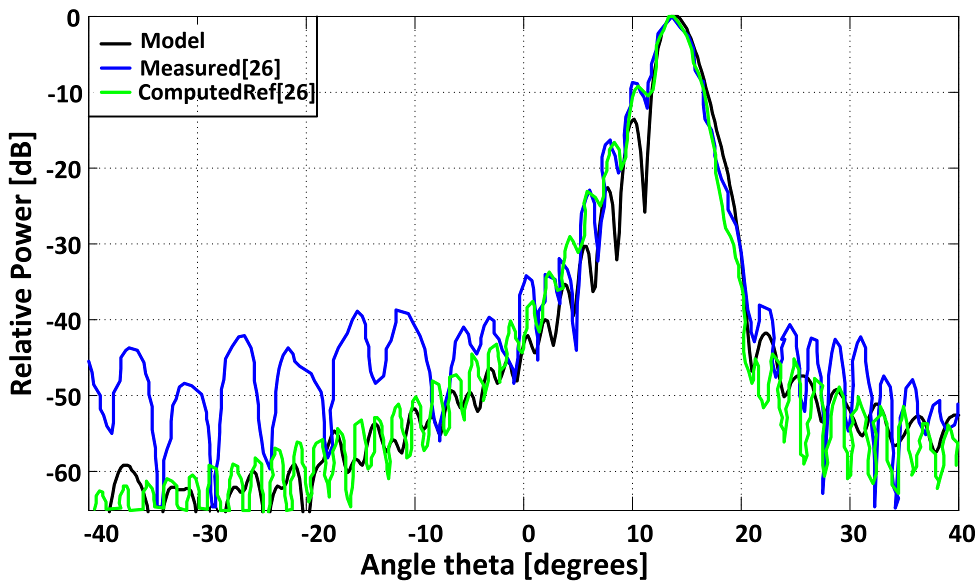

Fig. 7. Pattern of offset paraboloidal antenna without blockage effects.

Fig. 8 displays comparisons of the computational results due to the model of this paper with measurement and MoM-PO computational results published by Moumen and Ligthart [25]. The good agreements between the computational and measurement results in all three cases support the validation of the model.

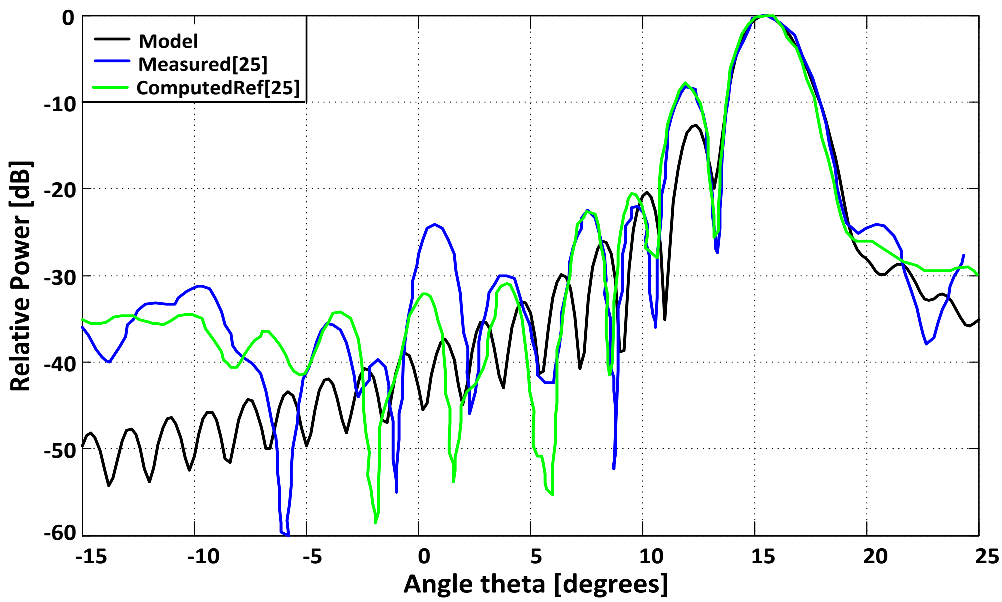

Fig. 8. Pattern of offset paraboloidal antenna with blockage effects.

4. CONCLUSIONS

A hybrid method of MoM-PO model has been developed in this paper, and utilized for determining the radiation zone field pattern characteristics of a general paraboloidal reflector antenna. Using a procedure suggested by Kerdemilidis in [2] for a purely PO model, the paper successfully modified the model due to Taylor formulation [20] such that the basis and weighting functions utilized for the MoM region become applicable for use in the PO region of the hybrid model. Computational results using the developed model have been shown -

through graphical comparisons - to be in very good agreement with the reported results using other models, suggesting that the model developed in this paper is valid.

It should be remarked that the paper's model did not consider antenna feed modeling and plane-wave excitation typical of MoM applications was utilized in all cases. One conclusion due to this observation is that improved results may be available if the model is modified to include feed modeling. This is the subject of an on-going investigation with special focus on the feed array structure (including a miniaturized dielectric rod antenna as array reflector element) described by Moumen in [26].

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