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A Data-Reuse Enhanced NSAF-NLMS Algorithm for Acoustic Echo Cancellation

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Abstract— This article presents a novel adaptive algorithm that combines the Normalized Subband Adaptive Filter (NSAF-NLMS) with a Data-Reuse (DR) strategy to address the challenges of Acoustic Echo Cancellation (AEC) and System Identification. These tasks are critical in modern communication systems, where effective echo suppression and accurate modeling of acoustic environments are essential for improving overall communication quality. Unlike conventional approaches that apply the data-reuse technique in full-band adaptive signal processing, the proposed method introduces data reuse within the subband processing framework. Specifically, the reuse of data is performed independently within each subband, enabling more efficient adaptation. Extensive simulations conducted under various signal and environmental conditions validate the performance of the proposed algorithm. The results demonstrate that the algorithm achieves notable improvements in convergence speed, misalignment, and tracking accuracy. In particular, the proposed method consistently outperforms the conventional Normalized least mean square (NLMS), DR-NLMS and NSAF-NLMS algorithms in dynamic and noisy scenarios, confirming its robustness and effectiveness.

Keywords— Adaptive filtering; Acoustic echo cancellation; Subband adaptive filter; NLMS; NSAF; Data reuse.

Nomenclature

AEC	Acoustic Echo Cancellation	NSAF	Normalized Subband Adaptive Filtering
DR	Data-Reuse	SNR	Signal-to-Noise Ratio
LMS	Least Mean Square	dB	Decibel
NLMS	Normalized Least Mean Square	MSE	Mean Square Error
RLS	Recursive Least Squares	MSD	Mean Squared Deviation
FNLMS	Fast Normalized Least Mean Square	USASI	United States of America Standards Institute
SAF	Subband Adaptive Filtering		

1. INTRODUCTION

Adaptive filtering plays a key role in modern signal processing applications such as speech enhancement [1, 2], speech recognition [3, 4], system identification [5], noise reduction [6, 7], and acoustic echo cancellation [8, 9]. Among these techniques, the Normalized Least Mean Square (NLMS) algorithm is widely used due to its simplicity, low computational complexity, and suitability for real-time processing [10]. However, its main drawback lies in

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the relatively slow convergence rate, particularly when the input signal is highly correlated, which can limit its effectiveness in practical scenarios.

To overcome this limitation, Subband Adaptive Filtering (SAF) algorithms have been introduced as an efficient solution to improve convergence performance [11-13]. In SAF, the input and desired signals are divided into several subbands that represent nearly independent frequency components. This decomposition enables more efficient adaptation and faster convergence. Nevertheless, the performance of SAF is still affected by band-edge effects [14, 15], caused by discontinuities between subbands, which can slow down the adaptation process. Specifically, these discontinuities arise from small eigenvalues in subband correlation matrices in oversampled schemes or from undesired aliasing components introduced by decimation in critically sampled schemes [15]. These structural issues increase estimation errors at subband boundaries, limiting the overall convergence rate. The NSAF algorithm avoids these problems, enabling faster and more reliable adaptation.

To mitigate this issue, the Normalized Subband Adaptive Filter (NSAF) algorithm was proposed [15–18]. Derived from the principle of minimum disturbance, NSAF minimizes the change in the filter weight vector while satisfying the output constraint, thereby achieving faster convergence and better tracking for correlated input signals [15]. Another effective strategy to enhance adaptive filters is the Data-Reuse (DR) technique, which reuses the same input data to update filter coefficients multiple times within a single iteration. This approach accelerates convergence and improves overall performance while maintaining low complexity. The DR concept was first introduced for the Least Mean Square (LMS) algorithm [19,20] and later extended to improved LMS [21], NLMS [22], Fast NLMS (FNLMS) [23], and Recursive Least Square (RLS)-type algorithms [24–25]. Overall, DR-based methods share the common strength of accelerating convergence and tracking adaptability. However, they also face limitations, such as increased computational cost in DR-RLS algorithms.

Although the NSAF-NLMS algorithm achieves better performance than the traditional NLMS, it still suffers from certain limitations. Specifically, the conventional NSAF-NLMS does not fully exploit the available input information, as each subband update is based on a single data instance per iteration, which limits its adaptation efficiency.

To address these limitations, this paper introduces a Data-Reuse enhanced NSAF-NLMS (DR-NSAF-NLMS) algorithm that incorporates a Data-Reuse (DR) mechanism into the subband structure, in contrast to traditional approaches that apply data reuse only in full-band adaptive filtering. The proposed approach applies the data-reuse mechanism independently within each subband to make better use of the input information. This iterative update process speeds up convergence improves tracking performance and enhances adaptability under non-stationary conditions. The remainder of this paper is organized as follows: Section 2 provides an overview of the adaptive AEC system. Section 3 reviews the NSAF-NLMS algorithm. Section 4 presents the proposed DR-NSAF-NLMS algorithm. Section 5 discusses the simulation results, and Section 6 concludes the paper.

2. ADAPTIVE ACOUSTIC ECHO CANCELLATION

Echo cancellation is a critical process that relies on adaptive filters to effectively eliminate unwanted acoustic echos, ensuring clear audio communication.

As demonstrated in Fig. 1, the input signal is mathematically represented by the vector: $\boldsymbol{x}(n) = [x(n)\ x(n-1)\ ...\ x(n-L+1)]^T$ and $\boldsymbol{w}(n) = [w_0(n)\ w_1(n)\ ...\ w_{L-1}(n)]^{\mathrm{T}}$ represents the estimated impulse response of the unknown system $\boldsymbol{h}$. The desired signal corresponding to this system is:

$$d(n) = y(n) + \mathrm{v}(n) \tag{1}$$

where

$$y(n) = \boldsymbol{x}^T(n)\boldsymbol{h} \tag{2}$$

where $v(\mathrm{n})$ denotes the additional background noise with unknown power σ_v^2 . The output system error is given by:

$$e(n) = d(n) - \boldsymbol{x}^T(n)\boldsymbol{w}(n-1) \tag{3}$$

The NLMS algorithm updates the filter coefficients through [10]:

$$\boldsymbol{w}(n) = \boldsymbol{w}(n-1) + \mu_{nlms}\frac{\boldsymbol{x}(n)e(n)}{\boldsymbol{x}^T(n)\boldsymbol{x}(n)+C_{nlms}} \tag{4}$$

where $0 < \mu_{nlms} < 2$ is the step size [10], and C_{nlms} is a small positive constant used to prevent potential division by zero.

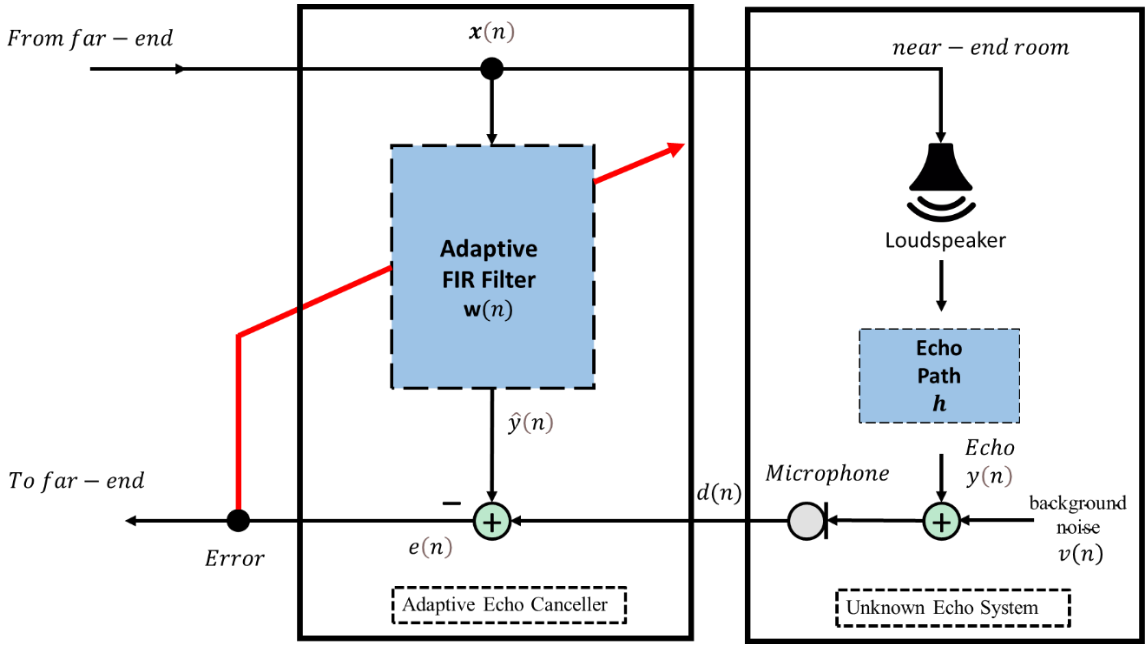

Fig. 1. Basic adaptive acoustic echo cancellation system.

3. THE NSAF-NLMS ALGORITHM

The core idea of the NSAF algorithm is to enhance convergence performance and mitigate aliasing effects between subbands in the SAF [15]. Figure 2 illustrates the multiband structure of NSAF.

The input and desired signals $\boldsymbol{x}(n)$ and $d(n)$, are partitioned into several subband signals via the analysis filter bank, namely: $d_i(n) = \boldsymbol{H}_i^T(n)\boldsymbol{d}(n)$ and $\boldsymbol{x}_i(n) = \boldsymbol{H}_i^T(n)\boldsymbol{x}(n)$ for $\mathrm{i} = 1,..,\mathrm{B}$. where B is the number of Subbands and $\boldsymbol{H}_i$ is the impulse response of the i -th analysis filter. The output signals of the Subbands $y_i(n)$ are obtained by filtering the input signals of the Subbands $\boldsymbol{x}_i(n)$ by means of an adaptive filter, where the weight vector is:$\boldsymbol{w}(k) = [w_0(k), w_1(k), \dots\dots, w_{L-1}(k)]^T$,where L is the filter size, then the signal $y_i(\mathrm{n})$ are decimated at every time instant k to give $y_{i,D}(k)$, where n refers to the original time index and k to the decimated time index. These decimated subband outputs are defined as follows [15]:

$$y_{i,D}(k) = \boldsymbol{x}_i^T(k)\boldsymbol{w}(k-1) \tag{5}$$

where the subband input vector $\boldsymbol{x}_i^T(k)$ is of size L. The decimation operation is performed using a sub-sampling factor D, where D is a positive integer referred to as the decimation factor. Similarly, the symbol I denotes the interpolation factor, which corresponds to an integer up-sampling operation applied to discrete-time signals.

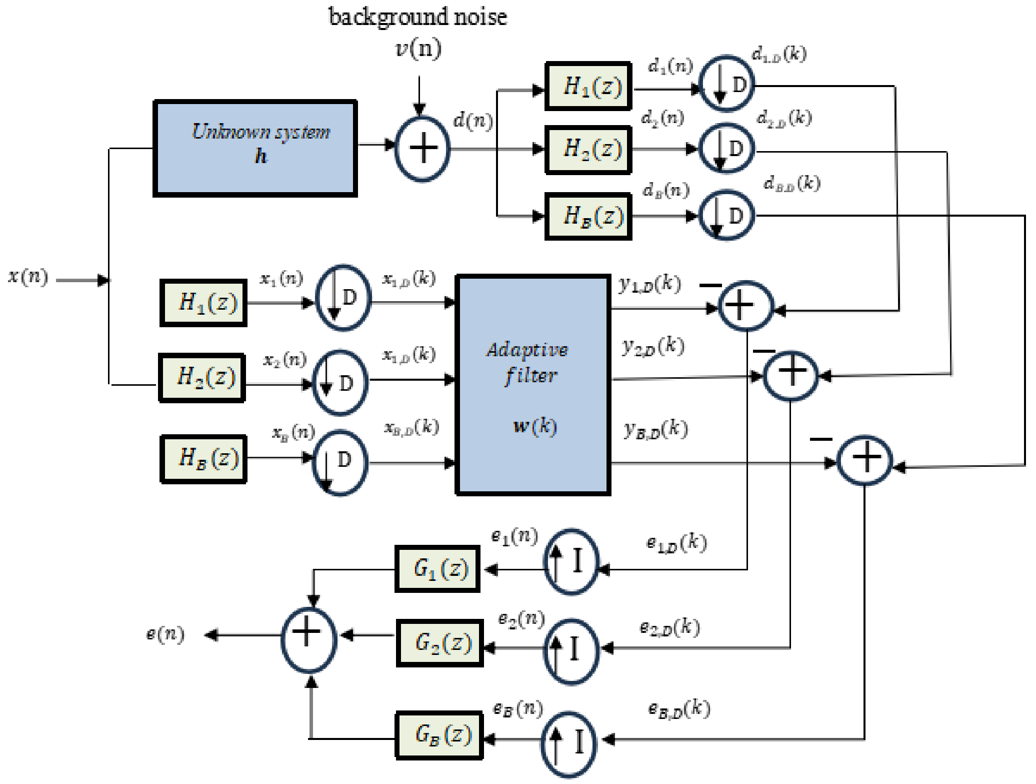

Fig. 2. Diagram of the NSAF structure [12].

The NSAF-NLMS algorithm is based on the principle of minimal disturbance and is formulated as a multi-constraint optimization problem, the principle of minimum disturbance states that, from one iteration to the next, the tap weights of an adaptive filter should be changed minimally, subject to a constraint imposed on the updated filter output. Based on this principle, a novel design criterion for the SAF has been developed as a constrained optimization problem, involving multiple constraints applied to the updated subband filter outputs [15].

Its primary goal is to minimize the squared Euclidean norm of the variation in the weight vector across iterations as follow:

$$f[\,\boldsymbol{w}(k)] \;=\; \|\boldsymbol{w}(k)-\boldsymbol{w}(k-1)\|^2 \tag{6}$$

The Lagrangian function is formed by applying the method of Lagrange multipliers:

$$J(k) = f[\mathbf{w}(k)] + \textstyle\sum_{i=0}^{B-1} \lambda_i \left[d_{i,D}(k) - \boldsymbol{x}_i^T(k)\boldsymbol{w}(k-1)\right] \tag{7}$$

where $\lambda_i$the Lagrange multipliers. These multipliers are introduced to incorporate multiple constraints into a quadratic cost function. By differentiating this cost function with respect to the tap-weight vector and setting the result to zero, an update expression for the weight vector is obtained, which depends on the Lagrange multipliers and the input signals, as follows [15]:

$$\boldsymbol{w}(k) = \boldsymbol{w}(k-1) + \tfrac{1}{2}\textstyle\sum_{i=0}^{B-1} \lambda_i\, \boldsymbol{x}_i(k) \tag{8}$$

To compute the unknown Lagrange multipliers, the constraints are reformulated by incorporating the relevant expressions. These are then arranged in matrix form, allowing the multipliers to be explicitly determined as shown in [15]:

$$\lambda = 2[\,\boldsymbol{x}^T(k)\boldsymbol{x}(k)]^{-1} e_{i,D}(k) \tag{9}$$

where $\lambda = [\ \lambda_{0,}\lambda_{1,},\ldots\ldots,\lambda_{B-1,}]^T$ is the $B\times 1$ Lagrange vector, vector, $\boldsymbol{x}(k) = [x_0(k), x_1(k), \ldots, x_{B-1}(k)]^T$ is the data matrix, and $e_{i,D}(k)$= $[e_{0,D}(\text{k}), e_{2,D}(\text{k}), \ldots, e_{B-1,D}(\text{k})]^T$is the error vector.

The off-diagonal elements of the matrix are negligible. If the frequency responses of the analysis filters do not overlap significantly, with this diagonal assumption, Eq. (9) essentially reduces to a simple form:

$$\lambda_i = 2\frac{e_{i,D}(k)}{\|\boldsymbol{x}_i(\mathrm{k})\|^2} \quad (10)$$

Combining the results of Eqs. (8) and (10), we obtain the recursive relation for updating the weight vector [15]:

$$\boldsymbol{w}(k) = \boldsymbol{w}(k-1) + \mu_{nlms}\sum_{i=0}^{B-1}\frac{\boldsymbol{x}_i(k)}{\|\boldsymbol{x}_i(k)\|^2 + C_{nlms}} e_{i,D}(k) \quad (11)$$

where $\|.\|$denotes the ℓ2 norm of a vector, the decimated subband error signals $e_{i,D}(k)$ is defined by:

$$e_{i,D}(k) = d_{i,D}(k) - \boldsymbol{x}_i^T(k)\,\boldsymbol{w}(k-1) \quad (12)$$

Algorithm 1. The NSAF-NLMS algorithm
Initialization and parameters.:
L= filter size,
M= length of analysis filter,
B=number of Subbands
$\mathrm{C_{nlms}} = 20\sigma_x^2$, $0 < \mu_{\mathrm{nlms}} < 2$,
$\boldsymbol{H}_i$, $\boldsymbol{G}_i$: linear-phase FIR filters
For time index n = 1, 2, ……………. . (**iterations**)
Analysis filters part
$x_i(n) = \boldsymbol{H}_i^T(n)\boldsymbol{x}(n) \qquad i = 0, \dots \dots . \dots, B-1$
$d_i(n) = \boldsymbol{H}_i^T(n)\boldsymbol{d}(n) \qquad i = 0, \dots \dots . \dots, B-1$
Filtering Error
For $\boldsymbol{k = 0, 1}$ … … … … … … … … …. . . , . (**iterations**)
$e_{i,D}(k) = d_{i,D}(\mathrm{k}) - \boldsymbol{x}_i^T(k)\,\boldsymbol{w}(k-1)$
Filter update
$\boldsymbol{w}(k) = \boldsymbol{w}(k-1) + \mu_{nlms}\sum_{i=0}^{B-1}\frac{\boldsymbol{x}_i(k)}{\|\boldsymbol{x}_i(k)\|^2 + C_{nlms}} e_{i,D}(k)$
Synthesis filters part
$e(n) = \sum_{i=0}^{B-1}\boldsymbol{G}_i^T\, e_i(n)$

4. THE PROPOSED DR-NSAF-NLMS ALGORITHM

In this section, we propose a DR-NSAF-NLMS Algorithm, which combines the principle of the NSAF algorithm and the DR technique to enhance the performance of adaptive filtering. The NSAF algorithm enhances convergence by decoupling adaptation across frequency bands through subband filtering, while the DR technique boosts convergence and tracking performance by iteratively updating the filter coefficients using the same input data within each iteration. The proposed algorithm combines these advantages, exploiting subband frequency selectivity and data reuse to achieve efficient adaptation without increasing input data requirements.

The decimated a posteriori subband error signal can be expressed, based on the adaptive filter coefficients, as follows:

$$e_{1,i,D}(k) = d_{i,D}(k) - \boldsymbol{x}_i^T(k)\boldsymbol{w}(k) \quad (13)$$

Using Eqs. (11), (12) and (13), the a posteriori error can be further expressed as:

$$e_{1,i,D}(k) = (1-\mu_{nlms})e_{i,D}(k) \quad (14)$$

Assuming that the NSAF-NLMS algorithm iterates several times over identical input $\boldsymbol{x}(n)$ and desired signal $d(n)$, this approach is referred to as the data reuse process [21-23]. Accordingly, by applying Eqs. (11), (12), and (14) in the first step, we obtain:

$$\boldsymbol{w}_1(k) = \boldsymbol{w}(k) + \sum_{i=0}^{B-1} \mu_{nlms} \frac{x_i(k)}{\|x_i(k)\|^2 + C_{nlms}} e_{1,i,D}(k) \tag{15}$$

$$\boldsymbol{w}_1(k) = \boldsymbol{w}(k-1) + \sum_{i=0}^{B-1} \frac{[1-(1-\mu_{nlms})^2]x_i(k)}{\|x_i(k)\|^2 + C_{nlms}} e_{i,D}(k) \tag{16}$$

Repeating this process yields for the second iteration:

$$e_{2,i,D}(k) = d_{i,D}(k) - \boldsymbol{x}_i^T(k)\boldsymbol{w}_1(k) \tag{17}$$

$$e_{2,i,D}(k) = (1-\mu_{nlms})^2\, e_{i,D}(k) \tag{18}$$

$$\boldsymbol{w}_2(k) = \boldsymbol{w}_1(k) + \sum_{i=0}^{B-1} \mu_{nlms} \frac{\boldsymbol{x}_i(k)}{\|\boldsymbol{x}_i(k)\|^2 + C_{nlms}} e_{2,i,D}(k) \tag{19}$$

$$\boldsymbol{w}_2(k) = \boldsymbol{w}(k-1) + \sum_{i=0}^{B-1} \frac{[1-(1-\mu_{nlms})^3]x_i(k)}{\|x_i(k)\|^2 + C_{nlms}} e_{i,D}(k) \tag{20}$$

Continuing this procedure, mathematical induction yields the general form of the a posteriori error after N reuse iterations:

$$e_{N,i,D}(k) = (1-\mu_{nlms})^{N+1}\, e_{i,D}(k) \tag{21}$$

Finally, the formula for the N-th order update is as follows:

$$\boldsymbol{w}(k) = \boldsymbol{w}(k-1) + \sum_{i=0}^{B-1} \frac{DR_N x_i(k) e_{i,D}(k)}{\|x_i(k)\|^2 + C_{nlms}} \tag{22}$$

where

$$DR_N = [1-(1-\mu_{nlms})^{N+1}] \tag{23}$$

where N is the order of data reuse, representing the number of iterations applied to the same set of input data.

This formulation illustrates that reusing data multiple times is mathematically equivalent to increasing the normalized step size, thus achieving faster convergence while maintaining stability under non-stationary conditions [21-23]. It is observed that the a posteriori error vanishes when $\mu_{nlms} = 1$, (by assuming that $e_{i,D}(k) \neq 0$) [22].

5. SIMULATION RESULTS

5.1. REPRESENTATION OF SIGNALS AND SYSTEMS

Throughout the simulations, we employed two types of input signals, each recorded at a 16 kHz sampling rate and represented with 16-bit precision. The first signal, known as USASI (United States of America Standards Institute) shown in Fig. 3, is a real, stationary, and correlated noise. Its spectrum closely resembles the average speech spectrum, featuring a spectral range of 32 dB and a power of σ_x^2 =0.32. Moreover, this signal exhibits a Gaussian probability distribution and is commonly employed in acoustic echo cancellation applications [8,9].

The second signal is a non-stationary real speech signal, shown in Fig. 4, it is generated by combining a male-spoken sentence and a female-spoken one. This signal has a spectral dynamic range of 46 dB, with an estimated speech power of σ_x^2 =0.16, and approximately follows the Laplace distribution in its probability characteristics.

Filtering the input signal with a 512-point real car acoustic impulse response produces the echo signal (see Fig. 5). To generate the desired signal, a white Gaussian noise component is added to the echo signal with a given Signal-to-Noise Ratio (SNR).

Algorithm 2. The proposed DR-NSAF-NLMS algorithm

Initialization and parameters:
L= filter size
M= length of analysis/synthesis filters
N= order of data reuse
B=number of Subbands
$C_{nlms} = 20\sigma_x^2$, $0 < \mu_{nlms} < 2$,
$\boldsymbol{H}_i$, $\boldsymbol{G}_i$: linear-phase FIR filters.
For time index n = 1, 2, …………
Analysis filters part
$x_i(n) = \boldsymbol{H}_i^T(n)\boldsymbol{x}(n) \qquad i = 0, \dots\dots, B-1$
$d_i(n) = \boldsymbol{H}_i^T(n)\boldsymbol{d}(n) \qquad i = 0, \dots\dots, B-1$
Filtering Error
For $k = 0,1 \dots\dots\dots\dots\dots\dots\dots$..(**iterations**)
$e_{i,D}(k) = d_{i,D}(k) - \boldsymbol{x}_i^T(k)\,\boldsymbol{w}(k-1)$

Data-reuse

$$DR_N = [1-(1-\mu_{nlms})^{N+1}]$$

Filter update

$$\boldsymbol{w}(k) = \boldsymbol{w}(k-1) + \sum_{i=0}^{B-1} \frac{DR_N \boldsymbol{x}_i(k) e_{i,D}(k)}{\|\boldsymbol{x}_i(k)\|^2 + C_{nlms}}$$

Synthesis filters part
$e(n) = \sum_{i=0}^{B-1} \boldsymbol{G}_i^T\, e_i(n)$

In our simulation, we employed a time-varying acoustic channel model. This channel is modified by multiplying the impulse response $w(n)$ with a gain function for a finite duration, specifically from 60,000 to 80,000 iteration samples. The amplitude of this function varies linearly between 1 and 3.5 in an increasing and then decreasing manner, as illustrated in Fig. 6. This scenario provides a more challenging condition than the abrupt change commonly used in literature, allowing for a more rigorous evaluation of the algorithm's ability to track time-varying system dynamics.

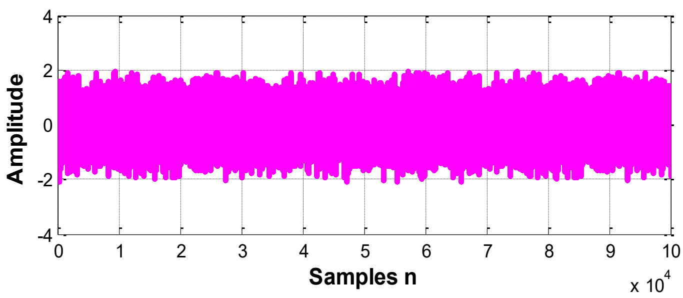

Fig. 3. USASI input signal.

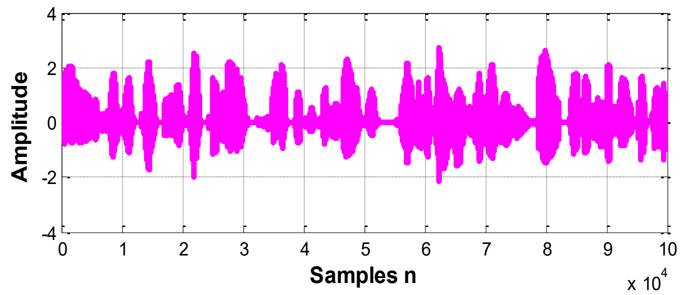

Fig. 4. Speech input signal.

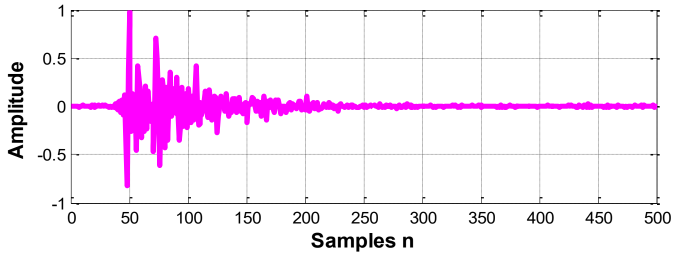

Fig. 5. real impulse response.

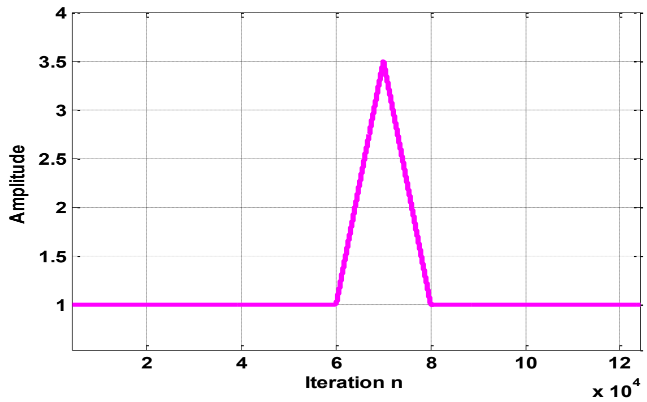

Fig. 6. Gain variation of time-varying change.

5.2. Representation of Subband Decomposition Filters

In this section, we present the subband signals obtained through decomposition using analysis and synthesis filter banks. To divide the input signal into 2, 4, and 8 frequency bands, we set the prototype filter length M to 16 for two subbands (Fig. 7), M to 32 for four subband (Fig. 8), and M to 64 for eight subband (Fig. 9).

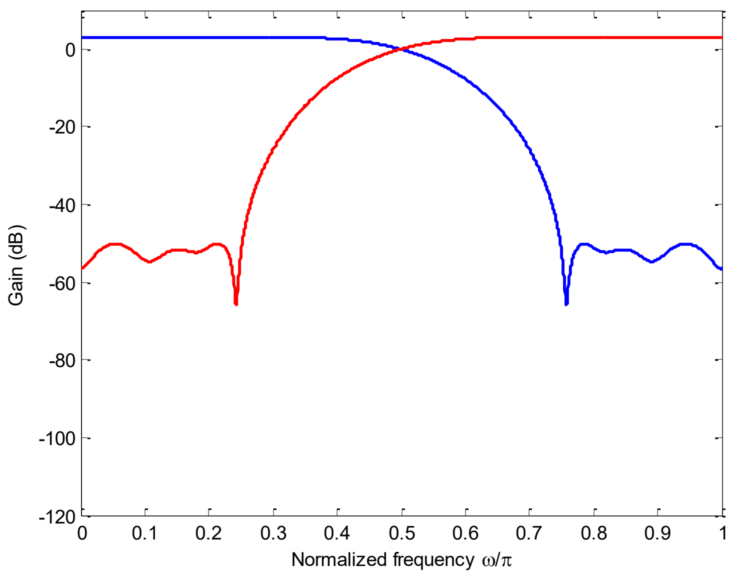

Fig. 7. Frequency response characteristics of analysis and synthesis filters synthesis filters for 2 Subband and M = 16.

5.3. Performance Criteria Description

The evaluation of adaptive algorithms for echo cancellation relies on several established performance metrics. In this study, we employ two specific criteria, namely:

5.3.1. *Mean Square Error Criterion (MSE)*

This criterion represents the temporal evolution of the MSE in decibels (dB) for comparing the performance of different algorithms, and it is determined by the following expression:

$$MSE_{dB}(n) = 10log_{10}(< e^2(n) >) \tag{24}$$

where the symbol $<.>$ represent temporal averages over 256, 512 and 1024 samples.

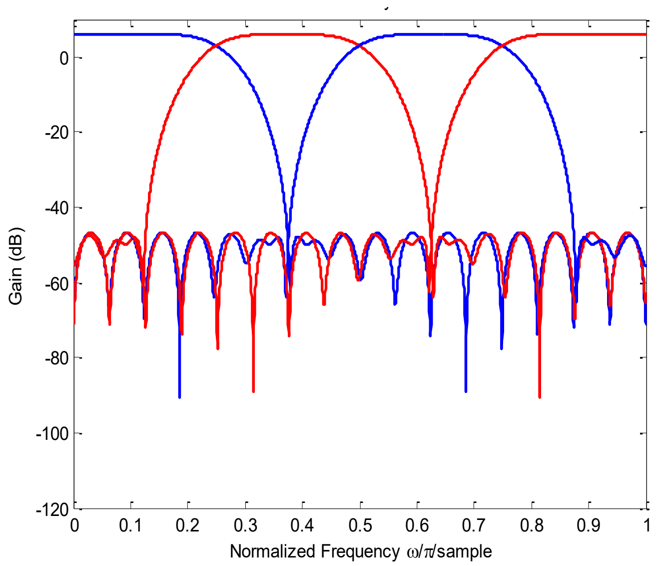

Fig. 8. Frequency response characteristics of analysis and synthesis filters synthesis filters for 4 Subband and M = 32.

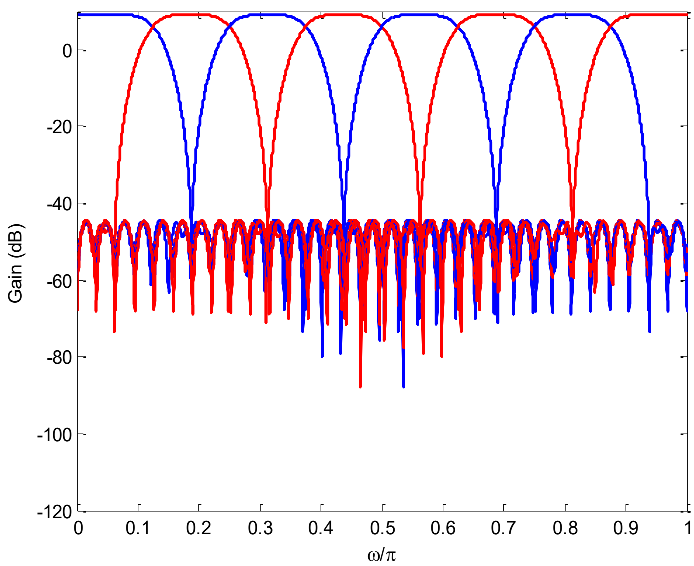

Fig. 9. Frequency response characteristics of analysis and synthesis filters synthesis filters for 8 Subband and M = 64.

5.3.2. *Mean Squared Deviation (MSD) or Misalignment Criterion*

The Misalignment criterion is a reliable metric for assessing the performance of adaptive algorithms. It measures the discrepancy between the coefficients of the actual impulse response and those of the estimated filter, expressed in decibels (dB). It is calculated using the following expression:

$$MSD_{dB}(n) = 10log_{10}\left[\frac{\|\boldsymbol{h}-\boldsymbol{w}(n)\|^2}{\|\boldsymbol{h}\|^2}\right] \quad (25)$$

5.4. Results And Discussions

5.4.1. Case of a Stationary System

In our simulations, we present comparative results for the NLMS algorithm in the full-band domain, the DR-NLMS algorithm with a data-reuse factor N=4, and the subband NLMS algorithm (8 subbands), alongside the proposed DR-NSAF-NLMS algorithm, which combines the subband NLMS of B=8, with a data-reuse method of factor order fixed in N=4.

In our simulations, we use two values for the step size parameter: $\mu_{nlms} = 0.2$ for low SNR scenarios and $\mu_{nlms} = 0.6$ for high SNR scenarios. Where in the absence of noise, we set the adaptation step size to one to achieve the fastest possible convergence speed. However, in noisy environments, it is generally advisable to select value less than one [26]. And we fixed the regularization constants as $\mathrm{C_{nlms}} = 20\sigma_x^2$ for all algorithms [23]. where $\mathrm{C_{nlms}}$ small constant is added to avoid division by zero and depends the variance of input signal σ_x^2[26].

Initially, we compare the proposed DR-NSAF-NLMS algorithm with the conventional NLMS, DR-NLMS(N=4) and NSAF-NLMS (B=8) algorithms based on the MSE criteria. The results, obtained using USASI noise input with L=256 and under two different signal-to-noise ratio (SNR) conditions of 20 dB and 50 dB, are illustrated in Figs. 10 and 11.

The results show that the proposed DR-NSAF-NLMS algorithm converges faster than the NSAF-NLMS (B=8), DR-NLMS (N=4), and conventional NLMS algorithms, while maintaining a steady-state MSE performance that is nearly identical.

A quantitative convergence time analysis further highlights this improvement. For instance, with SNR = 20 dB, the proposed DR-NSAF-NLMS algorithm converges in 15,360 iterations, which is approximately 6.3% faster than NSAF-NLMS (16,384 iterations), 45.5% faster than DR-NLMS (33,792 iterations), and 73.8% faster than the conventional NLMS algorithm (58,368 iterations). These results confirm the superior convergence speed of the proposed algorithm under stationary conditions and varying noise levels.

However, these results confirm the robustness and effectiveness of the proposed approach in stationary environments with different noise levels.

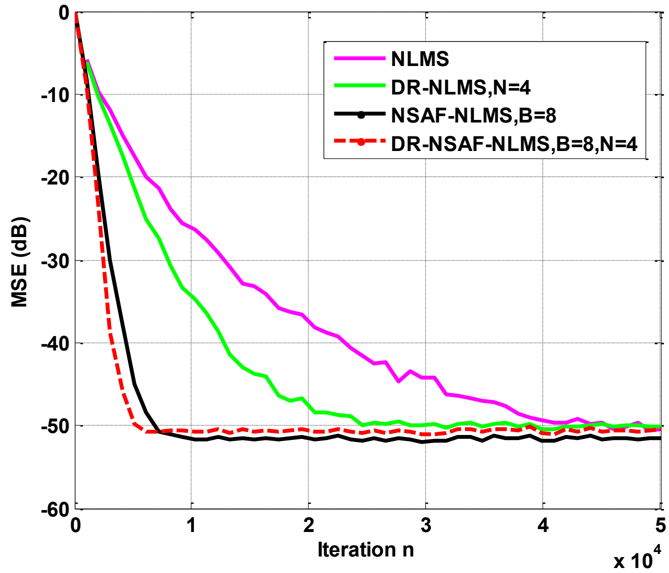

Fig. 10. MSE curves with USASI input and stationary system, L=256, SNR=50, $\mu_{nlms} = 0.6$.

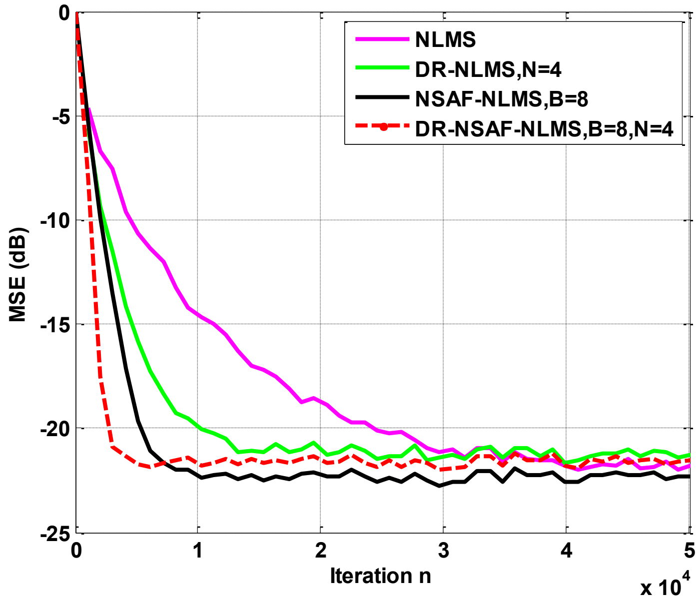

Fig. 11. MSE curves with USASI input and stationary system, L=256, SNR=20, $\mu_{\mathrm{nlms}} = 0.2$.

An additional experiment, illustrated in Figs. 12 and 13, assesses the algorithms under a non-stationary speech input with a 256-tap filter and SNR of 50 dB, using MSE and MSD as performance criteria. The results clearly indicate that the proposed algorithm delivers impressive performance, substantially exceeding the performance of both the conventional NLMS, DR-NLMS and NSAF-NLMS algorithms. It achieves the lowest MSE, a faster convergence rate, and superior misalignment performance. These findings confirm that the proposed algorithm offers enhanced echo cancellation capabilities, clearly surpassing both the full-band and subband NLMS algorithms, as well as the DR-NLMS algorithm.

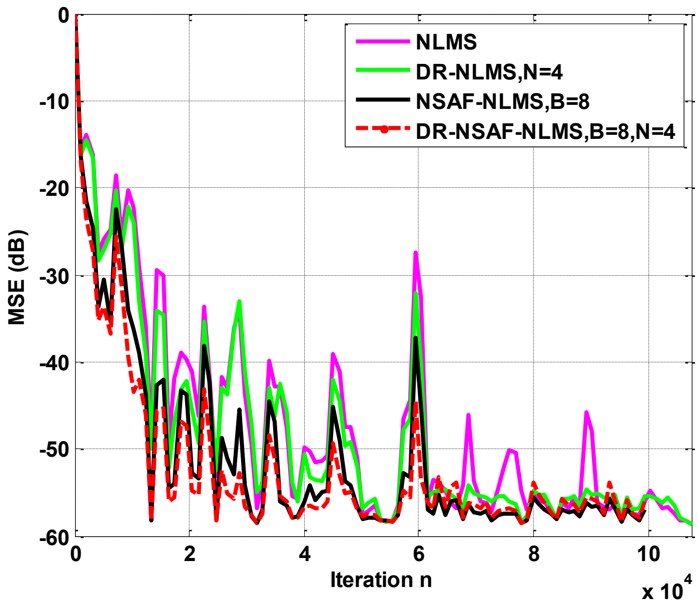

Fig. 12. MSE curves with Speech input and stationary system, L=256, SNR=50, $\mu_{\mathrm{nlms}} = 0.6$.

5.4.2. Case of a Non-Stationary System

In this case, we evaluated the proposed algorithm and compared its performance with the other algorithms with artificial non-stationary systems. This system exhibits time-varying changes, as shown in Fig. 6.

Figure 14 displays the results with an SNR=20 and $\mu_{nlms} = 0.2$ with MSE criteria. Notably, the proposed DR-NSAF-NLMS algorithm demonstrates significant performance improvements. Specifically, it yields an approximate gain of 6 dB compared to the proposed DR-NSAF-NLMS algorithm and the NSAF-NLMS with B=8.

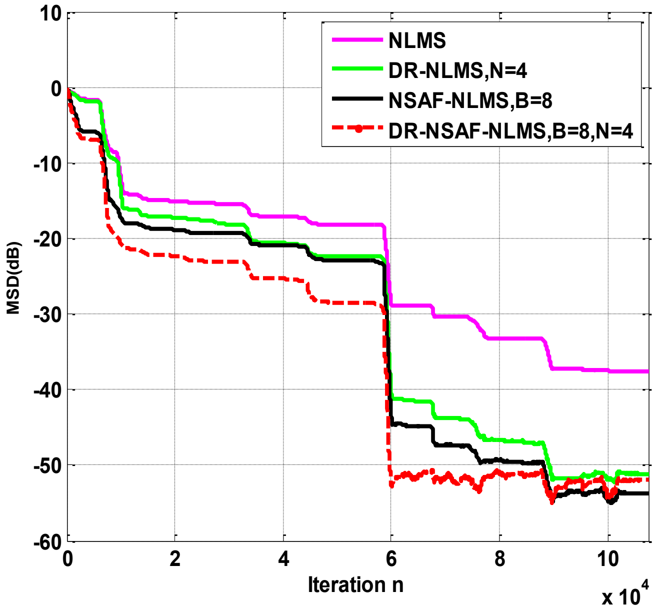

Fig. 13. MSD curves with Speech input and stationary system, L=256, SNR=50,$\mu_{nlms} = 0.6$.

Figures 15 and 16 show the results under time-varying conditions with SNR of 50 dB and $\mu_{nlms} = 0.6$, evaluated using the MSE and MSD criteria, respectively.

The results clearly show that the proposed algorithm offers superior tracking performance compared to the conventional NLMS and NSAF-NLMS algorithms. In both evaluation criteria, the proposed method consistently maintains lower error levels during time-varying system changes.

Specifically, it achieves a gain of approximately 4 dB over the NSAF-NLMS algorithm with B=8, highlighting its enhanced ability to adapt to dynamic environments.

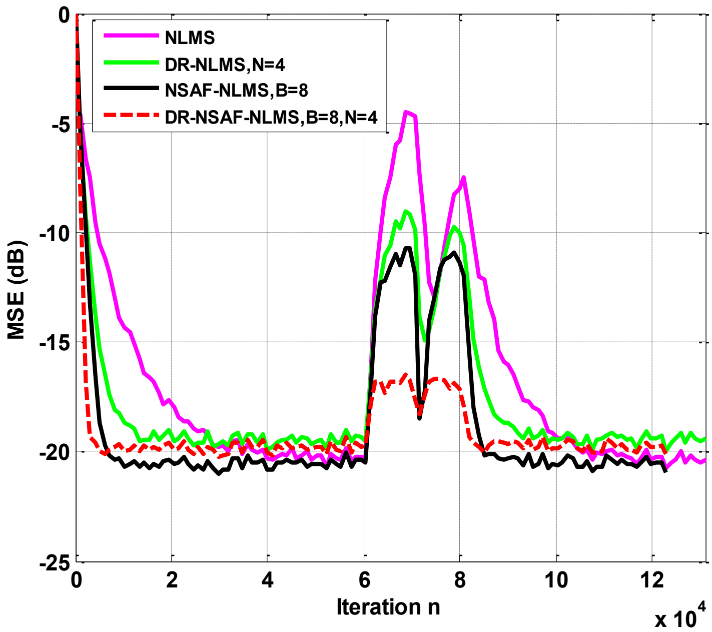

Fig. 14. MSE curves with USASI input and during time-varying system, L=256, SNR=20,$\mu_{nlms} = 0.2$.

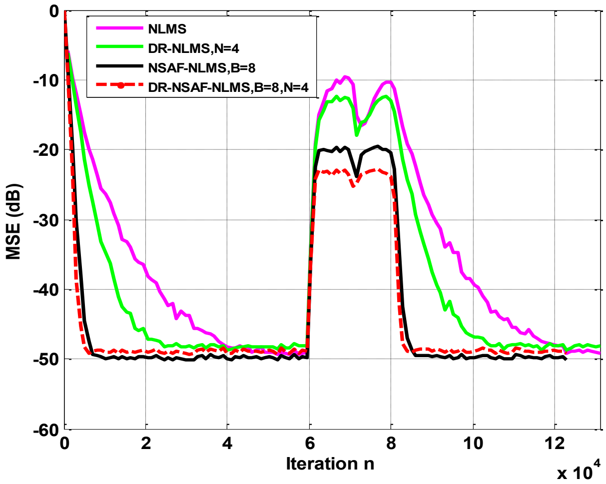

Fig. 15. MSE curves with USASI input and during time-varying system, L=256, SNR=50,$\mu_{nlms} = 0.6$.

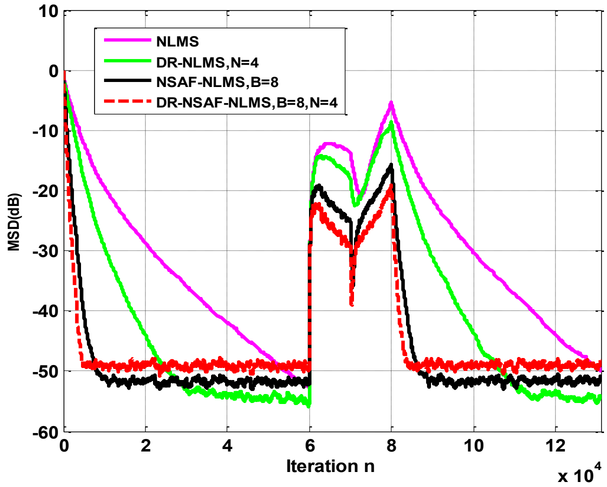

Fig. 16. MSD curves with USASI input and during time-varying system, L=256, SNR=50,$\mu_{nlms} = 0.6$.

5.4.3. Effect of Data-Reuse Order on DR-NSAF-NLMS Algorithm Performance

This section investigates the impact of the data-reuse order N on the performance of the proposed DR-NSAF-NLMS algorithm, as illustrated in Fig. 17. By varying the data-reuse order N, it becomes evident that the tracking capability of the proposed algorithm improves proportionally with the increase in N. Specifically, higher values of the data-reuse order result in more efficient use of past input data within each subband, which leads to noticeable improvements in tracking performance.

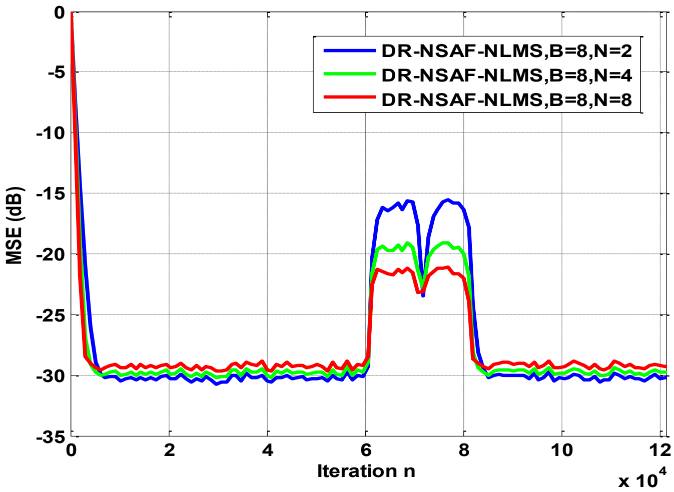

Fig. 17. MSE curves with USASI input and during time-varying system, L=256, SNR=30,$\mu_{nlms} = 0.2$.

5.5. Computational Complexities

Table 1 presents a comparative evaluation of the computational complexity of the NLMS, NSAF-NLMS, and the proposed DR-NSAF-NLMS algorithms in terms of the total number of multiplications and additions per iteration. The NLMS algorithm requires approximately $2L$ multiplications per iteration [10]. The NSAF-NLMS algorithm introduces an additional $3MB$ multiplications and approximately $3B(M-1)+3$ additions due to the analysis and synthesis filter banks [15], resulting in a total of $2L+3MB$ multiplications and $2L+3B(M-1)+3$ additions per iteration [15].

For the proposed DR-NSAF-NLMS algorithm maintains the same base complexity as the NSAF-NLMS but adds N extra multiplications related to the data-reuse computation and three additional scalar additions, yielding approximately $2L+N+\ 3MB$ multiplications and $2L+3B(M-1)+6$ additions per iteration. This increase in computational cost contributes to enhanced performance, particularly in tracking time-varying systems and improving convergence speed. From a theoretical complexity perspective, this increase is marginal. For example, with filter lengths typically used in AEC $L=256$, M=64, B=8 and N=4, the computational complexity per iteration of the NLMS, NSAF-NLMS, and the proposed DR-NSAF-NLMS algorithms is as follows: NLMS requires 513 multiplications and 515 additions; NSAF-NLMS requires 2048 multiplications and 2027 additions, the proposed DR-NSAF-NLMS requires 2052 multiplications and 2030 additions.

From a theoretical standpoint, the computational complexity of the algorithm remains linear with respect to the filter length L, meaning that the number of multiplications increases proportionally with L, rather than at a higher-order rate such as quadratic. This linear complexity ensures that the proposed algorithm remains computationally efficient and suitable for real-time applications.

Table1. Computational complexity evaluation.

Algorithm	Multiplication	Addition
NLMS	2L+1	2L+3
NSAF-NLMS	2L+3MB	2L+3B(M-1)+3
DR-NSAF-NLMS	2L+N+3MB	2L+3B(M-1)+6

6. CONCLUSIONS

In this paper, we presented the DR-NSAF-NLMS algorithm, a novel approach that combines the NSAF-NLMS with a DR technique to enhance the performance of adaptive filtering. In contrast to traditional full-band methods, our method implements data reuse independently within each subband, Specifically, data is reused independently within each subband, allowing for more efficient adaptation. By combining subband decomposition to accelerate convergence with independent data reuse to enhance algorithm efficiency, the proposed algorithm achieves significant performance gains.

Extensive simulations demonstrated that the DR-NSAF-NLMS outperforms the NLMS and DR-NLMS and NSAF-NLMS algorithms by achieving, faster convergence speed, and superior tracking capability. Notably, the performance gain increases with the order of data reuse, demonstrating the effectiveness of this strategy in dynamic acoustic conditions.

However, there exists a trade-off, as higher orders of data reuse increase computational complexity, and very large orders may provide only marginal additional benefits.

The algorithm also exhibits superior behavior under time-varying system scenarios, making it particularly suitable for real-time applications such as AEC. Despite the slight increase in computational complexity due to data reuse, the proposed algorithm remains close in complexity to the NSAF-NLMS. This balance between computational cost and performance is justified by the significant improvements in convergence and tracking particularly when smaller step sizes are used.

As future work, the algorithm can be extended to support multi-microphone systems, evaluated on real-world AEC datasets beyond simulations, and implemented in hardware (e.g., FPGA) with optimized computational resources for real-time or embedded applications. In conclusion, the proposed DR-NSAF-NLMS algorithm provides an efficient solution for enhancing adaptive filtering performance in stationery and time-varying environments.

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