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A New Algorithm for Reactive Power Compensation in Industrial Plant

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Abstract**—** This paper presents a method based on Lagrange multipliers for capacitor banks allocation at industrial plant networks. The aim of compensation is to avoid penalties in (JD/$) enforced on electricity bills due to low power factor, and to minimize the power losses at the plant's network. Capacitor banks will be allocated at the main board (MB) as a fixed type and at secondary distribution boards (SDB) as a regulated type. For this purpose, a mathematical formulation of the problems has been established and solved. The "equivalent consumed active energy" amount is calculated based on a newly devolved equation for finding the "equivalent working time" which is used to calculate the total amount of reactive power compensation. The proposed method is applied to the data extracted from the monthly electricity bill of an industrial plant network. A numerical example and discussion have been introduced to illustrate the effectiveness of the proposed method.

Keywords**—** Lagrange multipliers, Power factor improvement, Power loss reduction, Reactive power compensation.

I. INTRODUCTION

Reactive power is supplied to the electrical system to establish the magnetic flux of the magnetic core of transformers and induction motors, and to maintain the voltage level to deliver active power through transmission lines. The flow of reactive power can have harmful effects on the electrical equipment as well as on the electrical infrastructure since the current flowing through the electrical system is higher than it is necessary to perform the required work.

Industrial plants are usually operated under various electrical loads with various power factor (PF). The average PF in industrial plants generally varies between 0.55-0.80 [1], [2]. To reduce the harmful effects of low PF, most of the electrical utilities companies in different countries impose penalties on consumers whose average value of PF is less than a given threshold.

In Jordan, the Electricity Regulatory Commission (ERC) imposes additional payments on electrical bills as penalties for consumers whose PF is less than a certain value. These penalties are stipulated in categories and changed in discrete manner when the PF becomes low. These penalties are stated in special tariffs [3].To maintain the monthly average value of the PF within a certain category limit and to avoid the penalties enforced on electricity bills, a proper allocation of the capacitor banks into the plant's electrical network is required.

Various approaches have been used to improve the PF in electrical systems. These include various analytical, mathematical, heuristic and statistical methods [4]-[7]. The difference between these methods lies in the ways of calculation, simplicity, computational time and available data.

This paper presents an easy and straightforward inference method for determining the amount of capacitor banks required to avoid PF penalties and to reduce electrical power losses. The presented method has an advantage over other methods as it does need special mathematical tools or power flow programs. It can be easily implemented by the electrical plant engineer.

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In section 2, the industrial plant electrical network structure is presented. The power losses decoupling scheme is explained in section 3. In section 4, the mathematical model is formulated and solved using Lagrange multipliers and a new equivalent working time equation has been developed. The economic benefits gained from compensation along with illustrative examples demonstrating the application of the proposed method and conclusions are presented respectively in sections 5, 6 and 7.

II. NETWORK CONFIGURATION

Most of industrial plants networks configurations are similar as shown in Fig. 1. However, if any of the feeders of secondary distribution board (SDB-*n*) act as a main feeder for other feeders, the procedure remains the same, (i.e. one main feeder supplying several feeders).
More presumptions are considered; for example, lines parameters, symmetrical voltage and symmetrical load of each phase are identical. In this regard, the performed calculation is limited to one phase; and the achieved result will be applicable to the other two phases.

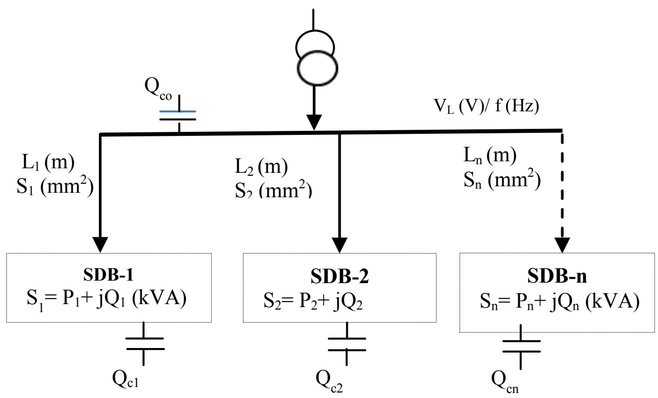

Fig.1. Industrial plant SLD

III. POWER LOSSES DECOUPLING SCHEME

The aim of this paper is to find the optimum size and location of the shunt capacitors to be installed at plant distribution boards. The solution attempts to reduce active power losses to the minimum possible level while improving the power factor. For the plant network presented in Fig. 1, one main feeder is supplying the power for several secondary distribution boards (SDB-*i*), (where: *i*=1-*n*).
The flows of the current (I_i) in each line resistance (R_{Li}) cause an active power loss (ΔP_i) as:

$$\Delta P_i = {I_i}^2 \cdot R_{Li} \tag{1}$$

The complex power (S) flowing through the line (i) has two components: active power (P_i) and reactive power (Q_i). The complex current (I) is equal to:

$$S = VI^* =\gg I^* = \frac{P_i + jQ_i}{V} \tag{2}$$

Substituting the value of the current (I) in (1) yields:

$$\Delta P_i = {I_i}^2 \cdot R_{Li} = \frac{P_i^2 + Q_i^2}{V^2} \cdot R_{Li} \tag{3}$$

Equation (3) can be decoupled into two parts as:

$$\Delta P_i = \frac{P_i^2}{V^2} \cdot R_{Li} + \frac{Q_i^2}{V^2} \cdot R_{Li} = \Delta P_{Pi} + \Delta P_{Qi} \quad (4)$$

The first part of (4) represents the real power losses (ΔP_{Pi}) due to the flow of the active power (P_i) in the lines' resistance (R_{Li}). It cannot be compensated. However, the second part represents the active power losses (ΔP_{Qi}) due to the flow of the reactive power (Q_i). This part is subject for compensation, where V is the network operating voltage.
The amount of (ΔP_{Qi}) in (4) can be written as:

$$\Delta P_{Qi} = \frac{1}{V^2} \cdot {Q_i}^2 \cdot R_{Li} \quad (5)$$

Adding an appropriate amount of leading reactive power (Q_{ci}) at the end of line (i) will compensate the flow of lagging reactive power (Q_i) of (5):

$$\Delta P_{Qi} = \frac{1}{V^2} \cdot \sum_{i=1}^{n} (Q_i - Q_{ci})^2 \cdot R_{Li} \quad (6)$$

Finding the optimum size of (Q_{ci}) in (6) that will be installed in MB and in each of SDB-i will be thoroughly discussed in further sections.

IV. THE PROPOSED METHOD

The total consumed active energy (A_p) over a certain period of time (usually one month) is equal to the area under the curve of the drawn active power ($P(t)$) over the working time ($T_{(w/m)}$) during the month as presented in Fig. 2, which is mathematically expressed as in (7).

$$A_p = \int_0^{T_{(w/m)}} P(t) \cdot dt = P_{avg} \cdot T_{(w/m)} \quad (7)$$

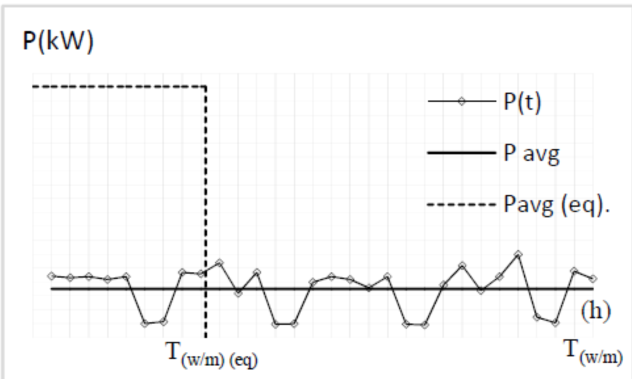

Fig. 2. The consumed active energy (A_p) for one shift industrial plant (kWh/Month)

The same area under the curve of (7) can be represented as an equivalent working time ($T_{(w/m)(eq.)}$) and equivalent drawn active power ($P_{avg(eq.)}$) as presented in Fig. 2 (dotted line) and (8).

$$A_p = \int_0^{T_{(w/m)(eq)}} P_{avg\,(eq)} \cdot dt = P_{avg\,(eq)} \cdot T_{(w/m)(eq)} \quad (8)$$

Hence, the areas under the curves in Fig. 2 (straight & dotted lines) are corresponding respectively to (7) and (8); and are equal. Therefore, if the value of the compensating power (Q_{ci}) in (6) is designed to deal with load ($P_{avg(eq.)}$), the designed Q_{ci} can cater any other load requirement during the time of the energy consumption. The value of A_p in kWh in (7) and (8) is the same; and it can be extracted from the monthly electricity bill. It is the sum of active energy consumed at day and night tariff over the month.
However, the values ($T_{(w/m)(eq.)}$) and ($P_{avg(eq.)}$) in (8) shall be determined as follows.

A. Determination of Equivalent Working Time

The equivalent working time over one month ($T_{(w/m)(eq.)}$) in hours (h), can be determined as the sum of the working hours during the time of operation (where the consumed energy is $A_{p(w)}$), the time out of operation (where the consumed energy is $A_{p\ (aw)}$) and the time of consumed energy during the holidays time ($A_{p\ (hd)}$). The three parts of time formulating the value of the equivalent working time are presented in (9).

$$T_{\left(\frac{w}{m}\right)eq.} = \frac{n_s}{3} \cdot \frac{D_{(w/y))}}{365} \cdot n_{(h/m)} + \frac{3-n_s}{3} \cdot \frac{D_{(w/y))}}{365} \cdot n_{(h/m)} \cdot \frac{A_{p\ (aw)}}{A_{p(w)}} + +(365 - D_{(w/y)}) \cdot \frac{A_{p\ (aw)}}{A_{p(w)}} \cdot \frac{24_{(h/d)}}{12_{(m/y)}} \tag{9}$$

Discussion on (9):

- If the plant is working in three shifts per day (n_s=3), that is 8h/shift with no stoppages over the year (like airports and emergency departments etc.), the number of working hours/year is equal $D_{(w/y)}$=365 day/year. Thus, if we apply this to (9), the second and third parts of (9) will equal zero; and only the first part will have a value.
- If the plant is working in three shifts (n_s=3) that is 8h/shift except during national holidays or/and shutdowns or/and weekends etc., when the load differs from that of normal operation day (like in chemical plants, big industrial factories etc.), the reaming parts will be the first and third parts since the value of the second part is zero. The second part value becomes zero due to n_s=3. However, the value of the third part in (9) can be interpreted as below:
  $(365 - D_{(w/y)}) \cdot \frac{A_{p\ (aw)}}{A_{p(w)}} \cdot \frac{24_{(h/d)}}{12_{(m/y)}}$, where (365-$D_{(w/y)}$)) is the number of working days/year (excluding holidays etc. -as above-); and the value $\frac{A_{p\ (aw)}}{A_{p(w)}}$ represents the ratio between the active energy consumed (day and night) after and during the time of operation, multiplied by the number of hours per day ($24_{(h/d)}$) and divided by the number of months per year ($12_{(m/y)}$) to obtain the value of this part in hours per month.
- If the plant works in one or two shifts (n_s= 1 or n_s= 2), 5 days/week except on public holidays, and have time for maintenance and shutdown (if any) like most public institutions, all the three parts of (9) will have their values as illustrated in the numerical example in section 6.

However, if substituting in (9); $n_{(h/m)}$= 730h, $\frac{n_{(h/m)}}{365} = 2$ and $\frac{24_{(h/d)}}{12_{(m/y)}} = 2$, we obtain:

$$T_{(w/m)eq.} = 2 \cdot D_{(w/y)} \cdot \left(\frac{n_s}{3} + \frac{3-n_s}{3} \cdot \frac{A_{p\ (aw)}}{A_{p(w)}}\right) + 2 \cdot (365 - D_{(w/y)}) \cdot \frac{A_{p\ (aw)}}{A_{p(w)}} \tag{10}$$

Equation (10) determines the equivalent working time over the month ($T_{(w/m)(eq.)}$) in hours, where:
$A_{p(w)}$– Consumed active energy (day and night) during the time of operation per month (kWh/m); $A_{p\ (aw)}$– Consumed active energy (day and night) outside the time of operation per month (kWh/m); $D_{(w/y)}$– Working days over the year (excluding holidays, shutdowns, weekends and etc.); $n_{(h/m)}$– Number of hours per month (i.e. year's hours/12 month=730h); n_s– Number of shifts during the time of operation; $T_{(w/m)(eq.)}$– Equivalent working time over the month in hours; $24_{(h/d)}$– Twenty four hours per day; $12_{(m/y)}$–12 months per year.

B. Determination of Equivalent Active Power

Knowing the amount of equivalent average power ($P_{avg(eq.)}$) drawn over the equivalent working hours over the month ($T_{(w/m)(eq.)}$) is necessary to determine the amount of the compensating capacitors banks (Q_c). Thus, (8) can be rewritten as:

$$P_{avg\,(eq)} = \frac{A_p}{T_{(w/m)\,(eq)}} \tag{11}$$

Substituting the value of $P_{avg(eq.)}$ obtained from (11) in (12), we determine the value of compensating reactive power (Q_c) that required to improve the power factor from $\cos\varphi_1$ to $\cos\varphi_2$.

$$Q_C = P_{avg\ (eq)} \cdot (tan.\,\varphi_1 - tan.\,\varphi_2\,) \tag{12}$$

The graphic illustrating the amount of (Q_c) in (12) is shown in Fig. 3, where S_1, S_2– Apparent power before and after compensation (kVA); Q_1, Q_2– Reactive power before and after compensation (kVAR); φ_1, φ_2– Load angle before and after compensation.
The amount of reactive power compensation shown in (12) is the total amount of reactive power required to achieve the desired PF value [8]. The problem now is: how to distribute the value of (Q_c) between the MB and SDB-*i* in such a manner that the benefits gained from active power loss reduction due compensation is maximum.

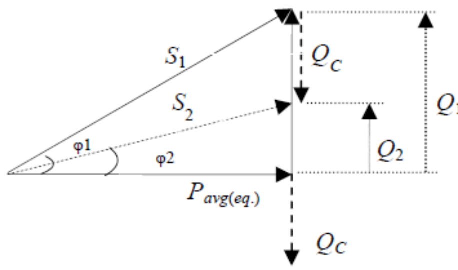

Fig. 3. Effect of adding capacitors

C. Mathematical Model Formulation

The objective function is to optimize the compensation amount (Q_{ci}) in (12); therefore, the problem can be formulated as follows:

$$\text{Find Min. } \{\Delta P_Q\} \tag{13}$$

Subject to:

$$Q_c = \sum_{i=1}^{n} Q_{ci} \tag{14}$$

With the following concentrate:

$$Q_{ci\,(Min.)} \leq Q_{ci} < Q_i \tag{15}$$

The concentrates in (15) are required to overcome the minimum standard size of the capacitor bank ($Q_{ci\,(Min.)} \leq Q_{ci}$) and to avoid over compensation at any node (SDB-i) of the plant's network; that is ($Q_{ci} < Q_i$). Equations (13)-(15) state the mathematical formulation of the problem.

D. Compensating Techniques

Lagrange multiplier (λ) is used to solve the objective function of (13) and to find the optimum solution of (14). The method of Lagrange multipliers [9], [10] is a strategy for finding the local maxima and minima of a function subject to equality constraints. It is similar to the mathematical model presented above.
The Lagrange function of (6) is then:

$$\mathcal{F} = \frac{1}{V^2} \cdot \sum_{i=1}^{n} (Q_i - Q_{ci})^2 \cdot R_{Li} + \lambda \cdot \sum_{i=1}^{n} Q_{ci} \tag{16}$$

To find the optimum values of Q_{c1}, Q_{c2} …. Q_{cn}, the partial derivatives of Lagrange function ($\mathcal{F}$) with respect to Q_{ci} shall be carried out; and the obtained result will equal zero, (i.e. $\frac{\partial \mathcal{F}}{\partial Q_{ci}} = 0$). The partial derivative of Lagrange function with respect to Q_{c1} is:

$$\frac{\partial \mathcal{F}}{\partial Q_{c1}} = -\frac{1}{V^2} \cdot 2 \cdot (Q_1 - Q_{c1}) \cdot R_{L1} + \lambda = 0 \Rightarrow\!\!> Q_{c1} = -\lambda \cdot \frac{V^2}{2} \cdot \frac{1}{R_{L1}} + Q_1 \tag{17}$$

Consequently, in the same way we will derive $\frac{\partial \mathcal{F}}{\partial Q_{c2}} = 0, \cdots \frac{\partial \mathcal{F}}{\partial Q_{cn}} = 0$. The result for Q_{cn} is:

$$\frac{\partial \mathcal{F}}{\partial Q_{cn}} = -\frac{1}{V^2} \cdot 2 \cdot (Q_n - Q_{cn}) \cdot R_{Ln} + \lambda = 0 \Rightarrow\!\!> Q_{cn} = -\lambda \cdot \frac{V^2}{2} \cdot \frac{1}{R_{Ln}} + Q_n \tag{18}$$

Having in mind (14), $Q_c = Q_{c1} + Q_{c2} + \cdots + Q_{cn}$ yields:

$$Q_c = -\lambda \cdot \frac{V^2}{2} \cdot \left(\frac{1}{R_{L1}} + \frac{1}{R_{L2}} + \cdots \frac{1}{R_{Ln}}\right) + Q \Rightarrow\!\!> \quad Q_c = -\lambda \cdot \frac{V^2}{2} \cdot \frac{1}{R_{eq.}} + Q \tag{19}$$

where the lines resistance $(R_{Li})\ in\ (\Omega)$:

$$R_{Li}(\Omega) = \frac{L_i\,(m)}{S_i(mm^2) \times \gamma(1/\Omega \cdot m)} \tag{20}$$

The inverse of the equivalent lines resistance ($\frac{1}{R_{eq.}}$) is equal to:

$$\frac{1}{R_{eq.}} = \frac{1}{R_{L1}} + \cdots \frac{1}{R_{Ln}} \tag{21}$$

Finally, the Lagrange factor (λ) in (19) can be written as:

$$\lambda = \frac{1}{V^2} \cdot 2 \cdot (Q - Q_c) \cdot R_{eq.} \quad (22)$$

Substituting the value of λ in (17) and (18):

$$\frac{1}{V^2} \cdot 2 \cdot (Q_1 - Q_{c1}) \cdot R_{L1} = \cdots = \frac{1}{V^2} \cdot 2 \cdot (Q - Q_c) \cdot R_{eq.} \quad (23)$$

If (23) is generalized for any number of lines (R_{Li}), we obtain:

$$\frac{1}{V^2} \cdot 2 \cdot (Q_i - Q_{ci}) \cdot R_{Li} = \frac{1}{V^2} \cdot 2 \cdot (Q - Q_c) \cdot R_{eq.} \quad (24)$$

Finally, (25) can be derived from (24). It presents the optimum size of Q_{ci} in each SDB-*i*:

$$Q_{ci} = Q_i - (Q - Q_c) \cdot \frac{R_{eq.}}{R_{Li}} \quad (25)$$

where Q_{ci}–The capacitor bank value at SDB-*i* in kVAR; Q_i– The reactive load at SDB-*i* in kVAR; Q– The sum of reactive power load of SDB-*i*-*n* in kVAR; Q_c– The total amount of compensating capacitors value obtained from (12) in kVAR.

V. COMPENSATION COST

The net revenue saving (S_R) achieved from the application of capacitors is the difference between the costs of the compensating capacitors against the value of the power factor penalties (PEN).

The cost of the three-phase standard unit compensating capacitors (C_c) is composed of the capital and installation cost, including accessories such as switches, control cables, circuit breakers, and erection cost etc. Thus, the monthly money saving is defined by:

$$S_R = \frac{\sum_{i=1}^{m} PEN.}{m} - \frac{1}{12} \cdot C_{c(year)} \quad (26)$$

where $C_{c(year)}$ is the yearly cost of the compensating capacitors [11], [12]:

$$C_{c(year)} = \frac{2 \cdot C_c \cdot (L+1-d)}{L(L+1)} \quad (27)$$

where L– Life time of the compensating capacitor in years (estimated 25 years for capacitors); d– The progression of the capacitor; it is one for the first year, two for the second and so up to L-year.

Another economic factor for evaluating the economic effect of compensation is the payback period (Pb). Pb is defined as the length of time required for an investment to recover its initial outlay in terms of profits or savings. It can be mathematically expressed as:

$$\text{Payback period } (Pb) = \frac{\text{Total cost (JD)}}{\text{PEN (JD/Month)}} \quad (28)$$

VI. NUMERICAL EXAMPLES

A. *Industrial Plant Data*

A.1. Plant input network data

An industrial plant having a network configured as shown in Fig. 1 is fed by copper cables, whose lengths (L) are in meters, cross section (S) in mm^2. The lines loads are as stated in Table 1.

TABLE 1
PLANT INPUT NETWORK DATA

SDB-i no.	P_i (kW)	Q_i (kVAR)	L_i (m)	S (mm^2)	R_{Li} (Ω)	$1/R_{Li}$(1/Ω)
1	78.57	104.66	150	35	0.0779	12.8333
2	125.71	128.23	70	70	0.0182	55.0000
3	47.14	50.76	230	90	0.0465	21.5217
4	94.29	90.83	250	150	0.0303	33.0000
Sum	345.71	374.47			122.35	0.00817

A.2. Plant consumed energy Data

The data extracted from the plant electricity bill for a certain month is as presented in Table 2.

TABLE 2
ELECTRICITY BILL DATA

A_p (day tariff) (kWh)	38,976
A_p (night tariff) (kWh)	58,464
Total A_p (kWh)	97440
A_Q (kVARh)	93,912
Power Factor (before compensation)	0.720
PEN in JD	1,051.62
Total Bill Amount JD	9,920.80

A.3. Plant operational data

- Cable conductivity; σ (S/m) at 20°C is equal to 35x10^6.
- Power factor after compensation shall be not less than 0.94.
- n_s=1, (n_s– number of shifts- 8 hours for each shift).
- $D_{(w/y)}$– Working days over the year (excluding holidays, shutdowns, weekends and etc.) are 5/7 working days per week plus 12 days per year as national holidays ($D_{(w/y)} \cong 248.7\ days$).

B. *Determination of Capacitors Size*

The achieved solution seeks to allocate the reactive power compensation amount Q_{ci} in MB and in each (SDB-i), and to achieve the maximum benefit on active power loss reduction using minimum loss criteria.

C. *Solution Algorithm*

The steps of problem solving are as follows:

1. Measure the consumed active energy during and after the working hours (obtained from the reading of active energy meter).

The measured quantity after the working hours is taken here for one day. The more is the measured days, the more accuracy of the achieved result is. The average value of the days shall be carried out. For the above numerical example, the value of the measured data is as follows:

$A_{p(w)}$= 3851kWh, $A_{p(aw)}$= 770kWh and $A_{Q(aw)}$= 639kVARh.

On basis of the above values, the power factor of the load after working hours $\cos\varphi_{(aw)}$ can be calculated as bellow:

$$tan.\varphi_{(aw)} = \frac{A_{Q(aw)}}{A_{p(aw)}} = \frac{639}{770} = 0.8298 \ \Rightarrow \ cos\varphi_{(aw)} = cos(tan.\varphi_{(aw)}{}^{-1}) \cong 0.77$$

The relation between the consumed energy after working hours $A_{p(aw)}$ and the energy consumed during the working operation $A_{p(w)}$ is presented as below:

$$\frac{A_{p\ (aw)}}{A_{p(w)}} = \frac{770}{3851} \cong 0.2$$

2. Calculate the equivalent working time as in (10):

$$T_{(w/m)eq.} = 2 \cdot 248 \cdot \left(\frac{1}{3} + \frac{3-1}{3} \cdot 0.2\right) + 2 \cdot (365 - 248) \cdot 0.2 = 278\ (h)$$

3. Calculate the equivalent active power as in (11):

$$P_{avg\,(eq)} = \frac{A_p}{T_{(w/m)(eq)}} = \frac{97440\ (kWh)}{278\ (h)} = 350.1\,9\ (kW)$$

4. Calculate the amount of reactive power compensation (Q_c) as in (12):

$$Q_C = 350.1\,9 \cdot (tan.(cos\,0.72)^{-1} - tan.(cos\,0.94)^{-1}) = 350.19 \cdot (0.9638 - 0.3629) = 210.43\ (kVAR)$$

5. Distribute the (Q_c) among the MB as fixed type and at secondary distribution boards (SDB-*i*) as a regulated type of capacitor banks:

5.1. Fixed type capacitor banks (Q_{c0}) value (kVAR)

The amount of (Q_{c0}) to be installed at the main board (MB) to cater the compensation amount after working hours and at the light load is as follows:

$$Q_{c(0)} = \frac{Ap_{(aw)}}{Ap_{(w)}} \cdot P_{avg\,(eq)} \cdot \left(tan.\varphi_{(aw)} - tan.\varphi_2\right) = \frac{770\ (kwh)}{3851\,(kWh)} \cdot 350.1\,9 \cdot \left(tan.(cos(0.77)^{-1}) - tan.(cos(0.94)^{-1})\right) = 32.64\ (kVAR)$$

5.2. The regulated type of capacitors bank value (Q_{cr}) (kVAR)

The amount of (Q_{cr}) to be installed at SDB-*i* (*i*=1-*n*) to cater the compensation amount during the working hours is as bellow:

$$Q_{c(r)} = Q_c - Q_{c(0)} = 210.43 - 32.64 = 177.79\ (kVAR)$$

Where

$$Q_c = Q_{c(0)} + Q_{c(r)}$$

5.2.1. Calculate the lines resistance using (20):

$$R_{li}\ (\Omega) = \frac{L_i\ (m)}{S_i(mm^2)\cdot (S/m)}$$

And calculate the equivalent lines inverse resistances using (21):

$$\frac{1}{R_{eq}} = \frac{1}{R_{l1}} + \cdots \frac{1}{R_{ln}}$$

The result is presented in Table 1.

5.2.2. Distributing the compensation amount (Q_{cr}) among (SDB-$i's$) by (25):

- As an example, the calculation is performed for Q_{c1}, (i.e.: for SDB-1):

$$Q_{c1} = 104.66 - (374.47 - 177.79)\cdot\frac{0.008173}{0.0779} = 84.05\ (kVAR)$$

- Round the calculated compensator amount to the nearest standard unit size:

$$Q_{c1(stnd.)} = 90\ (kVAR)$$

- Find the per phase capacitance:

$$Q_{(c1/phase)} = \frac{Q_{(c1)}}{3} = \frac{90}{3} = 30\ (kVAR)$$

$Q_{(c1/phase)} = \frac{Q_{ci}}{3} = \frac{V_{(ph)}^2}{X_{c1}} \Rightarrow X_{c1} = \frac{3\cdot V_{(ph)}^2}{Q_{(c1)}} = \frac{3\cdot 400^2}{90\cdot 10^3} = 5.333\Omega$ (for delta connected $V_{(ph)}=V_{(L)}$).

- Finally the capacitance (C) in Farad is equal to:

$$c_1 = \frac{1}{X_{(c1)}\cdot 2\cdot\pi\cdot f} = \frac{1}{5.333\cdot 2\cdot\pi\cdot 50} = 596.8\mu F/phase$$

$$c_{1(3\,phase)} = 596.8\mu F/phase\cdot 3 = 1790.5\mu F$$

- The results of the remaining compensators amount (Q_{ci}) in each of SDB-i and the capacitance (µF) are presented in Table 3.

Note: a negative sign (-) of Q_{ci} in (25) means that; with existing load and cables cross section relation, there is no enough influence to compensate in this SDB and the value of this -Q_{ci} shall be deducted proportionally from the remaining SDB, keeping in mind that concentrates of (14) and (15) shall be maintained.

TABLE III
COMPENSATION SUMMARY

SDB no.	$Q_{(ci)}$ (kVAR)	$Q_{(ci)\ std.}$ (kVAR)	$Q_{(ci)}$ (JD/kVAR)	$Q_{(ci)}$ (JD)	$C_{(capacitance/Phase)}$ (µF)	S_R (JD/year)	Pb (Month)
Q_{co}	32.64	35	25	875	696.08		
Q_{c1}	84.05	90	35	3150	1790.002		
Q_{c2}	39.90	40	35	1400	795,63		
Q_{c3}	16.19	20	35	700	397,8		
Q_{c4}	37.83	40	35	1400	795,63		
SUM	210.61	225	33.4	7525	1491.96	1003.4	7.15

6. Check the cost function

Let us assume that the installed cost (one kVAR) includes the protective device (breaker/fuse), installation labor and material; for fixed type, LV is around 20JD. For LV switched type is around 35JD. The net revenue saving (S_R) using (26) and (27) is:

$$C_{c(year)} = \frac{2 \cdot 7525 \cdot (25+1-1)}{25(25+1)} = \frac{376250}{650} = 578.8(\text{JD})$$

$$S_R = 1051.6 - \frac{1}{12} \cdot 578.8 = 1003.4(\text{JD/Month})$$

And the payback period (Pb) is as follows:

$$\text{Payback period } (Pb) = \frac{\text{Total cost (JD)}}{\text{PF. Penlty (JD/Month)}} = \frac{7525}{1051.6} = 7.15 \text{ (Month)}$$

VII. CONCLUSIONS

This paper presents a method based on Lagrange multipliers technique. The method aims to find the reactive power compensation amount in KVAR to avoid the penalties in JD/$ imposed on electricity bill due to low power factor. Capacitors banks are allocated at the main bus (MB) as a fixed type and at secondary distribution boards (SDB's) as a regulated type. The achieved results show that there is a monetary benefit due to power factor improvement; the payback period is less than eight months based on the existing price level of the energy and the compensation equipment.

An equation for the “equivalent working time” has been devolved. The proposed equation is used to find the equivalent active power flow through the lines of the system. It is also used to find the total reactive power required for compensation. Also, this equation can be very useful to use while designing of the electrical equipment (cables, transformers, etc.).

The presented algorithm is illustrated through a straightforward numerical example, which is simple to follow and does not need high mathematical tools or special software to solve the objective function. It is easy to implement by the electrical plant engineer; and allows the plant engineer to evaluate the whole cost and benefits due to compensation.

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