



AC Optimal Power Flow Using Whale and Sine-Cosine Optimization Algorithms

Moath Bani Fayyad^{1*}, Lina Alhmoud²

¹ Al-Ahliyya Amman University, Amman, Jordan

E-mail: m.banifayyad@ammanu.edu.jo

² Yarmouk University, Irbid, Jordan

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Abstract— This work presents two meta-heuristic optimization approaches, namely the Whale Optimization Algorithm (WOA) and the Sine-Cosine Algorithm (SCA), which are applied to solve the AC Optimal Power Flow (ACOPF) problem in the power system. The objective is to minimize the total fuel cost of power generation while satisfying power balance equations and operational constraints of generators and transmission networks. The proposed methods are evaluated on the standard IEEE 30-bus test system. The major benefit of the ACOPF is to find the optimal operating conditions of generators in a power system, subject to the generators' and system constraints across diverse scenarios. The performance of WOA and SCA is compared with that of a conventional Genetic Algorithm (GA) used as a benchmark. Simulation results demonstrate that the proposed algorithms achieve fuel cost values within the commonly reported range for the IEEE 30-bus AC-OPF problem, with WOA yielding a minimum fuel cost of 801.85 \$/h and transmission losses of 9.36 MW. The obtained generator dispatch patterns and power loss values are consistent with those reported in recent literature. The results confirm that WOA and SCA provide competitive and reliable solutions for the AC-OPF problem while preserving the standard problem formulation.

Keywords— AC optimal power flow; Optimization; Whale optimization algorithm; Sine-Cosine optimization algorithm.

1. INTRODUCTION

The AC Optimal Power Flow (ACOPF) is a rich field in power systems for achieving complete optimization of power operations. Research has shown interest in the application of optimization in power system domains [1]. Those domains are considered highly nonlinear and complex, with inflexible and tough rules: high-level modeling details, very large-scale, numerous details in complex systems. Demanding performance requirements must get a good solution quickly. The presence of renewables is one of the reasons that the power system grids are changing continuously. A smart load concept arises, meaning energy consumption responds to price signals and follows algorithms intelligently; this is also related to the so-called demand response—the idea of local generation also plays an important role in grid change. Consumers own their renewable generation assets, which impact the grid. Hence, power grids are changing, which is spurring significant interest in the main ACOPF-based optimization task used to operate them [2].

A power flow analysis is a steady-state analysis whose goal is to determine currents, voltages, real and reactive power flow, in a system under conveyed load conditions [3].

The optimal power flow problem was first formulated roughly 60 years ago [1]. The power system has generators, power lines, transformers, and Buses (nodes). Each bus has a load, i.e., a numerical demand for power. The objective is to meet demand at minimum cost, with costs incurred at the generators. Awareness of the solution boundary of the AC power flow problem is focal for evaluating strategies to improve robustness and determining the robustness of operating points. A method of exploring that solution boundary has been developed. Examples explain some of the possible forms that the solution boundary can expose [4]. It appears that quite difficult behavior is possible. This could extensively influence the formulation of algorithms to improve system robustness optimally. The ACOPF is a mathematical optimization problem; the classical objective function is to minimize the generation cost, i.e., the sum of the costs of all generators. Also, equality constraints, concerned that the power at each node is balanced, are represented by the power flow equations. The mathematical formulation of the ACOPF is based on the decision variables or control variables (u) such as at each generating units, the voltage and the active power output, the phase shifter (quad booster) taps position, position of the transformer taps, control of power electronics (FACTS, HVDC), status of switched reactors and capacitors, and amount of disconnected load [5]. Next, the ACOPF optimization problem has state variables (x) that describe the system's response to control variables changeset. Here, the state variables are the magnitudes of the voltages at each bus, except at the generating buses. Also, the state variables can be the angles of the voltages at each bus, except at the slack bus. The WOA is utilized to reduce the fuel cost with losses. In this work, the IEEE-30 bus test system is used to evaluate the validity of WOA in improving system performance [6]. The objective function is the fuel cost. Optimizing the control variables are the fundamental emphasis while addressing objective functions possess both equality and inequality constraints to be satisfied.

Two main constraints are enforced including an equality constraint that the generation must be balanced to the load and inequality constraints mean there are lower and upper limits on generating units' outputs. Here, the economic dispatch is limited in its application in power systems because, in practice, not all loads and generating units are allied to the same bus, resulting in unacceptable flows in transmission lines or unacceptable voltages at network buses. Also, the state variables can be the voltage angles at each bus, except at the slack bus.

The main ACOPF challenges are the size of the problem: some problems exceed 1000s of lines and hundreds of controls, and in some cases, the inequality constraints are binding. Other challenges stem from the problem of being nonlinear and nonconvex. Some variables are discrete based on the position of the phase shifter taps and the transformer and the condition of switched capacitors and reactors. In power systems communities, numerous studies have discussed ACOPF since the first publication in 1962 [1]. At the steady state operating point, minimizing the cost of electric power generation around it is the main objective of the ACOPF. While keeping the demand and the operating constraints adequately satisfied. Both the constraints and the objective function are nonlinear; hence, the problem is expressed as a nonlinear programming (NLP) problem [2].

The flow of power in transmission lines is depicted by using Kirchhoff's voltage law (KVL), and bus voltages and phase angles for computing real and reactive power to establish the ACOPF in a nonlinear function. Hence, in the manifestation of these nonlinear equality constraints in a nonconvex and feasible region. So, the ACOPF problem is characterized as a

large-dimensional, nonconvex, nonlinear optimization problem with many operational constraints [3].

In the literature, many optimization approaches are used to solve ACOPF problems. Such as quadratic programming, Newton-based techniques, sequential unconstrained minimization technique, and linear vs. nonlinear programming [2]. Generally, these approaches have drawbacks, such as algorithmic complexity and unstable convergence. These techniques have weaknesses accompanying with the piecewise-quadratic cost approximations and are difficult to use when the penalization issues become tremendously considerable. The latest Meta-heuristic optimization approaches, named SCA and (WOA) methods, are a new strategy to solve the ACOPF problem and have a flexible and well-balanced mechanism [7, 8].

This paper is a comparative investigation the use of the Whale Optimization Algorithm (WOA), and the Sine-Cosine Algorithm (SCA) to solve the AC Optimal Power Flow (ACOPF) problem. The objective was to compare these two algorithms based upon their ability to minimize fuel cost as well as their performance when using penalty function methods to handle constraints. Both algorithms were tested under the same conditions utilizing the IEEE 30-bus test case. In addition, each algorithm was run independently thirty times. The results from this research identified significant differences in the performance of these two algorithms relative to their robustness and rate of convergence and provide practical insight into the application of meta-heuristics in power system optimization problems.

2. ACOPF FORMULATION

The ACOPF can be expressed mathematically; it is formulated as follows [9, 10]:

$$C_i(PG_i) = \sum_{i=1}^{NG} \alpha_i + \beta_i PG_i + \gamma_i PG_i^2 \quad (1)$$

where

- C_i : The total cost function.
- PG_i : Power generation.
- NG : Number of generators respectively.
- i : number of generators.
- $\alpha_i, \beta_i, \gamma_i$: the coefficients of quadratic production cost function.

Equality constraints for ACOPF

1. Real power flow equation:

$$P_{Gi} - P_{Di} = \sum_{j=1}^{NB} V_i V_j [G_{ij} \cos(\theta_i - \theta_j) + B_{ij} \sin(\theta_i - \theta_j)] \quad (2)$$

$i=1$ to n (n = number of generators)

2. Reactive power flow equation

$$Q_{Gi} - Q_{Di} = \sum_{j=1}^{NB} V_i V_j [G_{ij} \sin(\theta_i - \theta_j) - B_{ij} \cos(\theta_i - \theta_j)] \quad (3)$$

$i=1$ to n (n = number of generators)

where:

- NB is the number of buses, P_{Gi} is the active power generation,
- Q_{Gi} is the reactive power generation, P_{Di} is the active load demand,
- Q_{Di} is the reactive load demand, G_{ki} and B_{ki} are the conductance and susceptance between bus i and bus k , respectively.

Inequality constraints:

1. Limits on the control variables

$$U^{min} \leq U \leq U^{max} \quad (4)$$

2. Operating limits on voltages

$$V^{min} \leq V \leq V^{max} \quad (5)$$

3. Operating limits on real power

$$P^{min} \leq P \leq P^{max} \quad (6)$$

4. Operating limits on reactive power

$$Q^{min} \leq Q \leq Q^{max} \quad (7)$$

5. Operating limits on voltage angle

$$\theta^{min} \leq \theta \leq \theta^{max} \quad (8)$$

3. OPTIMIZATION ALGORITHMS

3.1. Sine Cosine Algorithm (SCA)

The optimization algorithm based the SCA is adopted for solving optimization systems. Multiple initial random candidates` solutions are created by SCA. These candidates move toward or away from the current best solution using a position-update model governed by sine and cosine functions, with adaptive and random parameters embedded in the algorithm to manage exploration and exploitation across various optimization problems effectively, see Fig. 1 **Error! Reference source not found.** The SCA is a metaheuristic optimization algorithm stimulated by the trigonometric sine and cosine functions introduced by Mirjalili in 2016. It is a simple mechanism but prone to local optima. The algorithm dynamically balances exploration and exploitation through sine- and cosine-based position-updating mechanisms. It is based on population-optimization that uses sine and cosine functions to control the step size and direction of solutions, finding the optimal solution by enabling local and global search in the solution space. The procedure can be concise as follows: (i) initialize the population of candidate solutions randomly within the defined search space; (ii) evaluate the fitness of each individual in the population; (iii) identify the best solution Pbest found so far; (iv) update the position of each individual using sine or cosine based equations.; (v) update the best solution if a better solution is found and finally, repeat until the stopping criterion is met [12]. This technique introduces sigmoid-based nonlinear mapping to smooth updates, improves exploration- exploitation balance and convergence stability. The performance results achieves faster convergence, higher robustness and better global search. The main limitations observed that the convergence may slow in flat or narrow regions, the directions updates remain random with limiting adaptability [13].

Generally, the population is the first step in generating a set of random solutions based on optimization techniques. This solution is estimated using an objective function. Also, a set of rules is improved using the same solution. These rules are considered the core of an optimization technique. This selection of the random population is stochastic [11]. However, there is no warrant to find the solution in a single run. There should be an adequate number of optimization steps, sufficient iterations, and random solutions. Thus, the chance of obtaining the global optimum grows. There are many algorithms for stochastic populations in the field that differ from one another. The harmony between these algorithms can be divided into two phases: exploitation versus exploration. Regarding the former phase, to obtain promising regions of the search space at a high rate of randomness, the algorithm immediately

relates the random solutions from set [14]. During the exploitation phase, the random solution undergoes continuous changes. Keep in mind that the exploration phase has more random variations than the exploitation phase.

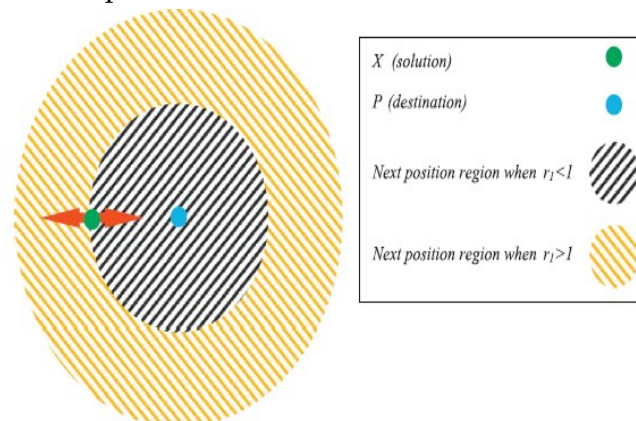


Fig.1. Effects of Sine and Cosine on the next position **Error! Reference source not found.**

The anticipated algorithm hypothetically can find out the global optimum of optimization problems due to:

- SCA creates and improves a set of random solutions for a given problem, so it inherently benefits from high exploration and local-optima avoidance compared to individual-based algorithms.
- The SCA algorithm smoothly transitions from exploration to exploitation using adaptive range in the sine and cosine functions.
- The best approximation of the global optimum is stored in a variable as the destination point and never gets lost during optimization.
- Since solutions always update their positions towards the best solution obtained so far, there is a tendency to converge to the best regions of the search space during optimization.

In stochastic systems, many algorithms can be combined with SCA to improve its performance. Finally, the SCA is very useful to solve the OPF problem.

3.2. Whale Optimization Algorithm (WOA)

In 2016, WOA was suggested via Ali Mirjalili and Andrew Lewis. It is a nature-inspired novel meta-heuristic optimization algorithm [15]. It simulates the social performance of humpback whales. Whales are elegant and massive animals. An adult whale can grow to 180 tons in weight and 30 m long. The fascinating thing about whales is that they are considered extremely rational and emotional animals, despite being considered predators. They never sleep completely; only half of their brain sleeps, because they must breathe close to the surface of the ocean. The hunting technique of the baleen whale is adopted in WOA. The bubble-net feeding strategy is an exceptional pursuit process. Whales' seeking strategy is to catch small fish near the surface in a circular path, as shown in the Fig. 2.

The WOA is a novel meta-heuristic algorithm that uses Whales, the largest mammals on Earth. They are intelligent due to the presence of spindle cells in their brains. Whales can mature their own dialects, and they live in groups. There are seven types of whales, and the humpback whale is one of these types [17]. It has an individual hunting mechanism called the bubble-net

feeding technique. This scavenging behavior concerns initiating a special spiral-shaped (9 shape) path.



Fig. 2. Bubble-net feeding behavior of humpback whales [16].

Humpback whales identify the position of their prey and encircle it; similarly, the best solution approximates the optimum. After identifying this agent, the remaining agents need to update their positions toward it, as shown in the following equation [15, 16]:

$$D = |C \cdot X'(t) - X'(t)| \quad (9)$$

$$X(t + 1) = X'(t) - A \cdot D \quad (10)$$

where A and C are coefficient vectors and can be calculated as the following equations; t represents the present iteration, X' is the position vector of the best solution acquired as such, X is the position vector, $| |$ is the absolute value and is a component by-component multiplication. Keep in mind that it is essential that X' should be updated in every iteration if there is a better solution [15, 16].

$$A = 2a \cdot r - a \quad (11)$$

$$C = 2 \cdot r \quad (12)$$

At investigation and exploitation stages, a is promptly shrunk from 2 to 0 through the span of emphases and r is an arbitrary vector in $[0,1]$. Eq. (10) permits any hunt operator to update its position throughout the existing best solution and remakes involving the prey [15, 16].

Two mechanisms are the sustaining behavior of humpback whales:

1. *Convergent Circling System*: This behavior is achieved by declining the approximation of a in the Eq. (11). Keeping that the variability extent of A is additionally lessened by a [17, 18].
2. *Spiral redesigning position*: In this mechanism, Once the separation between the prey's position and the whale's position is determined, then the characteristic helix-shaped motion of the humpback whale is constructed. As shown in the following equation [16]:

$$X(t + 1) = D' e^{bl} \cos(2\pi l) + X'(t) \quad (13)$$

where b is a constant, l is a arbitrary number in $[-1,1]$. $D' = |X'(t) - X(t)|$ is the distance between the best solution of prey and the i^{th} whale. The two aforementioned mechanisms reflect the natural circling patterns of humpback whales during hunting. In the algorithm, these behaviors are mathematically modeled. Both behaviors and updates each whale's position by selecting between them with a 50% probability, as follows [16]:

$$X(t+1) = \begin{cases} X'(t) - A.D, & p < 0.5 \\ D'e^{bl} \cos(2\pi l) + X'(t), & p \geq 0.5 \end{cases} \quad (14)$$

where p is an arbitrary number in $[0; 1]$.

During the exploration phase, humpback whale search agents randomly explore the search space to locate the optimal solution (prey) [19]. The prey's position is updated based on the positions of other whales. In this context, parameter A is assigned values exceeding 1 or less than -1 to enable a search agent to move away from a reference whale. Unlike the exploitation phase, the position of a search agent in the exploration stage is updated with respect to a randomly chosen agent rather than the current best solution [16, 20].

3.3. Genetic Algorithm (GA)

A genetic algorithm is an efficient and reliable optimization algorithm stimulated based on natural selection. It is a biological-inspired stochastic metaheuristic for combinatorial search and optimization. It allows for the evolution of possible solutions to difficult computational problems [21-23].

The procedure is starting with (i) create an initial population of random chromosome (solution); (ii) Score each chromosome in the population for fitness after define a fitness function, (iii) select the fittest members by creating a new generation of the population; (iv) use crossover and mutation to create a new and improved generation then repeat the last two steps. Finally, emit the fitness chromosome as the solution. Performance is measured using solution fitness, clock time to convergence, iteration to convergence, number of solutions explored, and inheritability.

4. STUDY CASE AND RESULTS

The proposed SCA and WOA algorithms is tested on the standard IEEE 30-bus with 6 generator units, 41 transmission lines and the load demand is 283.4 [MW] test system shown in Fig. 3.

3.4. IEEE 30- Bus Test System

The required data for this test system in Fig .3 are shown in Table 1 [23].

Table 1. Fuel cost function parameters.

Unit No.	Fuel Cost Coefficients			Generation Limits	
	a [\$/MW ² h]	b [\$/MWh]	c [\$/h]	p^{\min} [MW]	p^{\max} [MW]
1	0.00375	2.00	0	50	200
2	0.01750	1.75	0	20	80
5	0.06250	1.00	0	15	50
8	0.00834	3.25	0	10	35
11	0.02500	3.00	0	10	30
13	0.02500	3.00	0	12	40

3.5. RESULT USING ALGORITHMS

In all simulations, a population size of 30 individuals was used for each optimization algorithm, as this value provides a good balance between exploration capability and

computational cost for the IEEE 30-bus system. The maximum number of generations was set to 500 based on preliminary convergence tests showing negligible improvement beyond approximately 400 iterations, and the crossover and mutation probabilities for the Genetic Algorithm were set to their default values as provided by the MATLAB optimization toolbox; additional tests with different population sizes (20, 30, 50) and iteration limits (300, 500) confirmed the effectiveness of the selected settings.

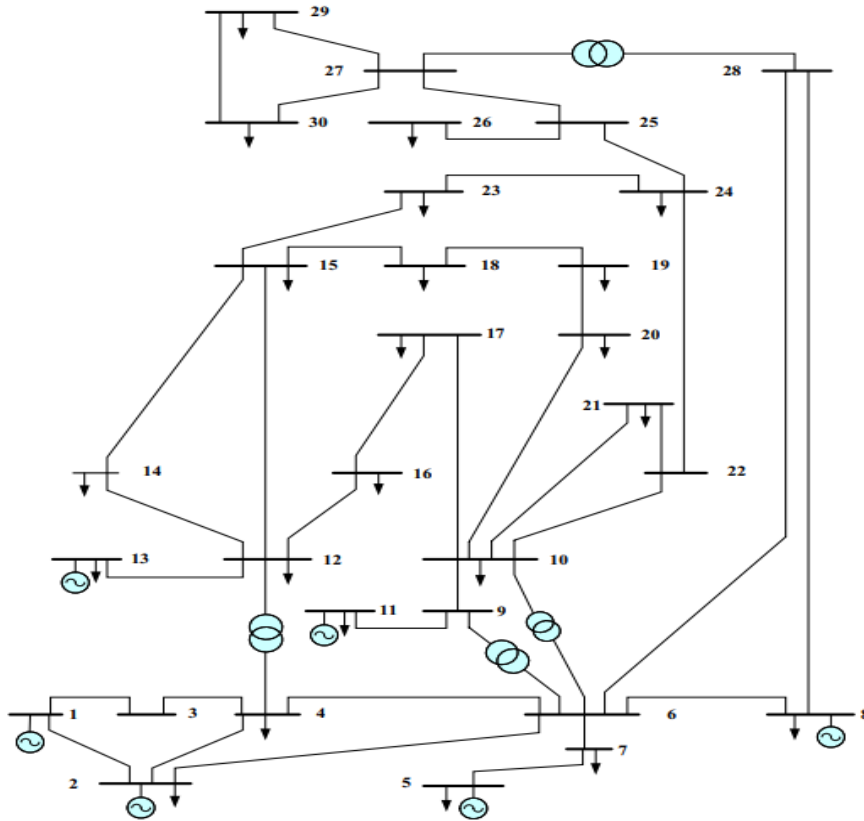


Fig.3. IEEE 30- bus test system [21].

Tables 2 and 3 present a comparative evaluation of the proposed WOA and SCA algorithms against classical and recent metaheuristic approaches on the standard IEEE 30-bus test system with six generators and a total load demand of 283.4 MW. As shown in Table 2, the fuel cost obtained by the proposed WOA (801.85 \$/h) and SCA (802.20 \$/h) lies within the commonly reported range of 801–803 \$/h for GA-based and recent metaheuristic solutions. Table 2. Mean and standard deviation of total fuel cost and transmission power loss obtained over 30 independent runs for the IEEE 30-bus AC optimal power flow problem. The associated transmission power losses are also comparable with published results, remaining within the typical range of 8–10 MW. Table 3 further demonstrates that the generator dispatch obtained by the proposed methods follows the same operating patterns reported in the literature, with generator outputs remaining within their prescribed limits and consistent with previously published IEEE 30-bus OPF solutions. Minor variations in dispatch and losses are expected due to the stochastic nature of metaheuristic algorithms and different constraint-handling strategies. Overall, the results confirm that the proposed approaches provide competitive and reliable AC-OPF solutions while preserving the standard problem formulation. In addition to the proposed methods, the results were compared with representative studies from literature using the same IEEE 30-bus test system. As shown in Tables 2 and 3, the performance ranges

reported for the Genetic Algorithm (GA) in [23] and for recent metaheuristic approaches such as GBO, SSA, and NGO in [21] are included for reference. The fuel cost and power loss values obtained by the proposed WOA and SCA fall within the same ranges reported in these studies. Moreover, the generator dispatch values obtained in this work are consistent with the operating ranges reported in [23] and [21], confirming the validity and competitiveness of the proposed approaches when compared with well-established and recent optimization techniques.

Table 2. WOA, SCA, and GA with power output of six generators, statistical comparison of fuel cost and power loss for IEEE 30-Bus AC-OPF (30 runs).

Optimization methods	Power losses P^{loss}	Total Cost (Fuel Cost)
	[MW] Mean±Std	[\$/h] Mean±Std
WOA (proposed)	9.36 ±0.03	801.852 ± 0.004
SCA (proposed)	9.67 ± 0.05	802.2031± 0.006
GA (benchmark)	9.40 ± 0.04	801.855 ± 0.005
GA [23]	9-10	801-803
GBO, SSA, NGO [21]	8-10	802-803

Table 3. Statistical generator output results for IEEE 30-Bus system (30 runs).

Optimization methods	P_1 [MW]	P_2 [MW]	P_5 [MW]	P_8 [MW]	P_{11} [MW]	P_{13} [MW]
WOA (proposed)	176.6 ± 0.4	84.7 ± 0.3	21.5 ± 0.2	22.3 ± 0.3	11.7 ± 0.2	12.0 ± 0.1
SCA (proposed)	181.6 ± 0.6	47.2 ± 0.4	22.5 ± 0.3	19.8 ± 0.4	10.0 ± 0.2	12.0 ± 0.1
GA (benchmark)	176.9 ± 0.5	49.2 ± 0.3	21.4 ± 0.2	21.9 ± 0.3	12.2 ± 0.2	11.2 ± 0.2
GA [23]	170-185	45-55	20-25	18-25	10-15	10-15
GBO, SSA, NGO [21]	170-190	45-55	20-25	18-25	10-15	10-15

5. DISCUSSION

The selected algorithms are up to the task and relevant to the system under study, but the future holds more approaches in heavy power system engineering and hard science. The findings of SCA and WOA are incredibly reasonable and certain within 500 iterations. From Tables 2 and 3, the power output of six generators in WOA and SCA is the best optimum output power from GA. The generator dispatches generated by the two proposed methods shown in Table 3 follow the same operating conditions as those described in prior studies; i.e., the generator output remains within their prescribed operating limits and are similar to other known IEEE 30-bus OPF solution results for generators. It is anticipated that there will be minor differences in generator output, system loss, etc., because of the inherent random nature of metaheuristics, and the differing ways in which constraints were treated in each case. Therefore, overall, it appears that the proposed approaches produce competitive and reliable AC-OPF solutions that also maintain the original problem formulations. The inclusion of literature-based ranges from GA [23] and GBO, SSA, and NGO [21] further demonstrates that

the proposed algorithms produce solutions that are consistent with benchmark and state-of-the-art OPF results. The IEEE 30-bus system was selected as the test case in this study because it is a widely used benchmark in AC optimal power flow research and allows direct comparison with numerous existing works. It includes the main characteristics of practical OPF problems, such as multiple generators and control variables. Although additional benchmark systems could further demonstrate scalability, this work focuses on the IEEE 30-bus system. Future studies will extend the proposed approach to larger systems, such as IEEE 57-bus and 118-bus networks.

6. CONCLUSIONS

In this work, the optimal solution for the ACOPF problem is verified using the SCA and WOA. These algorithms are tested on the standard IEEE 30-bus system with six generating units and a total load demand of 283.4 MW. The results show that both WOA and SCA achieve feasible and competitive solutions, with fuel cost, power loss, and generator dispatch values consistent with those reported in the literature. Although WOA produced a slightly lower fuel cost and power loss than SCA and GA, the differences among the methods are relatively small, indicating comparable performance. Overall, the findings confirm that WOA and SCA are effective and reliable techniques for solving the AC-OPF problem under standard operating conditions.

The proposed framework will be extended to larger power systems and multi-objective AC-OPF formulations incorporating emission and power-loss minimization. Future work will extend the proposed WOA and SCA approaches to larger benchmark systems, such as the IEEE 57-bus and 118-bus networks, in order to further evaluate their scalability and robustness.

REFERENCES

- [1] H. Dommel, W. Tinney, "Optimal power flow solutions," *IEEE Transactions on Power Apparatus and Systems*, vol. PAS-87, no. 10, pp. 1866–1876, 1968, doi: 10.1109/TPAS.1968.292150.
- [2] B. Risi, F. Riganti-Fulginei, A. Laudani, "Modern techniques for the optimal power flow problem: state of the art," *Energies*, vol. 15, no. 17, p. 6387, 2022, doi: 10.3390/en15176387.
- [3] S. Agrawal, S. Pandya, P. Jangir, K. Kalita, S. Chakraborty, "A multi-objective thermal exchange optimization model for solving optimal power flow problems in hybrid power systems," *Decision Analytics Journal*, vol. 8, p. 100299, 2023, doi: 10.1016/j.dajour.2023.100299.
- [4] A. Mohamed, Y. Mohamed, A. El-Gaafary, A. Hemeida, "Optimal power flow using moth swarm algorithm," *Electric Power Systems Research*, vol. 142, pp. 190–206, 2017, doi: 10.1016/j.epsr.2016.09.025.
- [5] X. Pan, M. Chen, T. Zhao, S. Low, "DeepOPF: a feasibility-optimized deep neural network approach for AC optimal power flow problems," *IEEE Systems Journal*, vol. 17, no. 1, pp. 673–683, 2023, doi: 10.1109/JSYST.2022.3201041.
- [6] S. Mohamed, N. Anwer, M. Mahmoud, "Solving optimal power flow problem for IEEE-30 bus system using a developed particle swarm optimization method: towards fuel cost minimization," *International Journal of Modelling and Simulation*, vol. 45, no. 1, pp. 307–320, 2025, doi: 10.1080/02286203.2023.2201043.
- [7] A. Maheshwari, Y. Sood, S. Jaiswal, "Investigation of optimal power flow solution techniques considering stochastic renewable energy sources: review and analysis," *Wind Engineering*, vol. 47, no. 2, pp. 464–490, 2023, doi: 10.1177/0309524X221124000.

- [8] S. Pandya, S. Ravichandran, P. Manoharan, P. Jangir, H. Alhelou, "Multi-objective optimization framework for optimal power flow problem of hybrid power systems considering security constraints," *IEEE Access*, vol. 10, pp. 103509–103528, 2022, doi: 10.1109/ACCESS.2022.3209996.
- [9] M. Abido, "Optimal power flow using particle swarm optimization," *International Journal of Electrical Power & Energy Systems*, vol. 24, no. 7, pp. 563–571, 2002, doi: 10.1016/S0142-0615(01)00067-9.
- [10] F. Capitanescu, J. Ramos, P. Panciatici, D. Kirschen, A. Marano Marcolini, L. Platbrood, L. Wehenkel, "State-of-the-art, challenges, and future trends in security constrained optimal power flow," *Electric Power Systems Research*, vol. 81, no. 8, pp. 1731–1741, 2011, doi: 10.1016/j.epsr.2011.04.003.
- [11] A. Attia, R. El Sehiemy, H. Hasanien, "Optimal power flow solution in power systems using a novel Sine-Cosine algorithm," *International Journal of Electrical Power & Energy Systems*, vol. 99, pp. 331–343, 2018, doi: 10.1016/j.ijepes.2018.01.024.
- [12] U. Mittal, U. Nangia, N. Jain, S. Gupta, "Optimal power flow solution using a learning-based sine-cosine algorithm," *The Journal of Supercomputing*, vol. 80, no. 11, pp. 15974–16012, 2024, doi: 10.1007/s11227-024-06043-7.
- [13] S. Gupta, N. Kumar, L. Srivastava, "Solution of optimal power flow problem using sine-cosine mutation based modified Jaya algorithm: a case study," *Energy Sources, Part A: Recovery, Utilization, and Environmental Effects*, vol. 47, no. 1, pp. 9803–9826, 2025, doi: 10.1080/15567036.2021.1957043.
- [14] M. Saddique, S. Habib, S. Haroon, A. Bhatti, S. Amin, E. Ahmed, "Optimal solution of reactive power dispatch in transmission system to minimize power losses using sine-cosine algorithm," *IEEE Access*, vol. 10, pp. 20223–20239, 2022, doi: 10.1109/ACCESS.2022.3152153.
- [15] S. Mirjalili, A. Lewis, "The whale optimization algorithm," *Advances in Engineering Software*, vol. 95, pp. 51–67, 2016, doi: 10.1016/j.advengsoft.2016.01.008.
- [16] R. Devarapalli, B. Rao, B. Dey, K. Kumar, H. Malik, F. Márquez, "An approach to solve OPF problems using a novel hybrid whale and sine cosine optimization algorithm," *Journal of Intelligent & Fuzzy Systems*, vol. 42, no. 2, 2022, doi: 10.3233/JIFS-189763.
- [17] S. Mirjalili, "SCA: a sine cosine algorithm for solving optimization problems," *Knowledge-Based Systems*, vol. 96, pp. 120–133, 2016, doi: 10.1016/j.knosys.2015.12.022.
- [18] T. Naidu, G. Balasubramanian, B. Rao, "Optimal power flow with distributed energy sources using whale optimization algorithm," *International Journal of Electrical and Computer Engineering*, vol. 13, no. 5, pp. 4835–4844, 2023, doi: 10.11591/ijece.v13i5.pp4835-4844.
- [19] K. Widarsono, A. Soeprijanto, R. Wibowo, "Improved whale optimization algorithm for dynamic optimal power flow with renewable energy penetration," *Engineering, Technology & Applied Science Research*, vol. 15, no. 1, pp. 20379–20387, 2025, doi: 10.48084/etasr.9662.
- [20] H. Saadat, *Power System Analysis*, New York: McGraw-Hill, 1999.
- [21] A. Diab et al., "Comprehensive analysis of optimal power flow using recent metaheuristic algorithms," *Scientific Reports*, vol. 14, 2024, doi: 10.1038/s41598-024-58565-y.
- [22] A. Ebrahimi, R. Haghighi, H. Yektamoghadam, M. Dehghani, A. Nikoofard, "Optimal power flow by genetic algorithm," in *Frontiers in Genetics Algorithm Theory and Applications*, 2024, pp. 121–136, doi: 10.1007/978-981-99-8107-6_7.
- [23] F. Prioste et al., "Optimal power flow using genetic algorithm," 15th Brazilian Congress on Computational Intelligence, 2021, doi: 10.21528/cbic2021-144.