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Stable Backstepping Sliding Mode Control Based on ANFIS2 for a Class of Nonlinear Systems

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Abstract— This paper presents an intelligent backstepping sliding mode control for a class of nonlinear systems. A new Adaptive Neuro Fuzzy Inference System (ANFIS), based on type-2 fuzzy sets (called ANFIS2) is used to approximate the conventional sliding mode control law. The proposed ANFIS2 method does not require prior information about the system; it also identifies the system's dynamics, as well as the estimated dynamics, used in the sliding mode controller. Moreover, the proposed ANFIS2 sliding mode control system - by tracking the control system's structure in the presence of uncertainty in a class of nonlinear systems - approximates the system's mathematical model momentarily. In order to compensate the control signal and to offer a better performance, a combination of a type-2 fuzzy system, backstepping method and sliding mode control is proposed. The backstepping method is used to improve the final threshold stability; and the sliding mode control is used to obtain robust response to uncertainty. The simulation results show that the proposed ANFIS2-based sliding mode control has better performance than the ANFIS-based one.

Keywords— Sliding mode control; Adaptive neuro fuzzy inference system; ANFIS2; Backstepping control; Stability analysis.

1. INTRODUCTION

One of today's challenges in control engineering is the change in system's parameters over time and during work. These changes are often so high that it may lead to instability and system's out of control. Therefore, it should either have a precise model with constant system's parameters or it must be able to monitor the system's model with a precise tool at any given moment. The first mode is not possible because the systems are becoming more complex and having a precise model with constant parameters is not possible. For the second mode, a powerful and precise tool for modelling systems is the type-2 fuzzy logic.

In the past decade, type-2 fuzzy logic was studied with more capability and more flexibility than type-1 fuzzy logic [1, 2]. In [3], the details of type-2 fuzzy logic have been discussed. In [4], different structures of type-2 fuzzy neural network in addition to new method for fuzzification and type reduction have been proposed. Type-2 fuzzy system has attracted much attention [5-11]. In [5], fuzzy clustering is used to update the antecedent parameters and gradient descent is used to update the consequent parameters. In [9], an integrated mechanism for discarding derogatory features and extraction of fuzzy rules based on an interval type-2 neural fuzzy system has been proposed. It has an evolving ability to efficiently identify the required network structure without requiring a basic initial structure for launching [9]. To learn the parameters in the network, the previous parameters and modulation parameters are taught to learn backward-player playback, and Kalman's filtered algorithm helps to fine-tune network accuracy by setting the parameters of the sequence section. In [12], a new method to reduce the order of type-2 fuzzy systems has been proposed. In [13], a new fault detection method for Takagi-Sugeno (TSK) fuzzy systems has

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been presented. Using interval type-2 fuzzy based model to control nonlinear networked control systems with packet dropouts and parameter uncertainties has been studied in [14]. In [15], the feedback and sliding mode control for type-2 fuzzy systems with heterogeneous membership functions have been studied. In [16-20], research works has been reported on type-2 fuzzy logic and its application.

Parameter learning is very important in soft computing techniques. Some of these methods for training type-2 fuzzy neural network are genetic algorithm [21] and Particle Swarm Optimization (PSO) [22]. Some applications of type-2 fuzzy systems are: time-series prediction [23], control of DC motor [24], equalization of nonlinear time-varying channels [25], sliding mode control [26], pattern recognition [27] and robot control [28]. A novel intelligent type reduction is presented in [29]. In [30], a type-2 fuzzy neural system was learned through its type-1 fuzzy counterpart through the integration and extension of type-1 fuzzy membership function. Then, a type-2 fuzzy neural system on a programmable gate chip was implemented. Two times learning and non-optimization of fuzzy rules are the drawbacks of this work.

Clearly, sliding mode control provides a powerful tool for controlling a nonlinear dynamic system with uncertainty. But, an alternating switch control that is used to resolve uncertainty often leads to chattering phenomenon on control input. Larger uncertainty increases the amplitude of the switching signal. In general, there is a lack of strength between the sections. To reduce chattering in sliding mode, several studies that incorporate combination of sliding mode control and fuzzy logic have been conducted [31-33]. In [34], a new sliding mode tracking control using a fuzzy logic system for a class of non-linear Multi-input Multi-output (MIMO) systems is proposed. In order to guarantee closed loop stability and timing of tracking errors, while estimating the dynamics of the controlled plant, the investigators combined a non-continuous sliding mode technique and a fuzzy logic system with an adaptive learning algorithm. In [35], a new algorithm prove to be feasible to create a fuzzy sliding controller that can easily provide stability and tracking problems for a class of non-linear MIMO systems in the presence of uncertainty and external disturbances. In this method, the fuzzy system is used to approximate the unknown functions in polynomial model. In [36], an adaptive fuzzy output feedback control tracking problem for a class of non-linear multivariate multi-valued multiplexing and undesired MIMO systems in the pure feedback form is considered. Various methods for controlling uncertain systems are reported [37-43]. They use fuzzy logic system to identify the uncertain nonlinear system and obtain a smooth function for the saturation approximation of the input and also a fuzzy state observer that estimates the non-measurable states. In [44], a feedback control method based on fuzzy adaptive output for a class of non-linear MIMO systems with non-measured states has been presented. It combines backstepping technique and fuzzy adaptive output feedback control to ensure the limitation for all signals and trace errors to a small neighborhood of origin. Some new works on fuzzy backstepping control for non-linear MIMO systems are reported [45, 46]. In [47], the combination of type-2 fuzzy logic with sliding mode control is used to match the resonance frequency and state separation from a micro-electro-mechanical systems (MEMS) z-axis gyroscope.

This paper presents backstepping sliding mode control based on type-2 fuzzy sets (ANFIS2), proposed by us in a previous work for a class of nonlinear systems [10]. The proposed method uses stable gradient descent with adaptive learning rate backpropagation for parameter learning phase in ANFIS2. This paper presents a novel combination of backstepping method and sliding mode control that helps compensating the control signal and getting a better performance. The backstepping method is used to improve the final threshold stability and uses the slider control to obtain high and unchangeable response to uncertainty. In this paper, we use the combination of backstepping and sliding mode control in a novel type-2 fuzzy system (ANFIS2). The fuzzy weights used in the consequent part of the proposed ANFIS2 are one of the novelties of the paper. The main contributions of this paper are as follows:

- Utilizing a novel type-2 fuzzy system (ANFIS2).
- Presenting a novel design of type-2 fuzzy sliding mode control that guarantees a fast convergence of the tracking error to zero.
- Utilizing type-2 fuzzy to eliminate the chattering effectively without losing the precision, applied to MEMS gyroscope.

2. TYPE-2 FUZZY LOGIC AND SYSTEMS

Ten years after presenting type-1 fuzzy logic by Zadeh, he introduced type-2 fuzzy logic for resolving some problems of type-1 fuzzy logic. Zadeh considered a fuzzy set, the membership function of which was a fuzzy set. He named it type 2 fuzzy set. Type-2 fuzzy sets can be used in cases, when determining the exact membership function is very difficult [48]. So using type-2 fuzzy systems for describing the behavior of these systems is very useful. The general formula of type-2 fuzzy sets is as follows [49]:

$$\tilde{A} = \int_{x\in X} \mu_{\tilde{A}}(x)/x = \frac{\int_{x\in X}\left[\int_{\mu\in J_x} \frac{f_x(\mu)}{\mu}\right]}{x} \tag{1}$$

where $\tilde{A}$ is a type-2 fuzzy set, $\mu_{\tilde{A}}(x)$ is the initial membership function, J_x is the sum of initial membership values for x ∈ X and $f_x(\mu) \in [0,1]$ is the secondary membership function. Providing a fuzzy type-2 set needs dealing with three dimensions, and computing in these systems is very complex. It is defined as a type-2 fuzzy alternative set. Fourier type fuzzy sets are special types of fuzzy sets of type-2 that are maintained under the conditions: $f_x(\mu) = 1, \forall \mu \in J_x \subseteq [0, 1]$.

Particular and simplified kind of general type-2 fuzzy sets are interval type-2 fuzzy sets. Two examples of fuzzy sets of type-2 are shown in Fig. 1. Fig. 1(a) exhibits a case of a fuzzy set with a Gaussian membership function with an average m and a standard deviation in the interval $[\sigma_1, \sigma_2]$. Fig. 1(b) depicts a fuzzy set with a Gaussian membership function with a constant standard deviation σ (but with an uncertain mean) taking values in $[m_1, m_2]$. In the case of an uncertain standard deviation, the Gaussian function center in m is constant, but the width of the variation of the function lies in the interval $[\sigma_1, \sigma_2]$. Fig. 1 shows the two cases of the Gaussian functions.

The region of uncertainty is called the Footprint of Uncertainty (FOU) type-2 fuzzy functions. Parts of FOU-1 are of type-2. In fuzzy type-2 systems, lower bounds and higher membership members are called low membership functions (LMFs) and high membership functions (UMFs), respectively.

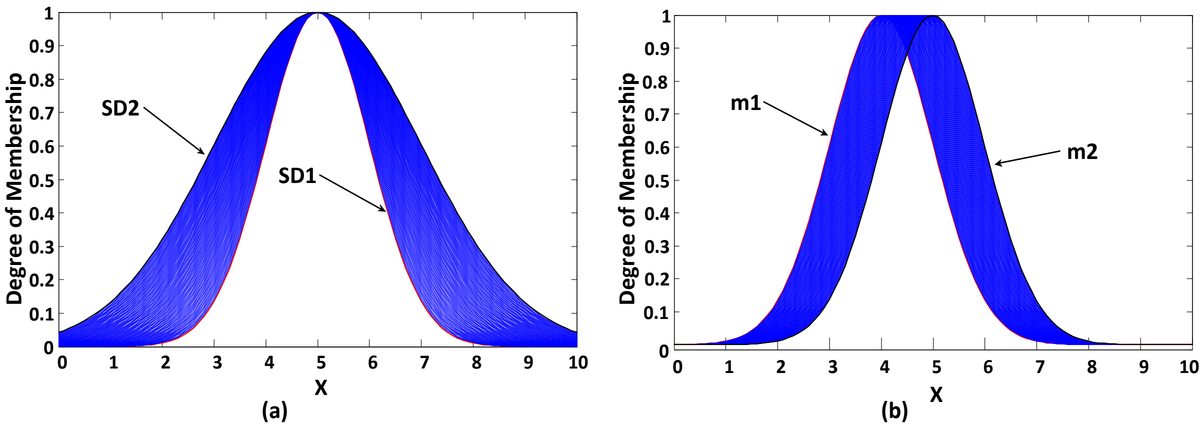

Fig. 1. Gaussian membership function with variable standard deviation (a) and variable mean (b).

The main difference between fuzzy systems of type-1 and type-2 is the type of their membership functions. Despite the fact that the output of a type-1 fuzzy system is a type-1 fuzzy set, the system output is discounted by the cost of the crunch. On the other hand, in type-2 fuzzy systems, outputs are type-2 fuzzy sets that are of defuzzification-type fuzzy sets. The method of converting type-2 into type-1 fuzzy sets is called "Type Reduction," which is a very important issue in the analysis of type 2 fuzzy systems [49]. The structure of type-2 fuzzy system is shown in Fig. 2. It is similar to fuzzy type-1 system, and as shown in Fig. 2, only in type-2 fuzzy system, the type reduction block is added.

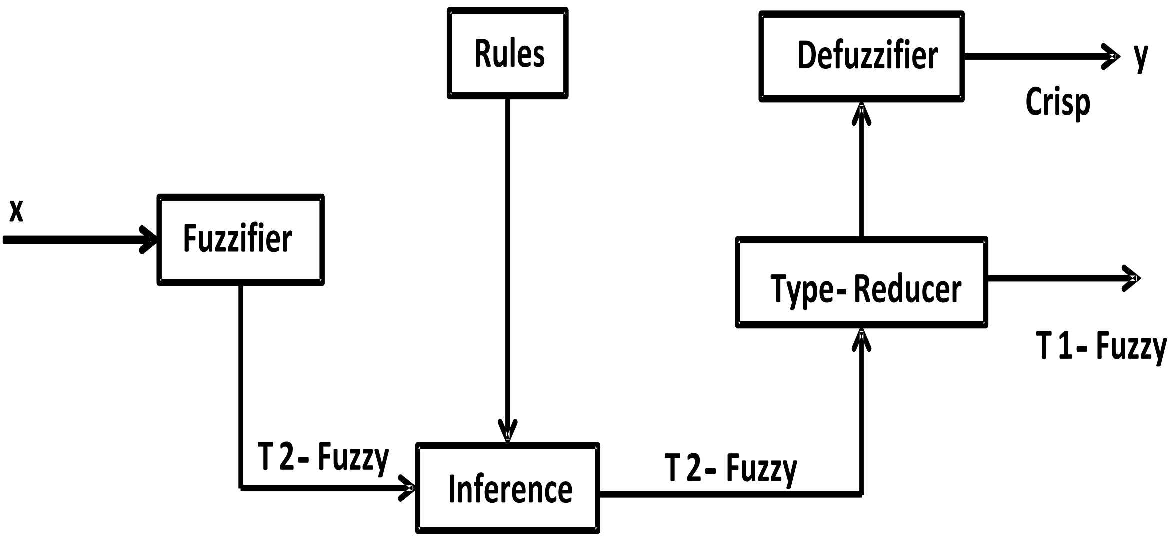

Fig. 2. Structure of type-2 fuzzy system.

3. ANFIS BASED ON TYPE-2 FUZZY SETS

ANFIS2 has been proposed by us in [10]. Fig. 3 shows its structure. The contributions of this structure are: i) using type-2 fuzzy sets as membership functions in the first layer and; ii) using type-2 fuzzy sets as linear coefficients in the consequent layer. More details are presented in [10].

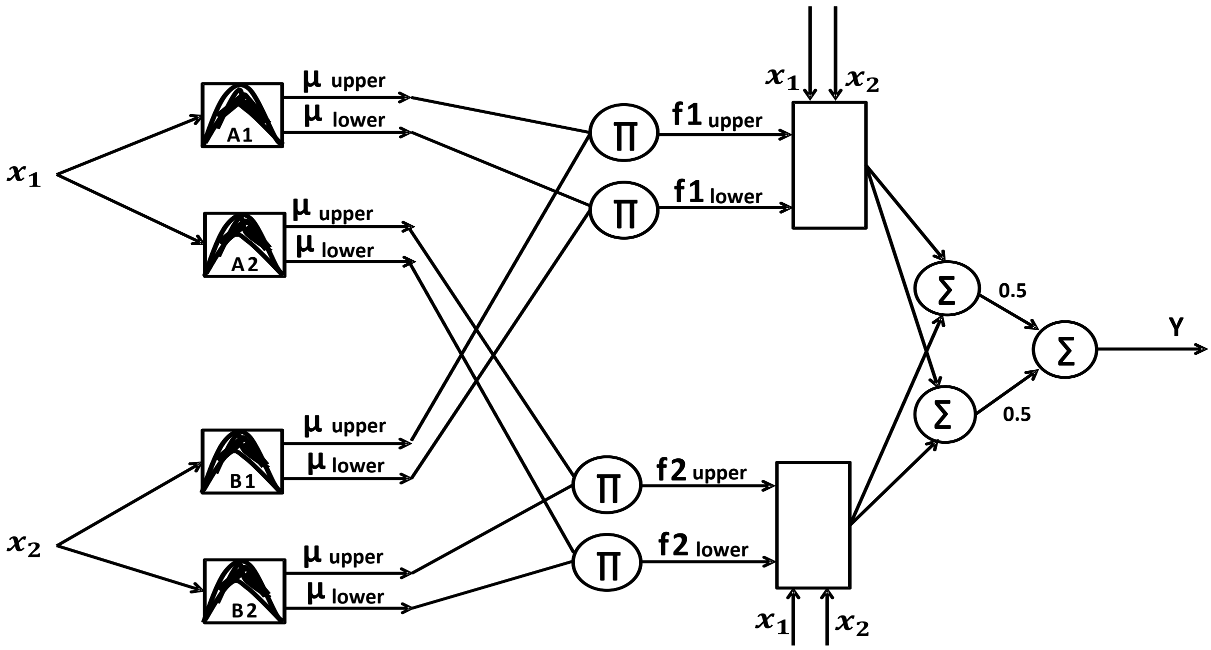

Fig. 3. The structure of ANFIS2.

4. ANFIS2 BASED SLIDING MODE CONTROL FOR A CLASS OF NONLINEAR SYSTEMS

A class of nonlinear systems is:

$$\ddot{y} = f(x) + g(x)u \tag{2}$$

where

$$f(x) = [f_1(x), \dots, f_m(x)]^T$$

$$g(x) = \begin{bmatrix} g_{11}(x) & \cdots & g_{1m}(x) \\ \vdots & \ddots & \vdots \\ g_{m1}(x) & \cdots & g_{mm}(x) \end{bmatrix}$$

$$u = [u_1, \dots, u_m]^T$$

$$y = [y_1, \dots, y_m]^T$$

$$x = [y_1, \dot{y}_1, y_2, \dot{y}_2, \dots, y_m, \dot{y}_m]^T$$

In the above equations, $f(x)$ and $g(x)$ are unknown nonlinear functions, u is control input, y is output vector and x is state vector.

If reference trajectory is $y_d = [y_{1d} \dots y_{md}]^T$; then the tracking error is:

$$e_1 = y_{1d} - y_1$$
$$\vdots$$
$$e_m = y_{md} - y_m$$

Define integral sliding surface as:

$$s_1 = k_1 e_1 + \dot{e}_1$$
$$\vdots$$
$$s_m = k_m e_m + \dot{e}_m$$

The time derivatives of the sliding surface are:

$$\dot{s}_1 = k_1 \dot{e}_1 + \ddot{y}_1 - \ddot{y}_{d1}$$
$$\vdots$$
$$\dot{s}_m = k_m \dot{e}_m + \ddot{y}_m - \ddot{y}_{dm}$$

The structure of the proposed backstepping-based recurrent type-2 fuzzy sliding mode control is shown in Fig. 4.

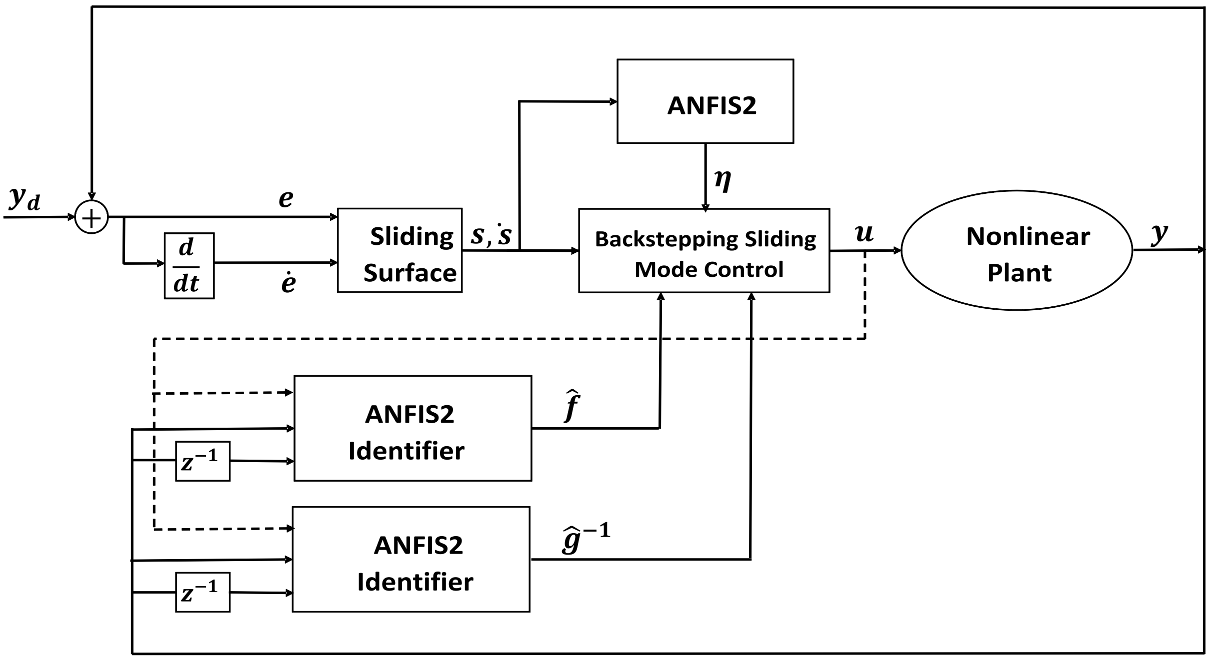

Fig. 4. Backstepping sliding mode control based on ANFIS2.

4.1. Stability of the Proposed Method

The first Lyapunov function is chosen as:

$$V_1 = v_1 + v_2 + \cdots + v_m$$

where

$$v_1 = \frac{(e_1)^2}{2} \quad , \dots , v_m = \frac{(e_m)^2}{2}$$

then; the time derivative of V_1 is:

$$\dot{V}_1 = e_1\dot{e}_1 + e_2\dot{e}_2 + \cdots + e_m\dot{e}_m$$

Define the following lyapunov function:

$$V_2 = V_1 + V_1' \tag{3}$$

where

$$V_1' = \frac{(s_1)^2}{2}, \dots, \frac{(s_m)^2}{2}$$

Differentiating Eq. (3) yields:

$$\begin{aligned}\dot{V}_2 &= \dot{V}_1 + \dot{V}_1' \\ &= e_1\dot{e}_1 + e_2\dot{e}_2 + \cdots + e_m\dot{e}_m + s_1\dot{s}_1 + s_2\dot{s}_2 + \cdots + s_m\dot{s}_m \\ &= e_1 s_1 + e_2 s_2 + \cdots + e_m s_m - k_1(e_1)^2 - k_2(e_2)^2 - \cdots - k_m(e_m)^2 \\ &\quad + s_1(k_1\dot{e}_1 + \ddot{y}_1 - \ddot{y}_{d1}) + \cdots + s_m(k_m\dot{e}_m + \ddot{y}_m - \ddot{y}_{dm})\end{aligned} \tag{4}$$

since

$$\begin{aligned}\ddot{y}_1 &= f_1(x) + g_1(x)u_1 \\ &\;\;\vdots \\ \ddot{y}_m &= f_m(x) + g_m(x)u_m\end{aligned} \tag{5}$$

Substituting Eq. (5) into Eq. (4), gives $\dot{V}_2$;

$$\dot{V}_2 = e_1s_1 + e_2s_2 + \cdots + e_ms_m - k_1(e_1)^2 - k_2(e_2)^2 - \cdots - k_m(e_m)^2$$
$$+s_1(k_1\dot{e}_1 + f_1(x) + g_1(x)u_1 - \ddot{y}_{d1}) + \cdots + s_m(k_m\dot{e}_m + f_m(x) + g_m(x)u_m - \ddot{y}_{dm}) \quad (6)$$

According to Eq. (5), a backstepping sliding mode control law u can be designed as:

$$u_1 = g_1{}^{-1}(x)(-f_1(x) - k_1\dot{e}_1 - e_1 + \ddot{y}_{d1} - \gamma_1s_1 - \eta_1 sgn(s_1))$$
$$\vdots \quad (7)$$
$$u_m = g_m{}^{-1}(x)(-f_m(x) - k_m\dot{e}_m - e_m + \ddot{y}_{dm} - \gamma_ms_m - \eta_m sgn(s_m))$$

where η and γ are positive constants. Substituting Eq. (7) into Eq. (6), gives $\dot{V}_2$ as:

$$\dot{V}_2 = -k_1(e_1)^2 - k_2(e_2)^2 - \cdots - k_m(e_m)^2 - \gamma_1(s_1)^2 - \gamma_2(s_2)^2 - \cdots - \gamma_m(s_m)^2 - \eta_1|s_1| - \cdots - \eta_m|s_m| \le 0$$

Define the following term

$$W(t) = k_1(e_1)^2 + k_2(e_2)^2 + \cdots + k_m(e_m)^2 + \gamma_1(s_1)^2 + \gamma_2(s_2)^2 + \cdots + \gamma_m(s_m)^2 \le -\dot{V}_2$$

then;

$$\int_0^\tau W(t)dt \le -\int_0^\tau \dot{V}_2\, dt = V_2(0) - V_2(\tau)$$

because $V_2(0)$ and $V_2(\tau)$ are bounded, so

$$\lim_{\tau\to\infty}\int_0^\tau W(t)dt < \infty$$

so the following result can be obtained

$$\lim_{t\to\infty} W(t) = 0$$

since k_i and γ_i for $i = 1, \ldots, m$ are positive, therefore $W(t)$ is positive. The $s_i(t)$ and $e_i(t)$ will converge to zero as $t \to \infty$, so $\lim_{t\to\infty} y_i(t) = y_{id}$.

To design control signals $u_1, \ldots, u_m$ all $f_1(x), \ldots, f_m(x)$ and $g_1(x), \ldots, g_m(x)$ and also $\eta_1, \ldots, \eta_m$ must be known. In practice generally they are unknown. Two MIMO recurrent type-2 fuzzy systems is used to approximate $f_1(x), \ldots, f_m(x)$ and $g_1(x), \ldots, g_m(x)$ and a simple two input one output type-2 fuzzy system is used to calculate $\eta_1, \ldots, \eta_m$. The input of recurrent MIMO type-2 fuzzy identifier for $f_1(x), \ldots, f_m(x)$ is x and the m outputs is $f_1(x), \ldots, f_m(x)$. The input of recurrent MIMO type-2 fuzzy identifier for $g_1(x), \ldots, g_m(x)$ is x and the m outputs is $g_1(x), \ldots, g_m(x)$. The input of recurrent MIMO type-2 fuzzy identifier for $\eta_1, \ldots, \eta_m$ is s and $\dot{s}$ and the m outputs is $\eta_1, \ldots, \eta_m$. Vector form of control signal is:

$$u = g^{-1}(x)(-f(x) - k\dot{e} - e + \ddot{y}_d - \gamma s - \eta sgn(s))$$

The goal of design $\eta_1, \ldots, \eta_m$ is that the following equation is satisfied:

$$s\dot{s} \le -\eta|s| < 0$$

The input fuzzy variables are the sliding surface s and the derivative of sliding surface $\dot{s}$ whereas the output fuzzy variable is the upper bound of the lumped uncertainty η.

5. MEMS TRIAXIAL GYROSCOPE

Generally, a typical MEMS vibratory gyroscope includes a proof mass suspended by spring beams, electrostatic actuations and sensing mechanisms for forcing an oscillatory motion and sensing the position and velocity of the proof mass. Dynamics of a MEMS gyroscope is derived from Newton's law in the rotating frame. Fig. 5 shows 2-D MEMS triaxial gyroscope.

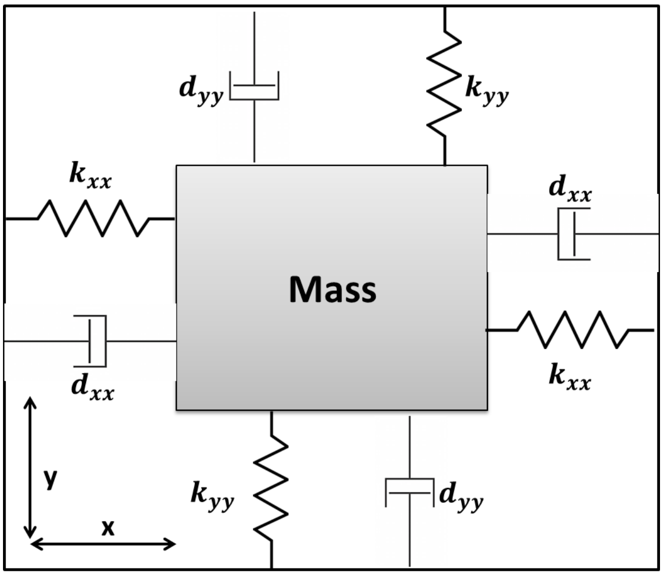

Fig. 5. The 2-D MEMS triaxial gyroscope.

The MEMS triaxial gyroscope is a MIMO system. The gyroscope undergoes rotations along x, y, and z axis. The mathematical model of the triaxial gyroscope system is given as follows [50]:

$$m\ddot{x} + d_{xx}\dot{x} + d_{xy}\dot{y} + d_{xz}\dot{z} + k_{xx}x + k_{xy}y + k_{xz}z = u_x + 2m\Omega_z\dot{y} - 2m\Omega_y\dot{z}$$
$$m\ddot{y} + d_{xy}\dot{x} + d_{yy}\dot{y} + d_{yz}\dot{z} + k_{xy}x + k_{yy}y + k_{yz}z = u_y - 2m\Omega_z\dot{x} + 2m\Omega_x\dot{z}$$
$$m\ddot{z} + d_{xz}\dot{x} + d_{yz}\dot{y} + d_{zz}\dot{z} + k_{xz}x + k_{yz}y + k_{zz}z = u_z + 2m\Omega_y\dot{x} - 2m\Omega_x\dot{y}$$

where: m is mass. Mock defects mainly relate to asymmetric spring terms k_{xy}, k_{xz} and k_{yz}, and symmetric damping terms d_{xy}, d_{xz}, d_{yz}. The spring terms in the x, y and z direction are k_{xx}, k_{yy} and k_{zz} , respectively. The damping terms in the x, y and z directions are d_{xx}, d_{yy} and d_{zz} , respectively. Angular velocity terms in the x, y and z directions are Ω_x, Ω_y and Ω_z , respectively. Control forces terms in the x, y and z directions are u_x, u_y and u_z , respectively.

6. SIMULATIONS

In this section, the simulation results of the backstepping type-2 fuzzy sliding mode for the MEMS triangular gyroscope are being demonstrated. The MEMS gyroscope parameters are: $m = 5.7 \times 10^{-9} kg$; $\Omega_x = 3\, rad/s$; $\Omega_y = 2\, rad/s$; $\Omega_z = 5\, rad/s$

$k_{xx} = 80.98\, N/m$	$k_{xy} = 5\, N/m$	$k_{yy} = 71.62\, N/m$
$k_{zz} = 60.97\, N/m$	$k_{xz} = 6\, N/m$	$k_{yz} = 7\, N/m$
$d_{xx} = 4.29 \times 10^{-7}\, Ns/m$	$d_{xy} = 4.29 \times 10^{-8}\, Ns/m$	$d_{yy} = 4.29 \times 10^{-8}\, Ns/m$
$d_{zz} = 8.95 \times 10^{-7}\, Ns/m$	$d_{xz} = 6.87 \times 10^{-8}\, Ns/m$	$d_{yz} = 8.95 \times 10^{-8}\, Ns/m$

Figs. 6-8 show the position tracking trajectory using backstepping-based recurrent type-2 fuzzy sliding mode control.

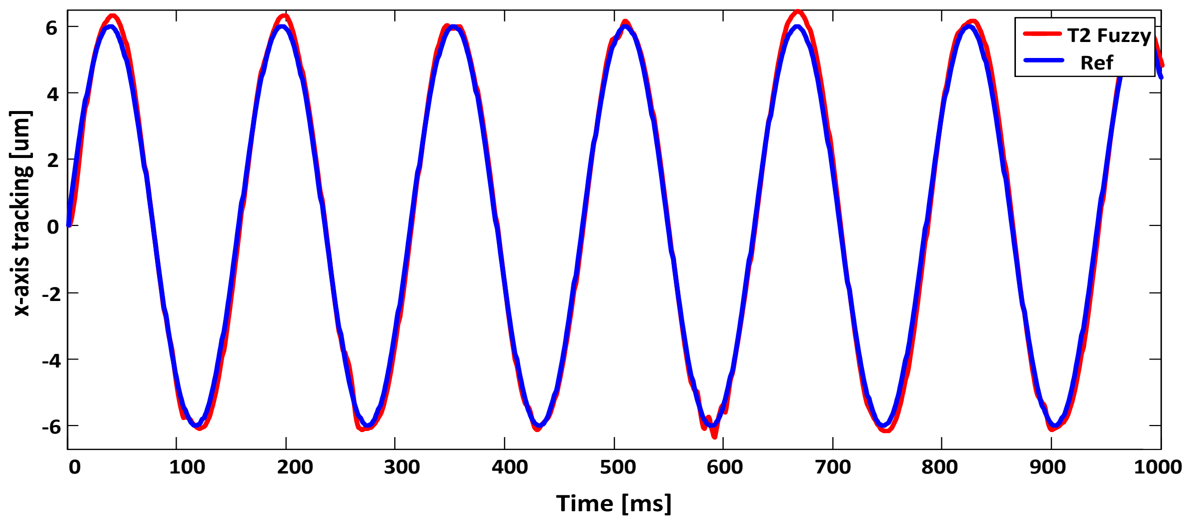

Fig. 6. The x-axis tracking by backstepping sliding mode controller based on ANFIS2.

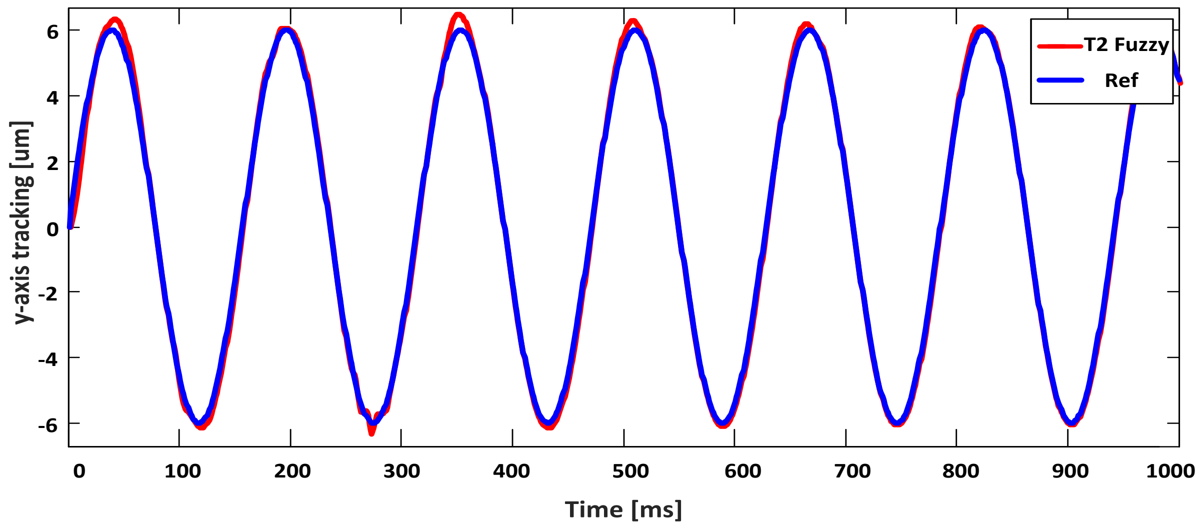

Fig. 7. The y-axis tracking by backstepping sliding mode controller based on ANFIS2.

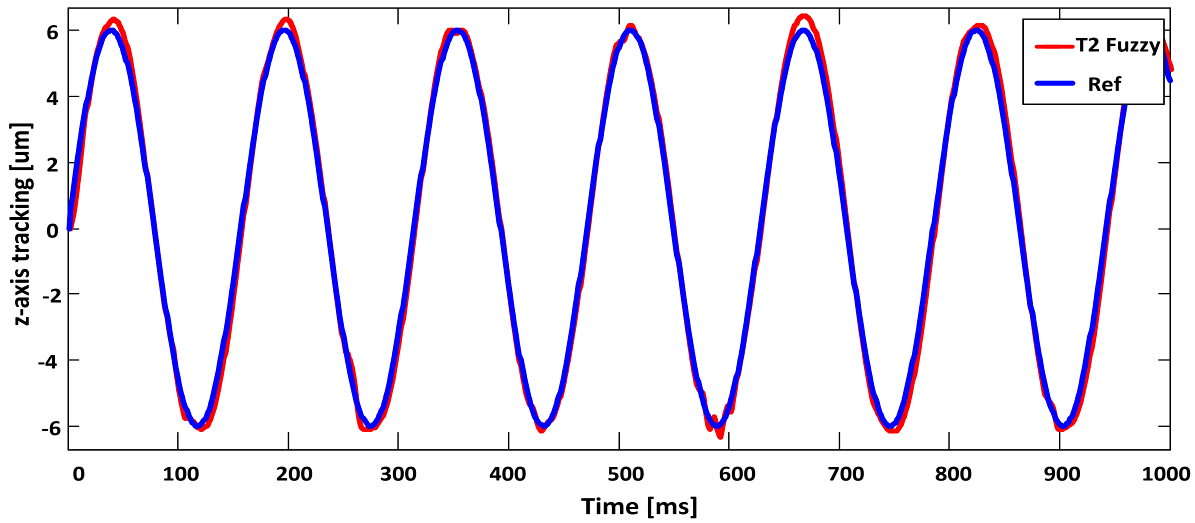

Fig. 8. The z-axis tracking by backstepping sliding mode controller based on ANFIS2.

±%15 change in spring and spring coefficient parameters with respect to nominal value and ±%15 magnitude change in the coupling terms are considered.

Figs. 9-11 show the effect of parameter uncertainty on backstepping sliding mode control based on recurrent nonlinear type-2 fuzzy TSK and type-1 fuzzy.

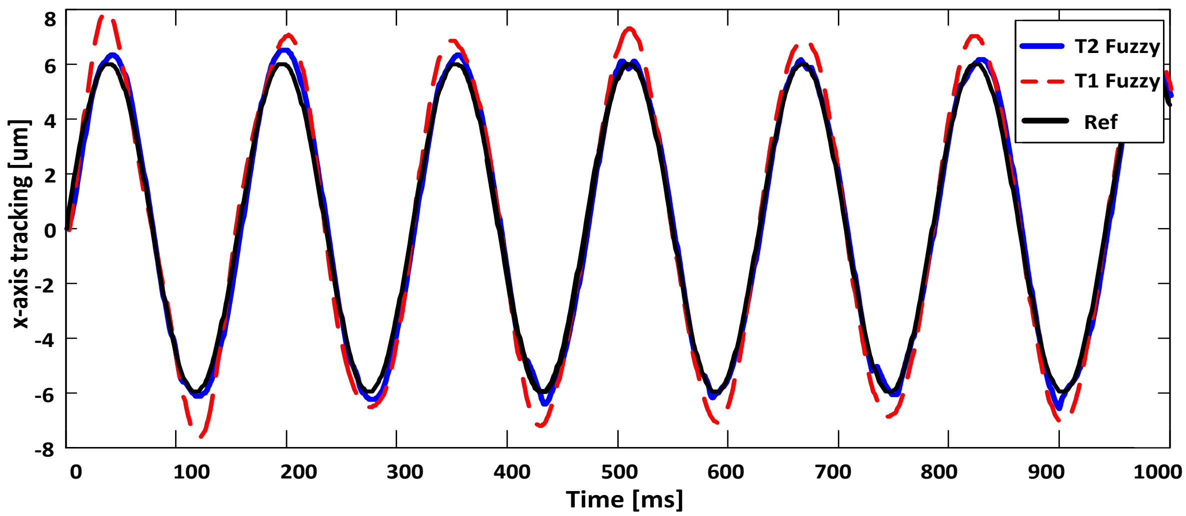

Fig. 9. The x-axis tracking by backstepping sliding mode controller based on ANFIS2 and ANFIS.

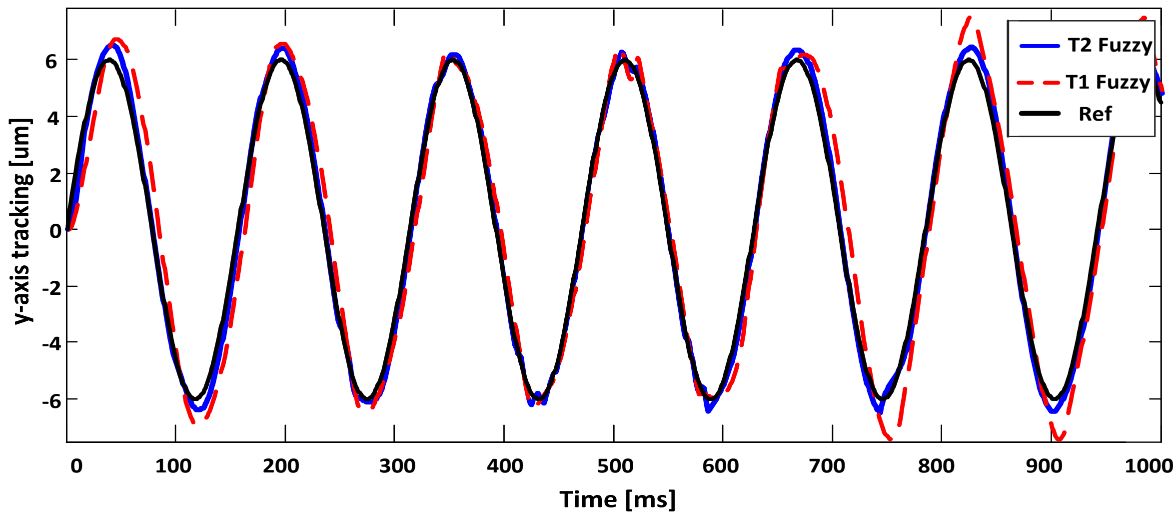

Fig. 10. The y-axis tracking by backstepping sliding mode controller based on ANFIS2 and ANFIS.

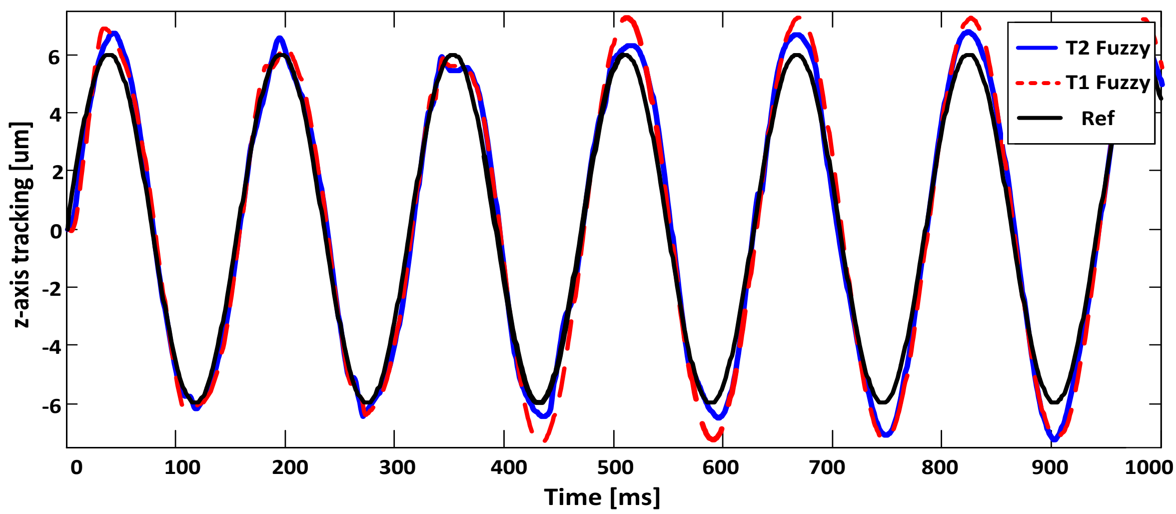

Fig. 11. The z-axis tracking by backstepping sliding mode controller based on ANFIS2 and ANFIS.

The simulation results shown in Figs. 9-11 reveal that the backstepping sliding mode control based on recurrent nonlinear type-2 fuzzy TSK system has better performance than the backstepping sliding mode control based on type-1 fuzzy system. The results of the proposed method are compared with results of [51] and [52] in Table 1.

Table 1. Comparison with results of [51] and [52].

Uncertainty	Method of [51]		Method of [52]		The proposed method	
	Fuzzy rules	RMSE	Fuzzy rules	RMSE	Fuzzy rules	RMSE
0	4	0.559	4	0.238	2	0.113
10%	5	0.936	4	0.394	3	0.285

Table 1 shows that our method has better performance than that of the methods reported in [51] and [52]: Type-2 fuzzy has more parameters so that it has more degree of freedom. The ANFIS2 is the same as the ANFIS, with the difference that it uses type-2 fuzzy logic. One of the benefits of ANFIS2 is that with fewer fuzzy rules, the results can be equal or even better than ANFIS. So the structure of ANFIS2 is easier and more understandable. On the other hand, previous studies have shown that type-2 fuzzy logic has more accuracy in the function approximation than type-1 fuzzy logic. Therefore, in the field of modeling and control, type-2 fuzzy logic is preferable to type -1 fuzzy logic. Therefore, ANFIS2 sliding mode control is better than ANFIS sliding mode control.

7. CONCLUSIONS

In this paper, a combination of type-2 fuzzy system, sliding mode technique and backstepping method was used to control a class of uncertain nonlinear systems. With this combination, all the disadvantages of sliding mode control were eliminated. Because the sliding mode control method is highly dependent on the mathematical model of the system, if the dynamic model is changed or inoperative during operation, it will not work properly. So in this paper, we used type-2 fuzzy neural network to approximate the system's mathematical model momentarily. In the simulation section, a discrete gyroscope was controlled by the proposed method. The simulation results showed the efficiency of the proposed method in controlling the MEMS gyroscope even though its parameters are uncertain. The proposed method has fewer fuzzy rules and less root mean squares error than other methods.

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