JJEE
Volume 5, Number 2, 2019
Pages 125-132
Jordan Journal of Electrical Engineering
ISSN (Print): 2409-9600, ISSN (Online): 2409-9619

Error Exponent for Cooperative Communications Under *α-μ* Multipath Fading

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Received: March 20, 2019 *Accepted: May 30, 2019*

Abstract— In this paper, we derive a novel analytical expression for random coding error exponent of a dual-hop amplify and-forward wireless communication system over independent and identically distributed α-μ fading channels. Random coding error exponent is an information theoretic performance measure which gives an insight into the fundamental trade-off between the achievable rate of information and the reliability of communication. It can be used to find the length of the code-word required to achieve a pre-determined probability of error at a rate below the capacity of the channel. Furthermore, the derived random coding error exponent expression is utilized to derive closed form expressions for the cutoff rate and the capacity of the system. Moreover, the derived expressions can be reduced to study the performance of the system over Nakagami-m, Weibull, One-sided Gaussian, Rayleigh, and Negative Exponential fading models which are included in the α-μ fading model as special cases. Numerical results are further presented to corroborate the analysis.

Keywords— Amplify and forward, Dual-hop, Random coding error exponent, α-μ fading.

I. INTRODUCTION

There have been recent growing interests in both academia and industry in integrating multi-hop relaying into the next generation infrastructure-based networks such as 5G technology and sensor networks. Multi-hop communications can be realized through the use of low-power, low-cost relays or through other wireless terminals in the network, where information is passed between two terminals (nodes) over multi-hop transmission.

Recently, a variant of multi-hop relaying, known as cooperative diversity or relay-assisted networks, has emerged. Generally, it is used in infrastructure-less based networks as a promising approach to increase spectral and power efficiency, network coverage, and reduce outage probability. Similar to multi-antenna transceivers, relays provide diversity by creating multiple replicas of the signal of interest. The basic idea behind relay-assisted networks is that various terminals/nodes in a relay network attempt to assist each other in moving information around the network instead of competing for system resources. The result is an improvement of the overall quality of services (such as the statistical measures of bit-error-rate, outage, throughput, and delay) at all the nodes and an associated increase in system spectral efficiency. Two common relaying techniques are the decode-and-forward (DF) and the amplify-and-forward (AF). In DF relaying, the relay terminal decodes a received signal; and then re-encodes it (possibly using a different codebook) for transmission to a destination. With the AF relaying, the relay terminal re-transmits a scaled version of the received signal without any attempt to decode it.

There have been recent results reported in the literature on the error rate performance in dual-hop transmission, which is a special case of a multi-hop transmission. In [1], the authors

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derived closed form expressions for the average bit error rate and the outage probability of AF dual-hop systems in α-μ channels. The same authors, in [2], derived closed form expressions for the capacity and outage capacity under the same scenario in [1].

Since a random coding error exponent serves as a tight lower bound for Shannon's error exponent which is also known as Shannon's reliability function. A random coding error exponent reveals the fundamental tradeoff between communication rate and reliability. It gives the best exponential decay in the probability of an error with the code-word length at a communication rate below the capacity of the channel. It shows that the probability of a decoding error decreases exponentially with the length of the code-word as well as the code rate; hence it can be used to find the length of the required code to achieve a pre-determined probability of an error at a rate below the capacity of the channel. Random coding error exponent expressions can be also used to derive expressions for the capacity, critical rate, and cut off rate of the system.

Performance analysis of AF dual-hop communication systems utilizing the error exponent has been studied in [3], [4]. In [3], the authors derive the random coding error exponent for a dual-hop AF system assuming an ideal relay, which is capable of inverting the channel of the independent previous hop over and identically distributed (i.i.d) Nakagami-m fading channels. The exact random coding error exponent for dual-hop AF systems with channel state information (CSI) assisted relay without avoiding the denominator noise figure has been derived in [4] which assumes independent but not necessarily identical Rayleigh fading channels. In [5], the authors derive exact expressions for a random coding error exponent of dual-hop AF systems over η−μ fading channels. In [6], the authors derive an analytical expression for a random coding error exponent of space time block code (STBC) systems over η-μ fading channels. In [7], the authors derive the random coding error exponent of AF systems in presence of arbitrary number of i.i.d interferers at the relay and destination. In [8], the authors provide analytical expressions for a random coding error exponent of multiple-input-multiple-output (MIMO)-STBC systems over both η-μ and block fading channels. In [9], the authors derive the random coding exponent for Rayleigh fading multi-input multi-output channel equipped with an automatic repeat request (ARQ) protocol.

Although there is a growing body of literature on the analysis of the random coding error exponent in relay communications, to the best of our knowledge, there has been no result reported in the literature which considers the random coding error exponent in α-μ channels. The α-μ distribution is a general fading distribution used to characterize small scale variations of fading signals in non-linear of sight communications [10]. The α-μ distribution assumes μ clusters of multipath signals propagating in a non-homogeneous medium; and it represents the envelope of the fading signal as a non-linear function of the amplitude of the multipath clusters. The uniqueness of the α-μ distribution over other general fading distributions like the η-μ and the κ-μ comes from the distribution parameter α which explores the non-linearity of the propagation medium. The Nakagami-m, Weibull, One-sided Gaussian, Rayleigh, and Negative Exponential distributions are special cases of the α-μ distribution. However, the α-μ distribution fits experimental data better than the previously mentioned distributions.

In this paper, we aim to fill this research gap and derive the random coding error exponent in α-μ channels. Our contributions in this work are summarized as follows: We first derive the random coding error exponent for AF dual-hop system operating over i.i.d α-μ fading channels. Then we use the random coding error exponent to derive closed form expressions for the cutoff rate and capacity of the system. It is noteworthy mentioning here that the derived expressions can be reduced to study the random coding error exponent of AF dual-hop systems over other fading channel models like Nakagami-m, Weibull, One-sided

Gaussian, Rayleigh, and Negative Exponential, a contribution that is missing from the literature.

II. SYSTEM AND CHANNEL MODELS

We consider a dual-hop wireless communication system with a single AF relay. AF multi-hop communication systems can be further divided into two major groups based on the amplification gain of the relay, namely, fixed gain relays and variable gain relays. In this paper, we will concentrate on variable gain relays which are capable of inverting the channel gain of the previous hop regardless of its magnitude. Given that, the end-to-end signal-to-noise ratio can be upper-bounded as in [11] by:

$$\gamma_{eq} = \frac{\gamma_1\gamma_2}{\gamma_1+\gamma_2} \tag{1}$$

where γ_i is the instantaneous signal to noise ratio (SNR) of the i-th hop link, which is assumed to be distributed according to the α-μ fading model with a probability density function (pdf) given as in [12] by:

$$f_\gamma(\gamma) = \frac{\alpha}{2\Gamma(\mu)}\left(\tfrac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^{\mu}\gamma^{\alpha\mu/2-1}\exp\left(-\mu\left(\tfrac{\gamma}{\bar{\gamma}}\right)^{\alpha/2}\right) \tag{2}$$

where $\bar{\gamma}$ is the average SNR; and $\Gamma(.)$ is the well-known Gamma function. The α-μ distribution is a general fading distribution that can be reduced to other fading models such as Nakagami-m (α= 2 and μ= m), Rayleigh (α= 2 and μ= 1), Weibull (α= β and μ= 1), and one-sided Gaussian (α= 1 and μ= 1).

Assuming that both links are independent and identically distributed then the pdf of the end-to-end SNR can be expressed as in [2], by:

$$f_{\gamma_{eq}}(\gamma_{eq}) = \frac{\sqrt{\pi}}{2^{2\mu-\frac{2}{\alpha}-2}\Gamma^2(\mu+\frac{1}{2}-\frac{1}{\alpha})}\left(\frac{\mu}{\bar{\gamma}^{\frac{\alpha}{2}}}\right)G_{1,2}^{2,0}\left(\frac{4\mu\gamma_{eq}}{\bar{\gamma}^{\frac{\alpha}{2}}}\middle|\begin{matrix}\mu-\frac{1}{\alpha}\\ \mu-\frac{1}{\alpha}-\frac{1}{2},2\mu-\frac{2}{\alpha}\end{matrix}\right) \tag{3}$$

where $G_{m,n}^{p,q}\left(x\middle|\begin{matrix}a_i\\ b_i\end{matrix}\right)$ is the Meijer's G-function as in [13]; and $\bar{\gamma}$ is the average SNR per hop.

III. RANDOM CODING ERROR EXPONENT ANALYSIS

The error exponent along with the codeword length imposes a tight upper bound on the probability of an error for a communication channel [14]-[17]. Hence, it can be used to indicate the coding requirements to achieve a predefined level of bit error rate at an information rate below capacity. The random coding error exponent serves as a lower bound for the reliability function which is also known as error exponent. The random coding error exponent for the predefined dual-hop AF system model, assuming Gaussian input distribution, can be expressed as in [14] by:

$$E_r(R) = \max_{0\le\rho\le1}\{E_o(\rho)-2\rho R\} \tag{4}$$

where

$$E_o(\rho) = -\ln\{E_{\gamma_{eq}}[(1+\tfrac{\gamma_{eq}}{1+\rho})^{-\rho}\} \tag{5}$$

and R is the transmission rate in nats/s/Hz.

Substituting (3) into (5) yields:

$$E_o(\rho) = -\ln\left[\frac{\sqrt{\pi}}{2^{2\mu-\frac{2}{\alpha}-2}\Gamma^2(\mu+\frac{1}{2}-\frac{1}{\alpha})}\left(\frac{\mu}{\bar{\gamma}^{\frac{\alpha}{2}}}\right)\underbrace{\int_0^\infty (1+\tfrac{\gamma_{eq}}{1+\rho})^{-\rho} G_{1,2}^{2,0}\left(\frac{4\mu\gamma_{eq}}{\bar{\gamma}^{\frac{\alpha}{2}}}\middle| \begin{matrix}\mu-\frac{1}{\alpha}\\ \mu-\frac{1}{\alpha}-\frac{1}{2}, 2\mu-\frac{2}{\alpha}\end{matrix}\right) d\gamma_{eq}}_{I}\right] \tag{6}$$

then we can express [18]:

$$(1+\tfrac{\gamma_{eq}}{1+\rho})^{-\rho} = \tfrac{1}{\Gamma(\rho)} G_{1,1}^{1,1}\left(\tfrac{\gamma_{eq}}{1+\rho}\middle| \begin{matrix}1-\rho\\ 0\end{matrix}\right) \tag{7}$$

Then, the integral I becomes

$$I = \tfrac{1}{\Gamma(\rho)}\int_0^\infty G_{1,1}^{1,1}\left(\tfrac{\gamma_{eq}}{1+\rho}\middle| \begin{matrix}1-\rho\\ 0\end{matrix}\right) G_{1,2}^{2,0}\left(\frac{4\mu\gamma_{eq}}{\bar{\gamma}^{\frac{\alpha}{2}}}\middle| \begin{matrix}\mu-\frac{1}{\alpha}\\ \mu-\frac{1}{\alpha}-\frac{1}{2}, 2\mu-\frac{2}{\alpha}\end{matrix}\right) d\gamma_{eq} \tag{8}$$

Using the Meijer's G-function properties, the integral I can be rewritten as follows:

$$I = \tfrac{2^{\alpha\mu-2}}{\Gamma(\rho)}\left(\tfrac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^{\frac{\alpha\mu}{2}-1}\int_0^\infty \gamma_{eq}^{\frac{\alpha\mu}{2}-1} G_{1,1}^{1,1}\left(\tfrac{\gamma_{eq}}{1+\rho}\middle| \begin{matrix}1-\rho\\ 0\end{matrix}\right) G_{1,2}^{2,0}\left(\frac{4\mu\gamma_{eq}}{\bar{\gamma}^{\frac{\alpha}{2}}}\middle| \begin{matrix}\mu-\frac{1}{\alpha}-\frac{\alpha\mu}{2}+1\\ \mu-\frac{1}{\alpha}+\frac{1}{2}-\frac{\alpha\mu}{2}, 2\mu-\frac{2}{\alpha}-\frac{\alpha\mu}{2}+1\end{matrix}\right) d\gamma_{eq} \tag{9}$$

which can be solved as follows [18]:

$$I = \tfrac{2^{\alpha\mu-2}}{(1+\rho)^{-\alpha\mu/2}\Gamma(\rho)}\left(\tfrac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^{\frac{\alpha\mu}{2}-1} G_{2,3}^{3,1}\left(\frac{4\mu(1+\rho)}{\bar{\gamma}^{\frac{\alpha}{2}}}\middle| \begin{matrix}\mu-\frac{1}{\alpha}-\frac{\alpha\mu}{2}+1, 1-\frac{\alpha\mu}{2}\\ \mu-\frac{1}{\alpha}+\frac{1}{2}-\frac{\alpha\mu}{2}, 2\mu-\frac{2}{\alpha}-\frac{\alpha\mu}{2}+1, \rho-\frac{\alpha\mu}{2}\end{matrix}\right) \tag{10}$$

Substituting the result (10) for the integral I into (6) yields:

$$E_o(\rho) = \begin{cases} -\ln\left[\frac{2^{2/\alpha}\sqrt{\pi}(1+\rho)^{\alpha\mu/2}}{\Gamma^2(\mu+\frac{1}{2}-\frac{1}{\alpha})\Gamma(\rho)}\left(\tfrac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^{\frac{\alpha\mu}{2}} G_{2,3}^{3,1}\left(\frac{4\mu(1+\rho)}{\bar{\gamma}^{\frac{\alpha}{2}}}\middle| \begin{matrix}\mu-\frac{1}{\alpha}-\frac{\alpha\mu}{2}+1, 1-\frac{\alpha\mu}{2}\\ \mu-\frac{1}{\alpha}+\frac{1}{2}-\frac{\alpha\mu}{2}, 2\mu-\frac{2}{\alpha}-\frac{\alpha\mu}{2}+1, \rho-\frac{\alpha\mu}{2}\end{matrix}\right)\right] & : 0 < \rho \le 1 \\ 0 & : \text{o.w.}\end{cases} \tag{11}$$

A. Special Cases from (11)

- Rayleigh Fading (α=2, μ=1): The result for Rayleigh fading model can be obtained by setting the appropriate fading parameters in (11), which results in:

$$E_o^{Rayleigh}(\rho) = \begin{cases} -\ln\left[\frac{2\sqrt{\pi}(1+\rho)}{\Gamma(\rho)\bar{\gamma}} G_{2,3}^{3,1}\left(\tfrac{4(1+\rho)}{\bar{\gamma}}\middle| \begin{matrix}\frac{1}{2}, 0\\ 0, 1, \rho-1\end{matrix}\right)\right] & : 0 < \rho \le 1 \\ 0 & :\text{o.w.}\end{cases} \tag{12}$$

- Nakagami-m Fading (α=2, μ=m): The result for the Nakagami-m fading can be obtained by setting α=2 in (11), and μ is the same as the fading parameter m.

$$E_o^{Nakagami}(\rho) = \begin{cases} -\ln\left[\frac{2\sqrt{\pi}(1+\rho)^{\mu}}{\Gamma(\rho)\Gamma^2(\mu)}\left(\tfrac{\mu}{\bar{\gamma}}\right)^{\mu} G_{2,3}^{3,1}\left(\tfrac{4\mu(1+\rho)}{\bar{\gamma}}\middle| \begin{matrix}\frac{1}{2}, 1-\mu\\ 0, \mu, \rho-\mu\end{matrix}\right)\right] & : 0 < \rho \le 1 \\ 0 & :\text{o.w.}\end{cases} \tag{13}$$

This is exactly the result reported in [3].

- Weibull Fading (μ=1): The result for Weibull fading can be obtained by setting μ=1 in (11); and $\frac{\alpha}{2}$ is the same as the fading parameter β:

$$E_o^{Weibull}(\rho)=\begin{cases}-\ln\left[\dfrac{2^{\frac{2}{\alpha}}\sqrt{\pi}(1+\rho)^{\alpha/2}}{\Gamma(\rho)\Gamma^2(\frac{3}{2}-\frac{1}{\alpha})\bar{\gamma}^{\frac{\alpha^2}{4}}}G_{2,3}^{3,1}\left(\frac{4(1+\rho)}{\bar{\gamma}^{\alpha/2}}\middle|\begin{matrix}2-\frac{1}{\alpha}-\frac{\alpha}{2},1-\frac{\alpha}{2}\\ \frac{3}{2}-\frac{1}{\alpha}-\frac{\alpha}{2},3-\frac{2}{\alpha}-\frac{\alpha}{2},\rho-\frac{\alpha}{2}\end{matrix}\right)\right] & :0<\rho\le 1\\ 0 & :\text{o.w.}\end{cases} \quad (14)$$

B. System Capacity and Cutoff Rate

The random coding error exponent can be used to find the capacity of the system which is defined as the maximum achievable rate. It can be expressed as in [14] by:

$$c=\frac{1}{2}\left[\frac{\partial E_o(\rho)}{\partial\rho}\right]_{|\rho=0} \quad (15)$$

$$c=\frac{1}{2}\int_0^\infty \ln(1+\gamma)f_{\gamma_{eq}}(\gamma)d\gamma \quad (16)$$

This can be solved in [2] as:

$$c=\frac{\sqrt{\pi}}{2^{2\mu-\frac{2}{\alpha}-2}\Gamma^2(\mu+\frac{1}{2}-\frac{1}{\alpha})\ln(2)}\left(\frac{\mu}{\bar{\gamma}^{\alpha/2}}\right)G_{3,4}^{4,1}\left(\frac{4\gamma\mu}{\bar{\gamma}^{\alpha/2}}\middle|\begin{matrix}-1,\mu-\frac{1}{\alpha},0\\ -1,\mu-\frac{1}{\alpha}-\frac{1}{2},2\mu-\frac{2}{\alpha},-1\end{matrix}\right) \quad (17)$$

Since the cutoff rate is defined as the maximum practical transmission rate for possible decoding strategies, thus for dual hop α−μ fading channels, the cutoff rate in nats/s/Hz can be given as

$$R_o=-\frac{1}{2}\ln\left[\frac{2^{\frac{2}{\alpha}+\frac{\alpha\mu}{2}}\sqrt{\pi}}{\Gamma^2(\mu+\frac{1}{2}-\frac{1}{\alpha})}\left(\frac{\mu}{\bar{\gamma}^{\alpha/2}}\right)^{\frac{\alpha\mu}{2}}G_{2,3}^{3,1}\left(\frac{8\mu}{\bar{\gamma}^{\frac{\alpha}{2}}}\middle|\begin{matrix}\mu-\frac{1}{\alpha}-\frac{\alpha\mu}{2}+1,1-\frac{\alpha\mu}{2}\\ \mu-\frac{1}{\alpha}+\frac{1}{2}-\frac{\alpha\mu}{2},2\mu-\frac{2}{\alpha}-\frac{\alpha\mu}{2}+1,\rho-\frac{\alpha\mu}{2}\end{matrix}\right)\right] \quad (18)$$

which follows directly from the fact that $R_o=\frac{1}{2}E_o(1)$. It can be easily shown that (18) can be directly simplified by substituting α=2 and μ=*m* for Nakagami-m fading channels [3].

IV. NUMERICAL RESULTS

In this section, we evaluate the random coding error exponent for different values of the fading parameters (α and μ) and for different values of the average signal-to-noise ratio (SNR) per hop. Fig. 1 shows how the random coding error exponent changes versus the rate *R* at an average SNR per hop ($\bar{\gamma}=20dB$) and (α=2) for different values of μ (Nakagami-*m* fading with μ=*m*). It is clearly seen from Fig. 1 that the random coding error exponent increases (i.e. probability of error decreases) with μ (less fading) at any rate below the capacity of the channel. It can be also seen from the figure that the same level of communication reliability (i.e. fixed random coding error exponent) can be achieved at higher rates for larger values of μ. For example, to achieve a random coding error exponent of 0.5 the rate has to be reduced from 0.8 nats/sec/Hz when α=2 and μ=1 (Rayleigh fading) to 0.4 nats/sec/Hz when α=2 and μ=0.5 (more fading than Rayleigh fading).

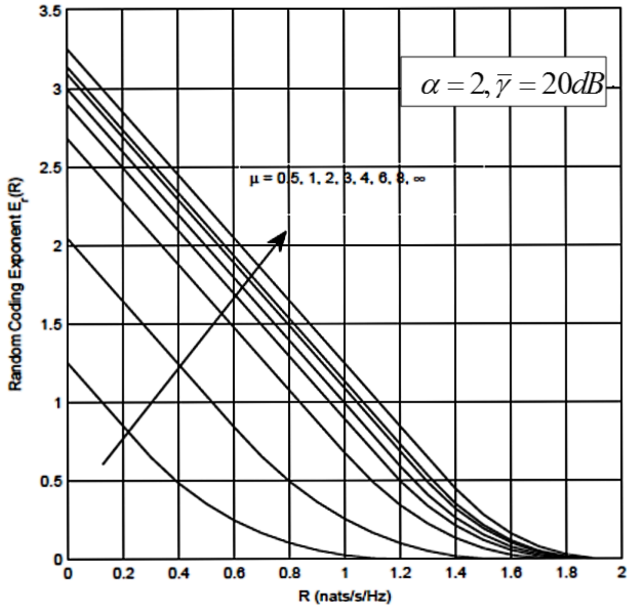

Fig. 1. Random coding error exponent versus R when α=2, $\overline{\gamma} = 20dB$, and for different values of μ

In Fig. 2, the random coding error exponent is plotted versus the rate R when α=3 and μ=1 (Weibull fading with β=1.5) and for different values of the average SNR per hop $\overline{\gamma}$. It is clearly seen from Fig. 2 that as the average SNR per hop increases, the random coding error exponent at rates below the capacity also increases. This also means a decrease in the probability of error. Fig. 2 shows that for the same value of the random coding error exponent (or bit error rate), higher rates (or longer code-words) can be achieved as the average SNR per hop increases. Fig. 3 shows the cutoff rate versus $\overline{\gamma}$ when μ=2 and α=2, 2.5, 3, and 4. The figure clearly shows that higher cutoff rates can be achieved at the same $\overline{\gamma}$ when α increases (i.e. less amount of fading). Fig. 3 also shows that the cutoff rate increases with $\overline{\gamma}$ for all values of α.

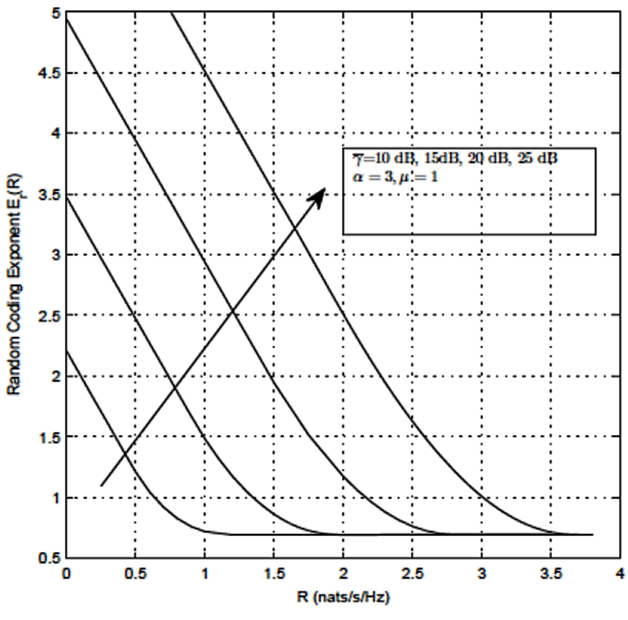

Fig. 2. Random coding error exponent versus R when α=3, μ=1, and for different values of $\overline{\gamma}$

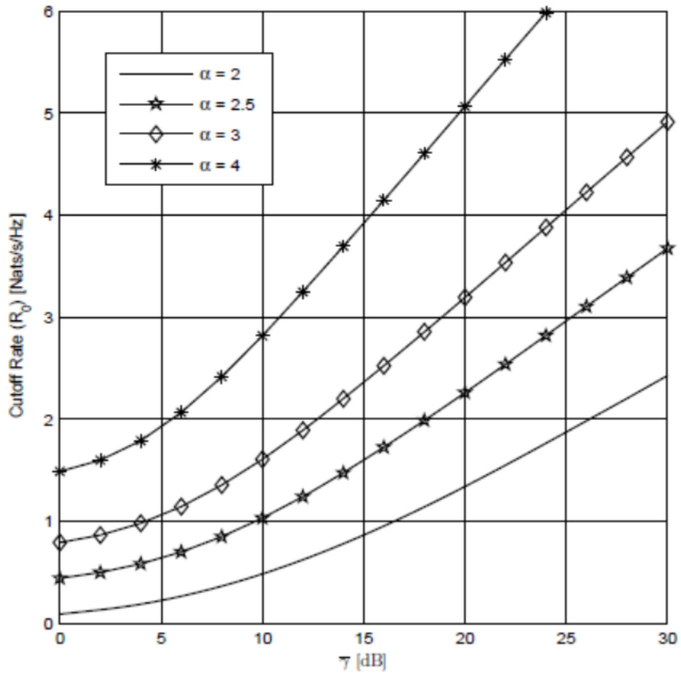

Fig. 3. Cutoff rate (R_0) versus $\overline{\gamma}$ when μ=2, and for different values of α

V. CONCLUSIONS

In this paper, we have derived novel analytical expressions for the random coding error exponent, cutoff rate, and capacity of dual-hop AF wireless communication systems over independent and identically distributed α-μ fading channels. The derived expressions were reduced to study the performance of the system over Weibull, Rayleigh, and Nakagami-m fading channels as special cases. Numerical results were provided to show the effects of the average per hop SNR and the fading parameters on the performance of the system.

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