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Detection of Multi-Class Epileptic Intracranial EEG Signals Based on Advanced Hybrid Time-Frequency and Machine Learning Technique

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Abstract— Epilepsy is one of the chronic brain disorders that affect the quality of life and well-being of millions of people around the globe. It is characterized by excessive electrical activity of the brain's cells that usually leads to recurrent seizures. Accurate, efficient, and robust techniques suitable for recent Internet of Medical Things (IoMT) devices to detect, classify, and diagnose epileptic seizures in a challenging multi-classification scenario and noisy environment are of paramount importance. Electroencephalograph (EEG) signals recorded even from the surface of the brain suffer from contaminated artifacts and noise from various sources, such as from EOG and EMG for eye-blinks and muscle artifacts, respectively. This work aims to address the challenges of multi-class classification and automatic seizure detection in intracranial EEG signals by developing a detection system suitable for real-world clinical settings. To achieve this, this work uses an effective feature extraction technique and efficient seizure detection methods based on a recent big data resource, along with advancements in deep machine learning techniques, to propose and develop robust hybrid models that combine conventional machine learning techniques and deep learning architectures to increase the performance of epileptic detection systems to levels that are close to acceptable for real-world applications. Firstly, a robust computationally efficient technique that characterizes different types of seizures with high precision and low latency of its onset was proposed. The system relies on an effective and low in complexity feature extraction approach based on the proposed advanced time-frequency Fourier Basel series Expansion based Flexible Time-Frequency Analytic Wavelet Transform (FBSE-FTFAWT) that extracts notable features associated with EEG seizure signals in a time-effective manner. Secondly, two noise robustness seizure detection techniques were developed to address the research question: can the hidden patterns in artifact-induced epileptic EEG data be identified and characterized? Stacked Auto Encoder based Support Vector Machine (SAE-SVM) and Deep Belief Network based Support Vector Machine (DBN-SVM) as hybrid classifiers are proposed with a novel feature extraction to classify various seizure and non-seizure class combinations. The proposed optimized SVM classifier, FBSE-FTFAWT /SAE-SVM, shows better detection accuracy, sensitivity, specificity, precision, and F1-score of 99.7%, 99.6%, 99.6%, 99.7%, and 99.6%, respectively, over the other two proposed models and the state-of-the-art methods in the literature.

Keywords— EEG; Epileptic Seizures; FBSE-FTFAWT; SAE-SVM; DBN-SVM.

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1. INTRODUCTION

Electroencephalography (EEG) has been extensively used in neurological research. It is a crucial tool in the diagnosis of brain illnesses, diseases, and mental health issues, as well as many treatment modalities in the medical and healthcare industries. EEG signals have been widely acknowledged as a leading approach in neurologically linked studies, notwithstanding the complexity of collecting and comprehending brain dynamics [1]. Electroencephalogram (EEG) signals are commonly utilized in neurological disease research and monitoring, including emotion recognition [2, 3], depth of anesthesia measurement [4], stroke-related disorders [5, 6], depression [7], Parkinson's diseases [7], sleep-related disorders [8, 9], brain death [10], and epileptic seizures [11]. Treatment for anomalies, attention issues, learning disabilities, behavioral disorders (like autism), and language delay can also benefit from it.

In epileptic patients, the EEG is measured from the scalp or intracranial (ECoG) and recorded using electrodes to distinguish between normal and abnormal situations, such as seizures and non-seizures [12]. Unprovoked seizures are the result of aberrant brain cell excitation in epilepsy, a persistent brain condition. Low blood sugar, deformities, and low oxygen levels after labor are some of the main causes of seizures [13, 14]. Seizures due to epilepsy can occur at any time and result in unconsciousness, injury, and even death. Since it is difficult for a patient to foresee when these seizures will occur, it is essential to predict when they will occur so that preventive actions can be taken to avoid loss of consciousness and occasionally even death [15,16]. Early identification and detection of these seizures is essential since it shields patients from the side effects of seizure activity and aids medical professionals in the diagnosis and treatment of patients. Ictal EEG is the EEG recorded during a seizure; however, because seizures are unpredictable, it is challenging to distinguish between seizure and non-seizure epileptic signals only from ictal EEG [17]. Because interictal EEG indicates the potential occurrence of epileptic seizures to aid in diagnosis, monitoring, and treatment, it is also utilized to differentiate epileptic seizures from other diseases [18, 19].

It is impossible to overstate the importance of integrating health care services for the monitoring, diagnosis, and analysis of various diseases, especially in light of the swift growth of intelligent internet of things (IoT) devices and the effective rollout of 5G networks [20]. The typical approach of expert detection and classification of epileptic EEG data involves manual visual inspection and analysis, which can be laborious, time-consuming, and error-prone. To address the issue of visual inspection and unpredictability in epileptic patients, research into automated techniques utilizing artificial intelligence is crucial. Nonetheless, the average outpatient with a seizure disorder records an EEG signal for roughly 20 minutes. This record is insufficient to capture the ictal activity for seizure or non-seizure detection. It can take several hours or even days to administer a multi-session EEG recording and a long-term length of recording from the inpatient service to assess, define, and diagnose epileptic patients [21]. This process is very costly and time-consuming. However, the need for automated, intelligent, wearable, portable, low-cost devices to improve patient diagnosis and the advent of generating massive amounts of data in the form of signals, texts, images, and sounds, among others, in health care management, have necessitated the research and development of robust feature extraction techniques.

Numerous techniques, including time-frequency techniques analysis [22], wavelet analysis [23], auto-regressive spectrum analysis [24], and multivariate technique analysis [25], have been studied and developed by researchers to identify and categorize epileptic episodes.

Other works investigated the localization of epileptogenic zone by detecting and classifying focal and nonfocal classes [26, 27]. Numerous encouraging findings on the identification and categorization of epileptic seizures have been documented in the literature; however, to date, the automated detection and prediction systems have not been widely adopted in the real world or as commercial products because of obstacles that must be surmounted to improve the detection system's sensitivity and accuracy. This work uses recent big data resources, along with advancements in deep machine learning techniques, to propose and develop robust hybrid models that combine conventional machine learning techniques and deep learning architectures to increase the performance of epileptic detection systems to levels that are close to acceptable for real-world applications. The suggested and developed hybrid robust feature extraction techniques based on sophisticated advanced time-frequency analysis, hybrid domain features, and non-linear features provide the input features for these models.

This work aims to conduct a thorough examination of several signal analysis domains for EEG epileptic seizure data to extract features for detection and classification. The sophisticated time-frequency technique, which merged other signal processing domains to serve as the foundation for robust feature extraction. To investigate and leverage the benefits of both systems, the extracted features were assessed using the suggested efficient detection methods, which are based on conventional machine learning and deep learning algorithms for detecting and predicting EEG seizure events. The work proposes a robust deep hybrid architectures that aim to improve the recognition accuracy by combining the SVM classifier with deep learning architectures to optimize the SVM classifier so that the computational complexity and cost would be reduced.

2. RELATED WORKS

Several works have been reported in the literature on detecting and classifying epileptic seizure signals using intracranial EEG signals. Conventional machine learning techniques and deep learning algorithms were developed to detect and predict the seizure events of normal, interictal, and ictal classes as contained in a popular and most commonly deployed database known as the University of Bonn dataset. Authors in [30,] employed Discrete Wavelet Transform (DWT) and experimented with several mother wavelets and statistical features with the genetic algorithm as the feature reduction approach. The classification was performed with conventional machine learning classifiers such as KNN, NB, SVM, and ANN, with ANN, reported to have the highest classification accuracy among the experimented classifiers. [31] utilized a five-level DWT to decompose EEG signals into sub-bands to extract features and train a Random Forest Classifier. The authors in [32] reported a method based on Continuous Wavelet Transform (CWT), extracted EEG waves' characteristics, and obtained the seizure boundaries from the data fusion applied to the CWT coefficients. A model for epilepsy diagnosis based on the DWT, Associated Petri Net (APN) methods, and minimum entropy approach was proposed by [33]. They experimented with different types of classifiers, such as Bayes Net (BN), decision tree, and SVM, and they reported a negative predictive value of 90.33% obtained using the APN model.

Extraction of significant features that characterize the epileptic EEG signals is a key to the successful detection and classification of epileptic seizures. Researchers proposed feature extraction techniques such as traditional time and frequency domain features [34,35], discrete and continuous wavelet transform features [31, 36], traditional statistical techniques, and higher order statistical analysis [30]. It is reported in [37] that a t-test and sequential forward feature selection play a significant role in the detection system's outcome. The authors built a reliable model with just five features and got outstanding results; KNN scored an amazing result of more than 99%, while SVM got flawless scores for accuracy, sensitivity, and specificity [37]. To multiclassify epileptic convulsions, the authors in [38] proposed a feature extraction technique from diverse domains. Ultimately, an optimal SVM classifier was used to build the model based on the retrieved characteristics and carry out a multi-classification of different types of seizures. They evaluated the proposed model using a recently available corpus dataset from Temple University Hospital (TUH), which included long-term seizure recordings.

Researchers have recently applied deep learning techniques to propose epileptic seizure detection models. These works include the one proposed by [39] using autoregressive moving average (ARMA) in the first part of the study and the pattern recognition method in the second part of the study. The methods are evaluated and report the accuracy of 93% and 94%, respectively. With the intracranial EEG signal, the Stockwell transform (S-transform) with Bi-LSTM was proposed in [40]. They obtained sensitivity and specificity of 98.09% and 98.69%, respectively. [41] presented an accuracy of 98.8% when they proposed the model based on convolutional long short-term memory (C-LSTM). Binary and three-class classification of different class combinations of the Bonn dataset was conducted by [42]. They proposed a deep convolutional neural network-based classifier using a signal image conversion technique from time-domain raw EEG signals to a time-frequency image representation as input to the classifier. Other works reported in the literature include [43], who proposed the CNN model [44, 45].

Most of the works reported in literature proposed techniques that are either suitable for binary classification, complexity in computation, require a large dataset, such as deep learning techniques, or do not consider the noisy clinical environment in their studies. To overcome the aforementioned challenges, this work proposed an advanced feature extraction technique that is low in complexity, however, it can characterize and explore the hidden information embedded in epileptic EEG signals. Also, the work validated the features in a challenging multi-class classification problem with a proposed hybrid machine learning and deep learning classifier.

3. SYSTEM METHODOLOGY

To develop an efficient automatic epileptic seizure recognition system, our proposed model involves some methodological steps. Figure 1 shows the block diagram of these steps: data acquisition, signal decomposition, feature extraction, feature ranking and selection, detection and classification, and performance evaluation of the obtained results.

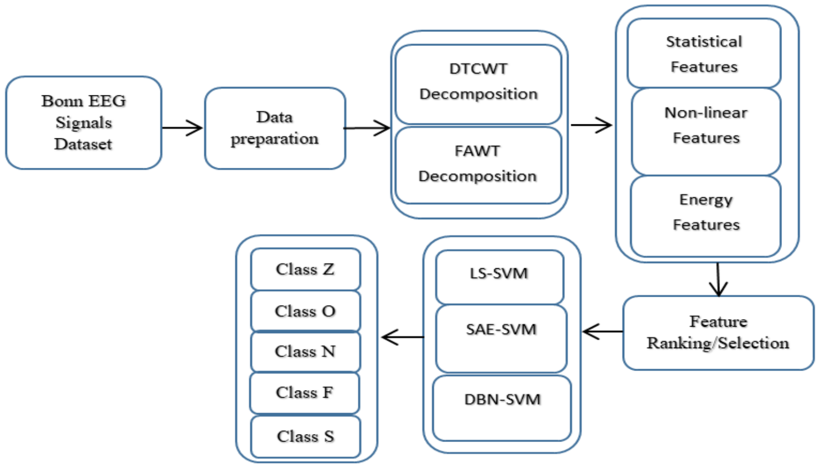

Fig. 1. Block diagram of the proposed model for multi-classification of five types of seizures from Bonn dataset.

3.1. The University of Bonn Dataset Description

The data used in this study were a database made available by the Department of Epileptology at Bonn University in Germany. It was a well-known and extensively used database that was made available online. Dr. Ralph Andrzejak of the Epilepsy Center at the Bonn University Hospital of Freiburg provided a thorough explanation and description of this dataset [28].

The usual 10-20 electrode placement system was used to record the data during the signal acquisition. A 12-bit A/D converter was used to digitize the 23.6-second EEG signals, which have a sample frequency of 173.6 Hz. This dataset is split into five classes or groups, each consisting of 100 single channels, based on five distinct criteria. Z, O, N, F, and S or A, B, C, D, and E are the groups. Additional details regarding the dataset are available at [29], and Table 1 describes the epileptic EEG dataset.

Table 1. Description of University of Bonn EEG dataset.

S/N	Set	Recording stage	Electrode type	No. of segments	Duration of segments	Predetermined class
1	Z	Open Eyes	Surface	100	23.6 s	normal or healthy
2	O	Closed eyes	Surface	100	23.6 s	normal or healthy
3	N	From hippocampal half sphere	Intracranial	100	23.6 s	preictal or seizure-free
4	F	From epileptic zone	Intracranial	100	23.6 s	preictal or seizure-free
5	S	During seizure	Intracranial	100	23.6 s	ictal or seizure

3.2. Feature Extraction

The accuracy and precision of classifiers must be taken into consideration for the successful detection and classification of epileptic seizures, and this depends on the quality of features extracted that characterize the signal's properties. To achieve the aim of this work, a highly robust feature extraction method was proposed.

The features are appropriately fit for Computer-Aided Devices (CAD) devices due to their low in complexity, portability, and low-cost, especially in our current Internet of Medical Things (IoMT) devices and networks. For this technique, a combination of energy, entropy, and higher statistical features is used.

The model consists of signal decomposition using Dual Tree Complex Wavelet Transform (DTCWT), which is a modified version of DWT, and the proposed advanced time-frequency signal processing method, which is a modification and improved version of flexible time-frequency coverage analytic wavelet transform (FTFAWT) known as Fourier Basel Series Expansion-based FTFAWT (FBSE-FTFAWT) that uses FBSE instead of the Fourier Transform. It is highly suitable for non-linear and non-stationary signals such as EEG because it provides double frequency resolutions compared to FTFAWT. After the decomposition, energy, non-linear, and statistical parameters were computed to explore the nonlinearity and complex nature of the EEG brain signals and reduce the signal dimension for low complexity and redundancy reduction.

3.2.1. Flexible Analytic Wavelet Transform

Despite the successful implementation of wavelet transform and the promising results obtained in the detection, classification, and localization of epileptic seizures over the years. Some limitations of this technique still exist, such as decreasing the resolution of sub-bands decomposed at higher frequency scales. Others are the issue of shift variance and adopting an equal time-frequency partition to cover all the scales. Therefore, recently, an FAWT approach has been proposed to eliminate the restrictions of DWT and its variants and efficiently process non-linear and non-stationary signals such as EEG. FAWT uses a set of filter banks to decompose the Epileptic EEG signals with high-pass and low-pass filters with arbitrary sampling rates. The ability to decompose the signals with flexible partitioning of time-frequency, as opposed to a constant one in DWT and other variants, leads to better tunable oscillatory scales and shift-invariance. Earlier works that utilized the efficacy of FAWT to analyze complex oscillatory signals, such as the classification of motor imagery [46], Emotion recognition [47], ECG signals detection and classification [48], sleep stage classification [49], and alcoholism detection [50].

FAWT can generate pairs of atoms using the Hilbert transform and allow adjusting parameters such as the redundancy control factor, dilation control factor, and quality factor [51]. It consists of high-pass and low-pass filters, and flexibility is achieved by adjusting the parameters of these filters. e, f, are the low-pass filter up sampling and down sampling factors, respectively, while g, h, are the equivalent high-pass filter up sampling and down sampling factors, respectively. The quality factor (Q_f) can be described as:

$$Q_f = \frac{2-\beta}{\beta} \tag{1}$$

where β is a constant and is positive.

The raw EEG signals are decomposed at level J_{th}, and at each level, two high-pass filters are used to deal with positive and negative frequencies. While a single low-pass filter decomposed the signal into J_{th} level with a frequency response $H(w)$ expressed in Eq. (2) as [51]:

$$H(w) = \begin{cases} \sqrt{ef}, & |w| < |w_e| \\ \sqrt{ef}\theta\left[\frac{(w-w_e)}{(w_s-w_e)}\right], & w_h \le w \le w_e \\ \sqrt{ef}\theta\left[\frac{(\pi-w+e)}{(w_h-w_e)}\right], & -w_h \le -w \le e \\ 0, & |w| \ge |w_h| \end{cases} \tag{2}$$

Also, the frequency response for the high pass channels *G(w)* can be described in Eq. (3) as:

$$G(w) = \begin{cases} \sqrt{2gh}\,\theta\left[\frac{(\pi-w+w_0)}{(w_1-w_0)}\right], & w_0 \le w < w_1 \\ \sqrt{2gh}, & w_1 \le w < w_2 \\ \sqrt{2gh}\theta\left[\frac{(w-w_2)}{(w_2-w_3)}\right], & w_2 \le w < w_3 \\ 0, & w \in [0, w_0) \cup ([w_3, 2\pi] \end{cases} \tag{3}$$

where the parameters in Eqs. (2) and (3) were expressed in Eqs. (4) – (9) as:

$$w_e = \big((1-\beta)\pi + \varepsilon\big)/e, \tag{4}$$
$$w_s = \pi/f, \tag{5}$$
$$w_0 = \big((1-\beta)\pi + \varepsilon\big)/g, \tag{6}$$
$$w_1 = (e\pi/fg), \tag{7}$$
$$w_2 = (\pi - \varepsilon/g), \tag{8}$$
$$w_3 = (\pi + \varepsilon/g), \tag{9}$$

$\varepsilon \le (e - f + \beta f)/(e + f)$, while, the $\theta(w)$ can be described in Eq. (10) as:

$$\theta(w) = \frac{[2-\cos(w)]^{1/2}[1+\cos(w)]}{2}, \text{ for } w \in [0,\pi] \tag{10}$$

The following conditions must be met by the adjustments parameters *e, f, g, h, β,* and *θ(w)* to obtain a perfect reconstruction as [51]:

$$\begin{cases} \theta|\pi - w|^2 + |\theta(w)|^2 = 1 \\ \left(1 - \frac{e}{f} \le \beta \le \frac{g}{h}\right) \end{cases} \tag{11}$$

The dilation factor d determines the wavelet size, and its frequency resolution is controlled by Q_f as defined in Eq. (1), its redundancy is donated as *R* and defined as:

$$d = \frac{e}{f} \text{ with } \beta \le 1, \text{ and } \mathrm{R} = \left(\frac{g}{h}\right)\frac{1}{1-d} \text{ with } \mathrm{R} > \frac{\beta}{1-d}$$

Based on the conditions and constraints mentioned above, the parameters *e, f, g, h,* and *β* can be selected quickly with the given d, Q_f, and *R*. An example of a 3-level FTFAWT decomposition of the signal is shown in Fig. 2.

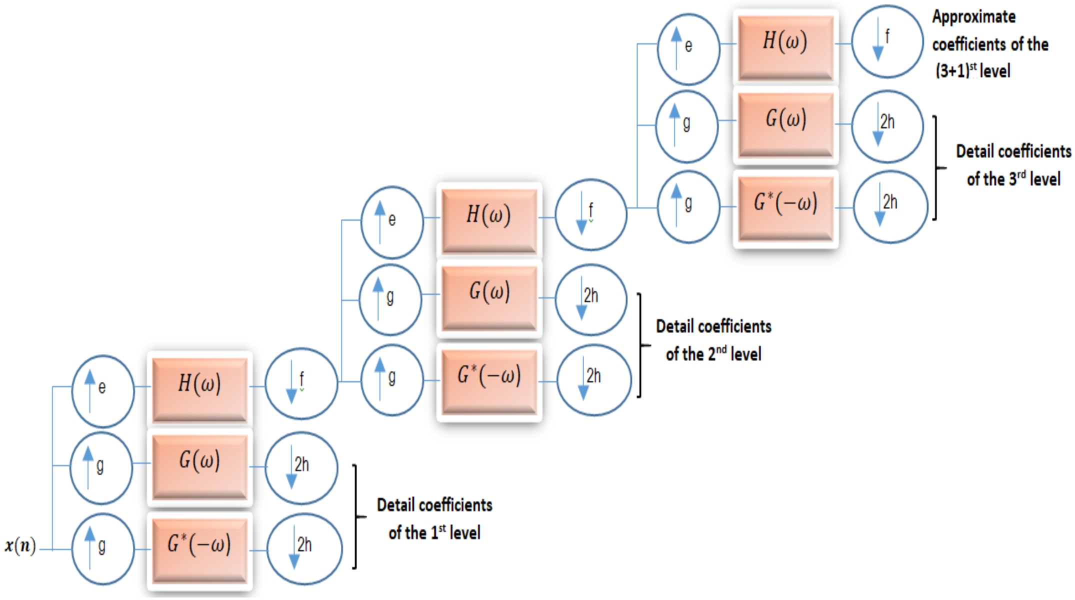

Fig. 2. Signal decomposition using 3-level FAWT decomposition.

The FAWT implementation approach consists of low and high pass filters, as mentioned above and mathematically described in Eqs. (2) and (3), which are iterated using Fourier Transforms as in [51]. At the same time, our proposed FBSE-FTFAWT approach employed FBSE, which is based on a zero-order Bessel function during the iteration as opposed to Fourier Transforms. It is described as:

$$x(n) = \sum_{i=1}^{N} C_i J_0\left(\frac{\delta_i n}{N}\right) \qquad n = 1,2,\dots,N-1 \tag{12}$$

where $x(n)$ is the discrete time signal. While C_i is the FBSE coefficient which is defined as:

$$C_i = \frac{2}{N^2 (J_1(\delta_i))^2} \sum_{i=1}^{N} n x(n) J_0\left(\frac{\delta_i n}{N}\right) \tag{13}$$

In this proposed approach, Newton Raphson's approach is used to compute the positive roots of the Bessel function. For the zero order $J_0(\delta) = 0$, the positive roots are donated as δ_i with i as the order of the coefficients and sampling frequency f_s is described as [52, 53]:

$$\delta_i \approx \frac{2\pi f_s N}{f_s} \tag{14}$$

Therefore, Eq. (14) can be simplified and written as [53-55].

$$i \approx \frac{2 f_s N}{f_s} \tag{15}$$

To cover the entire signal bandwidth, i should be varied from 1 to N, as shown in (15). For the implementation of this proposed method as a modified version of FTFAWT, the designed wavelet filter banks were used for the synthesis and analysis, performed with Eqs. (12) and (13). Also, from Eqs. (2) and (3), only the positive frequencies have been used that correspond to the frequency responses from the FBSE coefficients in Eq. (13). The new modified and proposed frequency responses based on positive frequencies for low-pass $R(w)$ and high-pass $S(w)$ are described as:

$$R(w) = \begin{cases} \sqrt{ef}, & w < w_p \\ \sqrt{ef}\,\theta\left[\frac{(w-w_p)}{(w_s-w_p)}\right] & w_p \le w < w_s \\ 0, & w \ge w_s \end{cases} \tag{16}$$

$$S(w) = \begin{cases} \sqrt{gh}\,\theta\left[\frac{(\pi-(w-w_0))}{(w_1-w_0)}\right], & w_0 \le w < w_1 \\ \sqrt{gh}, & w_1 \le w < w_2 \\ \sqrt{gh}\theta\left[\frac{(w-w_2)}{(w_2-w_3)}\right], & w_2 \le w \le w_3 \\ 0, & w \in [0, w_0) \cup ([w_3, \pi] \end{cases} \tag{17}$$

Equations (16) and (17) show that the low-pass and high-pass constraints and conditions remain the same, as suggested in FTFAWT. The sub-band signals from the ictal and non-ictal EEG signals of the Bonn dataset were generated using the proposed FBSE-FTFAWT. The adjustment parameters of *e, f, g, h,* β are selected in this work as e = 3, f = 4, g = 1, h = 2, and $\beta = (0.8g)/h$, while the decomposition level is selected as 10.

This FBSE-FTFAWT method was selected due to its superior time-frequency resolution, adaptability to signal dynamics, and its capacity for capturing subtle patterns often associated with seizure activity. Unlike traditional wavelets, FBSE-FTFAWT offers more precise localization and flexibility in shaping frequency bands, which is crucial for the accurate characterization of EEG features.

Recognizing that FBSE-FTFAWT can be sensitive to parameter selection, we implemented a careful parameter tuning process to enhance stability and performance. A range of parameter configurations—such as the Bessel function order, windowing parameters, and scaling factors—were empirically evaluated on a validation dataset. Parameters were chosen based on their consistency across multiple trials and minimal sensitivity to perturbations. Furthermore, we performed robustness checks to assess the impact of parameter variations, confirming that the selected configuration provided stable and reliable feature extraction. Wherever feasible, parameter settings were adapted based on signal-specific characteristics, such as sampling rate and dominant frequency ranges, to ensure the transform remained flexible and data-driven. These steps allowed us to mitigate the risk of overfitting and ensured reproducibility of the feature extraction process.

After the successful decomposition of epileptic EEG signals using advanced time-frequency decomposition approaches, the improved versions of the DWT and FTFAWT approaches were obtained. Robust features are extracted from the spatial and temporal features formed from advanced non-linear, statistical, time, and frequency domains. The features highlighted in 3.2.2 below were selected based on the statistical computation of their p-value using the SPSS statistical analysis tool, to identify and retain features that showed a significant difference between seizure and non-seizure classes. This approach offers a straightforward and interpretable method for preliminary feature ranking, which aligns with similar methodologies in existing literature and enables easier comparison of results. and they were found to be statistically significant. These selected non-linear features were combined with advanced higher-order statistical features.

3.2.2. Nonlinear Features

Entropy and energy features were extracted from DTCWT and FBSE-FTFAWT decomposed sub-bands of EEG signals to explore the non-stationarity and non-linear dynamics of the signals. The following are the entropy and energy parameters selected in this study.

1. Stein's Unbiased Risk Estimation and Threshold Entropies (SURE)

For EEG signal $x(n)$, SURE entropy can be described in Eq. (18) as:

$$SURE_{en} = N - \#\{i \text{ such that } |x_i| \le \in\} + \sum_i min\left(x_i^2, \in^2\right) \tag{18}$$

where N is the number of samples of the signals

$$TH_{en}(x_i) = \begin{cases} 1, & |x_i| > \varepsilon \\ 0, & otherwise \end{cases} \tag{19}$$

Therefore,

$$TH_{en} = N - \#\{i \text{ such that } |x_i| > \varepsilon\} \tag{20}$$

where, x_i are the signal's sample, ε represents the positive threshold

The average values of $SURE_{en}$ and TH_{en}, it is calculated from the decomposed signal in each EEG signal sub-band.

2. Shannon Wavelet Entropy

Shannon wavelet entropy ShW_{en}, it is calculated as follows:

$$ShW_{en} = \sum_n x_i(n) log x_i(n) \tag{21}$$

3. Centered Correntropy

Correntropy measures the correlation in the non-linear domain. It extracts the EEG signal characteristics from the decomposed components. It can be defined as:

$$CC(k) = \frac{1}{N-k+1} \sum_{n=k}^{N} k_\sigma(x(n) - x(n-k)) \tag{22}$$

where N is the length of x(n), k is the delay factor, and k_σ, is the Gaussian kernel function for σ bandwidth. The meaning correntropy is given as:

$$CC_{mean} = \frac{1}{N^2} \sum_{k=1}^{N} \sum_{n=k}^{N} k_\sigma(x(n) - x(n-k)) \tag{23}$$

Therefore, centered correntropy CCk can be computed as:

$$CC_k = CC(k) - CC_{mean} \tag{24}$$

4. Mean Teager-Kaiser Energy (MTKE)

The ability of MTKE to track minor fluctuations in EEG signals for both amplitude and frequency, which can extract meaningful information from the signals, makes it a suitable choice parameter to extract signal properties in this study. MTKE can be mathematically expressed as:

$$MTKE = \log\left(\frac{1}{N} \sum_t |x(t)^2 - x(t+1) * x(t-1)|\right) \tag{25}$$

5. Log Energy Entropy

Taking the square of the signal enables us to obtain complete information about the signal. Therefore, Log energy entropy helps us better discriminate our EEG signals. Log energy entropy is defined as:

$$LE_{en} = \sum_t \log{(x(t)^2} \tag{26}$$

6. Higher order statistics

Higher order statistics analysis has been employed in various EEG signal processing analyses and applications such as emotion recognition [56], classification of EEG signals [57], [58], classification of glaucoma [59], detection of focal EEG signal [60], among others. This work uses a third-order cumulant to reveal the hidden nonlinear changes in the epileptic EEG signals in higher-dimensional space.

The advantage of this method is preserving the original signal information, which consists of harmonic information at a higher dimension. Equation (27) describes the ith order cumulant with X_j denoted as the discrete-time random process of zero-mean

$$C_X^i[\ell_1, \ell_1, \dots, \ell_{i-1}] = M_{i,X}[\ell_1, \ell_2, \dots, \ell_{i-1}] - m_{i,G}[\ell_1, \ell_2, \dots, \ell_{i-1}] \tag{27}$$

where $M_{i,Y}$ and $m_{i,G}$ are denoted as ith order moment function of X_j, and an equivalent Gaussian random process, respectively. The ith order cumulant for a stationary random process is a function of $(i-1) lags \ell_1, \ell_2, \dots, \ell_{i-1}$, and j, as well as $(i-1)$ for a non-stationary random process. The complexity order can be determined by the number of lags [61]. Let's define the cumulant for the orders of i = 1, 2, and 3 as follows:

First order cumulant

$$C_X^1 = M_{1,X} = \varepsilon\{(X_j)\} \tag{28}$$

Second order cumulant

$$C_X^2[\ell_1, \ell_2] = M_{2,X}[\ell_1] - M_{1,X}^2 = M_{2,X}[-\ell_1] - M_{1,X}^2 = C_X^2[-\ell_1] \tag{29}$$

Third order cumulant

$$C_X^3[\ell_1, \ell_2] = M_{3,X}[\ell_1, \ell_2] - M_{1,X}\{M_{2,X}[\ell_1] + M_{2,X}[\ell_2] + M_{2,X}[\ell_1 - \ell_2]\} + 2M_{1,X}^3 \tag{30}$$

The expansion operator is denoted as ε. Figure 3 shows the third-order cumulant's contour and mesh plots of each set of the Bonn dataset. It can be observed that the third-order cumulant has various symmetries.

3.3. Classification

3.3.1. SAE-SVM Classifier

Sparse Autoencoder

An Autoencoder is a deep neural network that replicates the input value at the output when trained in an unsupervised fashion using a back-propagation algorithm. The output of AE is reconstructed from the spatial representation of the compressed input. AEs consist of two parts: encoder and decoder.

For an input x, the encoder maps x to a new representation z. The decoder at the output decoded z back to reconstruct the input x'. AEs are divided into input, hidden, and output layers [62]. The new representation and reconstructed input can be described mathematically as follows:

$$Z = h(Wx + b) \tag{31}$$

$$X' = g(W'z + b') \tag{32}$$

Where h and g represent the activation function, W and W', are the weight matrices, while b and b', are the bias vectors for the encoder and decoder, respectively.

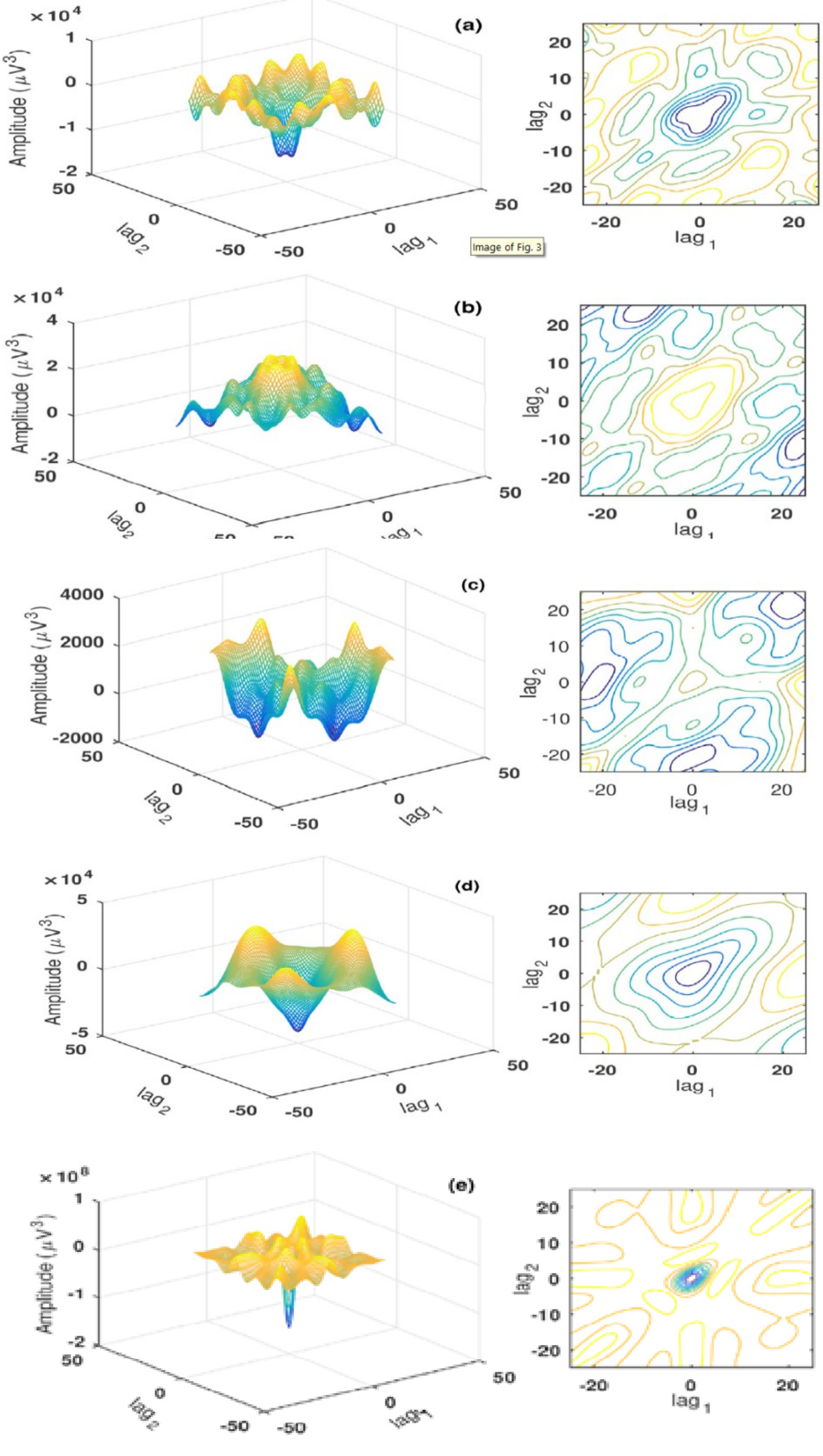

Fig. 3. Third-order cumulants' mesh and contour plots of (a) set Z, (b) set O, (c) set N, (d) set F, and (e) set S of Bonn dataset.

In conventional AE, mean square error (MSE) is the cost function which is given as:

$$J_{AEcost}(U) = J_{MSE}(U) = \frac{1}{m}\sum_{i=1}^{m}\left[\frac{1}{2}\|y_i - x_i\|^2\right] \tag{33}$$

where m, x_i, y_i, and U are the number of samples, input vector, output vector, and network parameters, respectively.

To optimize the error between the input and the reconstructed input x'. The following Equation should be solved as follows:

$$min_{(W,b,W',b')} = \sum_{i=1}^{n}\|x_i - x_i'\|^2 \tag{34}$$

To get around the redundancy in the features the AE has learned, we can raise the regularization limit of *L1* in the AE. To get around the duplication in encoding and decoding, a sparse constraint is also used. By adding sparse restrictions to the cost function and tightening the constraints on each hidden layer's response, the goal is to let some neurons fire while inhibiting the majority of neurons. The autoencoder cost function, as stated in Eqs. (35), (36), and (37), can be supplemented by the following sparse constraints.

$$J_{SAEcost}(W) = J_{MSE}(W) + J_{sparce}(W) \tag{35}$$

$$J_{sparce}(W) = \beta\sum_{i=1}^{2} KL(\rho|\rho_j) \tag{36}$$

$$KL(\rho|\rho_j) = \rho log\frac{\rho}{\rho_j} + (1-\rho)log\frac{1-\rho}{1-\rho_j} \tag{37}$$

where ρ, ρ_j, β, KL is the sparsity constraint level, average activation of the hidden layer unit neurons, weight of sparsity penalty term, and the divergence, respectively. The cost function becomes more minor as ρ and ρ_j, are closer to each other.

In this work, we propose a deep SAE (DSAE) network to reduce the dimensionality of the epileptic EEG signals by eliminating the redundant information in the signal. The architecture of DSAE consists of multiple layers of sparse autoencoders stacked; each layer's outputs are wired to the inputs of each successive layer.

This network is based on the mechanism of brain neurons' excitability. We decoded the original data after encoding it to the greatest extent possible. The advantage of DSAE is that it does not easily fall into the local minima during the training stage and has a quick convergence speed. In this work, we designed DSAE with two-hidden-layer SAEs.

Another classifier considered and proposed in this work is the hybrid structure of the SAE and SVM classifier, as shown in Fig. 4. In this classifier, the features were extracted by the SAE part, while the classification was performed on the SVM part. At the SAE stage, the optimization function that needs to be minimized to find the weight matrix W_l^q and bias vector b_l^q for the lth subbands with $l \in \{\delta, \theta, \alpha, \beta, \gamma\}$ is given as [63, 64]:

$$J(W_l, b_l) = \sum_{i=1}^{Tr}\left\|\tilde{S}_l^i - S_l^i\right\|_2^2 + \frac{\lambda_1}{2}\left\|W_l^q\right\|_2^2 + \lambda_2\sum_{j=1}^{\theta_{Tr}} KL\left(\frac{\rho^l}{\tilde{\rho}_j^l}\right) \tag{38}$$

where λ_1, λ_2 are denoted as regularisation indices, S_l^i and $\tilde{S}_l^i$, represents the actual and predicted sub-bands of ith EEG signals. *Tr* is the total number of training instances. *KL* is defined as the Kullback-Leibler divergence, ρ^l, $\tilde{\rho}_j^l$, are the desired and estimated sparcity of the lth subband and the jth hidden neuron of the lth sub-band.

At level 1, which consists of the input layer to the hidden layer, q is set as 1; at level 2, with the hidden layer to the output layer, q is set as 2. The feature vector H_l, at the hidden layer for the lth layer, can be obtained after calculating the weight matrices W_l^1 *and* b_l^1 for the lth sub-band during the training of SAE. H_l, can be expressed as:

$$H_l = f\left(W_l^T S_i + b_l\right) \tag{39}$$

Sigmoid is used as activation in both levels. After extracting features from SAE, various kernel functions were experimented to classify focal and non-focal EEG signals. The dimension for the weight matrix is $W_l \in R^{456\times4096}$, and $b_l \in R^{456}$.

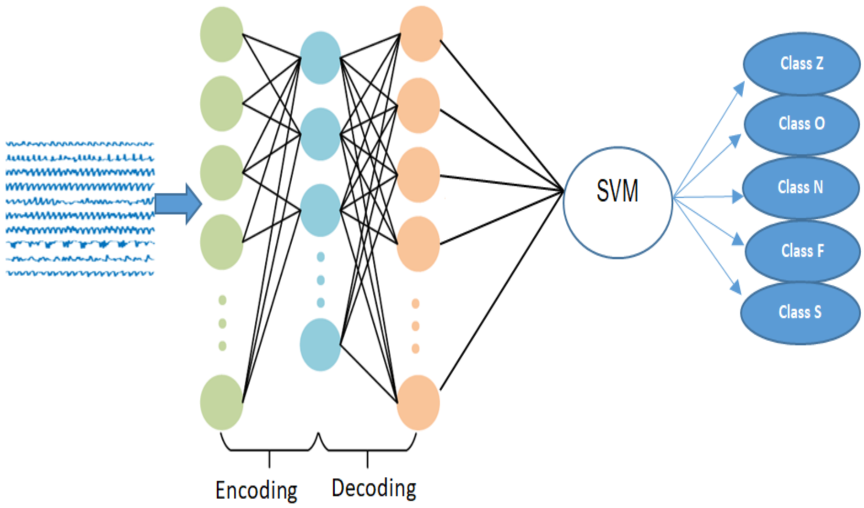

Fig. 4. Architecture of SAE network with SVM to form optimized SAE-SVM classifier.

3.3.2. DVN-SVM Classifier

Using the features extracted from modified FTFAWT and DTCWT, we combined DBN and SVM to characterize EEG signals from the Bonn dataset to enhance the performance of SVM classifiers further and examine the performance of combined conventional machine learning classifiers with deep neural network architectures. By extracting the features from the probabilistic generation model and deep structure, the DBN structure removes the training difficulty problem with traditional multilayer neural networks. Layers of Restricted Boltzmann Machines (RBM) with concealed variable input data distribution comprise the DBN structure. The RBM has two levels: lower and higher, with the output of the lower level acting as the input to the higher-level RBM. Since SVM has a better convergence rate than DBN, the DBN-SVM structure improves the convergence rate. DBN, conversely, is robust in mapping arbitrarily complicated non-linear relations and has fault tolerance and a greater non-linear fitting capacity [65, 66].

Both supervised and unsupervised learning were used in the DBN-SVM model, with SVM acting as a classifier and DBN handling unsupervised learning. Four hidden layers make up DBN, and the input layer nodes are five times the channels chosen, as determined by the empirical formula given in Eq. (40):

$$y = \sqrt{z+y} + a \tag{40}$$

Where y and z are the number of output layers' nodes and the previous layers; nodes respectively. While a is a constant. To classify ictal and non-ictal EEG signals, the output layer of the DBN is connected to the SVM via each BBN node.

The DBN-SVM model proposes in this work is used to train and test the FBSE-FTFAWT features as follows: first, the weighted parameters are computed from the FBSE-MEWT features that are trained by DBN unsupervised training. Next, SVM performed the supervised training on the connection of the DBN top-level output. Then, the error is feedback to the DBN top layer using a backpropagation algorithm, and therefore, the DBN top layer and the SVM layer weights are adjusted. Finally, the RBM, which is below the DBN top-level layer, is fine-tuned using the tuning process.

4. RESULT AND DISCUSSIONS

This study addressed the multi-classification problem using the features extracted from our proposed feature extraction technique and the classification of various classes of the Bonn dataset using the proposed hybrid classifiers. As mentioned in 3.1, the dataset consists of five classes corresponding to Z, O, N, F, and S classes. Each EEG class consists of 100 observations, with the length of each EEG signal L equal to 4096. Therefore, for the several classification problems conducted in this work, we can define each classification problem as n = 100 x k, where k represents the number of classes.

We developed a robust feature extraction technique using advanced time-frequency domains, statistical, and non-linear parameters in this work. To assess the efficacy of the decomposition approach proposed in this thesis, a popular decomposition technique called DTCWT, the modified version of DWT, is compared with the proposed advanced decomposition approach known as FAWT. As described in the section, we proposed the modified version of this approach, known as FBSE-FTFAWT. After decomposing the epileptic EEG signal using DTCWT and FBSE-FTFAWT, statistical, non-linear, and entropy parameters were investigated, computed, and ranked to select the most relevant and significant features. Feature ranking and selection methods are also employed to select the most optimized and robust parameters among the parameters under study. 500 EEG records from two normal, preictal, and ictal classes were downloaded as .txt files and distributed randomly to classify normal and seizure beats. Approximately 70% is used for training, and the remaining is used for validation and testing. The classification performance is considered in terms of sensitivity, specificity, accuracy, precision, and F1-score. The used performance measures are defined as:

$$\text{Sensitiivity} = \frac{TP}{\text{TP+FN}}$$

$$\text{Specificity} = \frac{TN}{\text{TN} + \text{FP}}$$

$$\text{Accuracy} = \frac{TP + TN}{\text{TP} + \text{TN} + \text{FP} + \text{FN}}$$

$$\text{Precision} = \frac{TP}{\text{TP} + \text{FP}}$$

$$\text{F1\_Score} = 2\,\frac{\text{Precision} * \text{ Sensitivity}}{\text{Precision} + \text{ Sensitivity}}$$

Various classifications between different groups have been performed as follows:

- Binary classification between ictal (class S) and normal (class Z)
- Binary classification between ictal (class S) and nonictal class (Z, O, N,F)
- Three class multi-classification between ictal (class S), interictal (class F), and normal (class Z)
- Five class multi-classification between ictal (class S), interictal (class F and N), and normal (class Z and O)

In this stage, we evaluated the proposed models in this work based on the hybrid methods of traditional machine learning classifiers and deep learning networks. As mentioned before, LS-SVM was chosen among the conventional classifiers after several experiments on various types of traditional classifiers, such as KNN, random forest, etc. However, different

SVM kernel functions were also tested on different types of SVM, such as SMO-SVM, QP-SVM, and LS-SVM. The kernel functions used in this work are linear, quadratic, RBF, and polynomial. Finally, we found that LS-SVM with RBF kernel functions performs better than other SVMs and kernel functions.

Part of this work aims is to evaluate the performance of traditional hybrid classifiers with deep learning networks to assess and quantify the benefits or otherwise of combining the efficacy of both techniques, as very few researchers have recently started exploring this area of research in the analysis of epileptic EEG signals.

Several features from various domains were extracted to analyze the effects and benefits of using these features. We have experimented with and tested features from many domains, such as differential energy, fractal dimension, entropy, and higher-order statistical features. Feature selection was performed, and the features that passed the statistical test are utilized in our proposed models FBSE-FTFAWT-LS-SVM, FBSE-FTFAWT-SAE-SVM, and FBSE-FTFAWT-DBN-SVM, as well as DTCWT-LS-SVM, DTCWT-SAE-SVM, and DTCWT-DBN-SVM for the binary and multi-classification of epileptic EEG signals using the small and single channel Bonn dataset.

4.1. Performance of Binary Classification of the Seizure (class S) vs. Normal (Class Z) Group

Binary classification of the seizure class and the normal class was performed using our proposed developed models that combined deep learning networks and traditional machine learning to improve the detection and classification of epileptic seizure signals. The proposed models are compared with the recent models developed by other researchers that utilized the same dataset. Firstly, we computed and plotted the confusion matrix of each developed model to extract the values of TP, TN, FP, and FN used in calculating the performance parameters of accuracy, sensitivity, specificity, precision, and F1-score.

Table 2 shows the comparison of the two time-frequency decompositions proposed in this work. FBSE-FTFAWT proved effective in all the advanced features, including energy, entropy, and higher statistical domains. Also, hybrid features outperform standalone features among the different domain features investigated and tested. The advanced energy features yield the accuracy, sensitivity, specificity, precision, and F1-score of 95.5%, 94.4%, 95.8%, 95.0%, 94.7%, and 95.9%, 96.1%, 97.8%, 96.3%, and 96.2% for DTCWT and FBSE-FTFAWT, respectively, using the SAE-SVM classifier. In contrast, the hybrid features classified with the same SAE-SVM classifier yield the performance of 100%, 98.6%, 99.7%, 99.4%, 99.0%, and 100%, 99.7%, 99.8%, 99.2%, 99.4% for both DTCWT and FBSE-FTFAWT time-frequency decomposition techniques.

4.2. Performance of Binary Classification of the Seizure (class S) vs. non-Seizure (Class Z, O, N, F) Group

In this section, we have evaluated our proposed models based on the detection and classification of seizure class and non-seizure classes; the non-seizure classes in the Bonn dataset are Z, O, N, and F classes combined as one class. Despite the data imbalance issue in this classification problem, our proposed hybrid method shows a very good performance with DTCWT-SAE-SVM and FBSE-FTFAWT-SAE-SVM, as shown in Table 3.

Table 2. Binary classification of normal Z vs. seizure S based on Time-frequency decomposition and optimized SVM classifier.

Features		DTCWT			FBSE-FTFAWT		
	Models	Acc [%]	Sen [%]	F-Sc [%]	Acc [%]	Sen [%]	F-Sc [%]
Advanced Energy Features	LS-SVM	94.9	93.5	93.3	95.1	95.6	95.4
	SAE-SVM	95.5	94.4	94.7	95.9	96.1	96.2
	DBN-SVM	95.0	95.2	96.1	97.2	98.9	98.9
Advanced Entropy Features	LS-SVM	93.8	93.2	92.6	94.4	95.1	96.2
	SAE-SVM	96.8	98.9	97.6	99.1	99.1	98.9
	DBN-SVM	97.3	98.2	98.2	99.1	99.3	99.4
Higher Order Statistical Features	LS-SVM	89.8	95.1	95.6	98.2	93.0	95.1
	SAE-SVM	97.3	96.9	97.8	98.1	96.3	97.1
	DBN-SVM	98.2	96.1	96.7	99.0	98.8	98.6
Hybrid Features	LS-SVM	99.2	99.2	99.0	99.1	99.1	98.7
	SAE-SVM	100	98.6	99.0	100	99.7	99.4
	DBN-SVM	98.3	100	99.8	100	99.8	99.5

Table 3. Binary classification of seizure S vs. combine non-seizure classes of Z, O, N, and F based on Time-frequency decomposition and optimized SVM classifier.

Features		DTCWT			FBSE-FTFAWT		
	Models	Acc [%]	Sen [%]	F-Sc [%]	Acc [%]	Sen [%]	F-Sc [%]
Advanced Energy Features	LS-SVM	93.6	94.6	94.4	95.6	94.8	95.4
	SAE-SVM	95.3	93.9	94.4	96.1	96.1	96.2
	DBN-SVM	95.5	94.5	95.5	97.2	98.1	98.9
Advanced Entropy Features	LS-SVM	94.2	93.7	93.6	95.3	95.6	96.2
	SAE-SVM	96.0	97.6	97.2	99.3	99.6	98.9
	DBN-SVM	98.4	98.2	98.5	99.4	99.3	99.4
Higher Order Statistical Features	LS-SVM	87.7	91.5	91.8	93.2	92.5	95.1
	SAE-SVM	98.0	97.9	98	93.8	95.4	97.1
	DBN-SVM	97.1	97.3	97.7	99.3	99.2	98.6
Hybrid Features	LS-SVM	98.4	98.8	98.9	99.3	99.2	98.7
	SAE-SVM	99.8	99.1	99.1	100	99.8	99.4
	DBN-SVM	99.5	100	99.6	100	99.7	99.5

4.3. Performance of Three Class Multi-Classification Between Ictal (class S), Interictal (Class F), and Normal (Class Z)

We covered the three-class multi-classification problem between three different classes of EEG signals in the Bonn dataset. These classes are the ictal, interictal, and normal classes; we chose one class from the interictal and normal classes, as each consists of two classes in the dataset. The corresponding classes of these classes used in this subsection are the S, F, and Z classes. The proposed model shows a promising result in the multi-class problem with deep hybrid learning and a machine learning classifier. The conventional machine learning classifier, such as SVM, is popularly known as a very effective classifier for binary classification. Our results indicate that the optimized SVM classifier that performed multi-class classification would be handy in epileptic seizure detection, which is suitable for the IoMT framework due to its simplicity and lower complexity cost. Table 4 depicts the performance of different domains and proposed time-frequency decomposition features, classified with LS-SVM, SAE-SVM, and DBN-SVM.

Table 4. Three-class multi-classification of seizure S vs. interictal F vs. normal Z based on Time-frequency decomposition and optimized SVM classifier.

Features	Models	DTCWT			FBSE-FTFAWT		
		Acc [%]	Sen [%]	F-Sc [%]	Acc [%]	Sen [%]	F-Sc [%]
Advanced Energy Features	LS-SVM	85.7	87.8	88.5	91.5	90.5	91.6
	SAE-SVM	91.2	89.4	90.7	93.9	90.9	92.9
	DBN-SVM	91.0	92.1	91.9	92.6	89.8	92.1
Advanced Entropy Features	LS-SVM	83.3	83.2	85.9	89.4	91.9	91.6
	SAE-SVM	86.7	86.2	87.8	91.7	91.4	90.7
	DBN-SVM	93.7	92.6	92.8	93.8	93.8	94.4
Higher Order Statistical Features	LS-SVM	81.7	85.2	85.7	82.8	90.5	88.8
	SAE-SVM	87.8	86.9	87.0	88.0	91.3	92.0
	DBN-SVM	86.4	88.1	90.1	93.4	90.0	90.7
Hybrid Features	LS-SVM	89.3	89.6	90.6	98.9	98.9	98.9
	SAE-SVM	98.9	98.6	99.0	100	98.9	99.0
	DBN-SVM	99.5	99.2	99.0	99.7	99.7	99.8

4.4. Performance of Five Class Multi-Classification Between Ictal (class S), Interictal (Class F and N), and Normal (Class Z and O)

In this subsection, we carried out a more challenging task of detecting and classifying five classes in the Bonn dataset. As explained in 3.1, these five classes are for ictal, two interictal, and two normal classes. These classes correspond to S, N, F, O, and Z. Despite its challenges, classifying all five classes in the dataset has great potential in clinical applications, as it helps develop real-time applications. For example, classification between interictal classes N and F corresponds to the same brain state. This will also help in seizure localization because both signals are acquired from the same patient, although from different brain regions. Therefore, it is imperative to investigate and develop models that can successfully detect and classify this problem with high accuracy. Table 5 shows the performance result for the five-class multi-classification problem.

4.5. Performance of Different Combinations of Classes in Multi-Classification of Three, Four, and Five Class Problem

To further validate our proposed models, we have conducted several experimental studies and analyzed different group combinations of classes in the Bonn dataset. The results of the three, four, and five-class multi-classification problems using the proposed hybrid features and SAE-SVM classifier are provided in Tables 6, 7, and 8 below.

The proposed models involve the decomposition using the proposed FBSE-FTFAWT, computing the features using the hybrid features method, and classifying with the proposed SAE-SVM classifier. A confusion matrix was presented for some classification combinations, such as class O vs. N vs. S, which stands for the normal vs. interictal vs. seizure class. We first computed the confusion matrix for all our performance values presented in this work. Then we extracted the values of TP, TN, FN, and FP to compute the average values of accuracy, sensitivity, specificity, precision, and F1-score.

The performance results of the proposed approach for the various combinations of multi-class classification for eleven different combinations of three-class multi-classification, five different combinations of four-class multi-classification problem, and finally, five-class multi-classification that includes all the class sets of the Bonn dataset are shown in Figs. 5, 6, Table 6, 7, and 8.

Table 5. Five-class multi-classification of seizure S vs. interictal N, F vs. normal Z, O based on Time-frequency decomposition and optimized SVM classifier.

Features	Models	DTCWT Acc [%]	DTCWT Sen [%]	DTCWT F-Sc [%]	FBSE-FTFAWT Acc [%]	FBSE-FTFAWT Sen [%]	FBSE-FTFAWT F-Sc [%]
Advanced Energy Features	LS-SVM	84.6	85.8	84.9	85.9	83.2	83.0
	SAE-SVM	91.9	89.4	90.7	94.9	94.1	93.8
	DBN-SVM	90.6	92.2	92.4	92.7	93.9	94.1
Advanced Entropy Features	LS-SVM	83.8	83.6	83.2	84.6	84.2	84.1
	SAE-SVM	91.9	92.2	91.8	93.3	94.4	93.6
	DBN-SVM	92.1	92.8	92.9	94.2	93.9	93.7
Higher Order Statistical Features	LS-SVM	83.9	83.2	83.2	84.2	85.1	84.6
	SAE-SVM	90.8	93.9	93.3	94.4	93.0	93.5
	DBN-SVM	91.4	91.7	91.7	93.0	92.8	92.6
Hybrid Features	LS-SVM	85.3	86.2	86.0	89.4	90.2	89.7
	SAE-SVM	93.5	94.1	94.2	99.2	98.7	98.8
	DBN-SVM	94.6	95.2	94.7	98.9	97.3	97.8

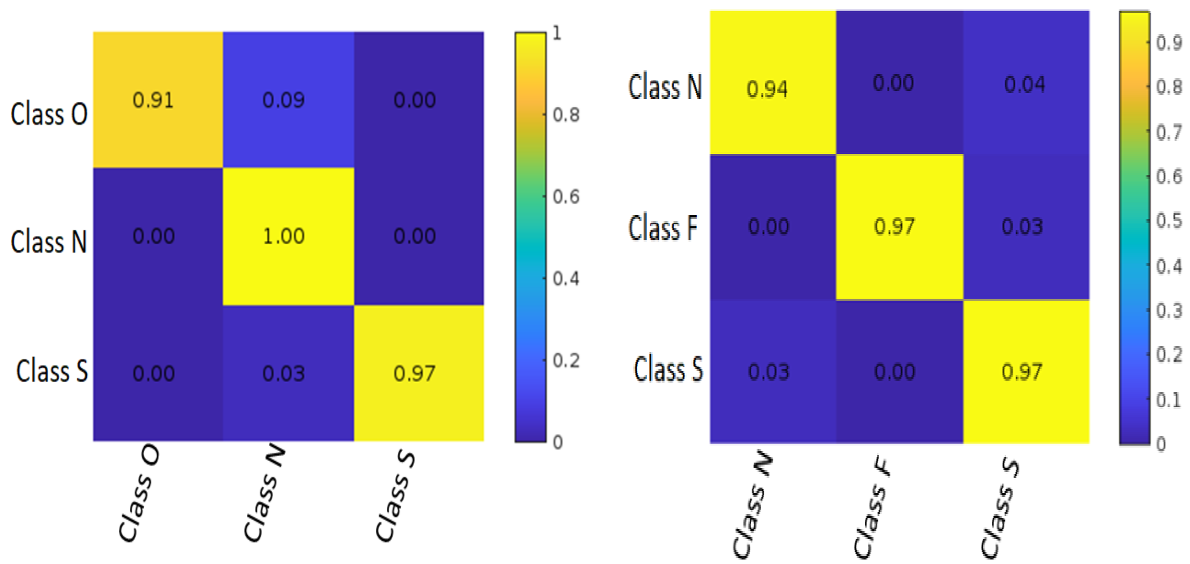

Fig. 5. Confusion matrix for three-class combinations of Bonn dataset using FBSE-FTFAWT-SAE-SVM model.

Table 6. Performance of three-class multi-classification combinations of the Bonn dataset.

Group	Class	Acc / %	Sen / %	Spe / %	Pre / %	F1-Sc / %
Normal vs. Normal vs. Preictal	Z vs O vs N	96.2	96.7	95.1	98.3	97.5
	Z vs O vs F	95.2	95.8	97.3	96.1	95.9
Normal vs Normal vs ictal	Z vs O vs S	97.7	95.8	96.1	97.3	96.5
Normal vs Preictal vs Preictal	Z vs N vs F	97.8	96.7	98.6	98.0	97.3
	O vs N vs F	97.1	98.9	97.1	97.1	98.0
Normal vs Preictal vs ictal	Z vs N vs S	98.7	99.4	99.1	99.3	99.3
	Z vs F vs S	99.2	98.8	98.3	99.1	98.9
	O vs N vs S	99.7	99.2	99.1	99.3	99.2
	O vs F vs S	98.8	98.7	99.6	99.0	98.8
	ZO vs FN vs S	99.8	99.2	99.1	99.5	99.3
Preictal vs Preictal vs ictal	N vs F vs S	99.1	98.7	99.1	98.9	98.8

Table 7. Performance of Four-class multi-classification combinations of the Bonn dataset.

Group	Class	Acc [%]	Sen [%]	Spe [%]	Pre [%]	F1-Sc [%]
Normal vs. Normal vs. Preictal vs. ictal	Z vs. O vs. N vs. E	96.5	95.8	97.3	96.8	96.3
	Z vs O vs F vs E	97.0	96.6	97.1	98.9	97.7
Normal vs Normal vs preictal vs preictal	Z vs O vs N vs F	95.6	97.2	97.0	96.2	96.7
Normal vs Preictal vs Preictal vs ictal	Z vs. N vs. F vs. E	96.9	96.9	96.3	97.0	96.9
	O vs N vs F vs E	95.5	97.3	97.1	96.2	96.7

Table 8. Performance of five-class multi-classification combinations of the Bonn dataset.

Class	Acc [%]	Sen [%]	Spe [%]	Pre [%]	F1-Sc [%]
Z vs O vs N vs F vs E	98.9	97.3	97.3	98.4	97.8

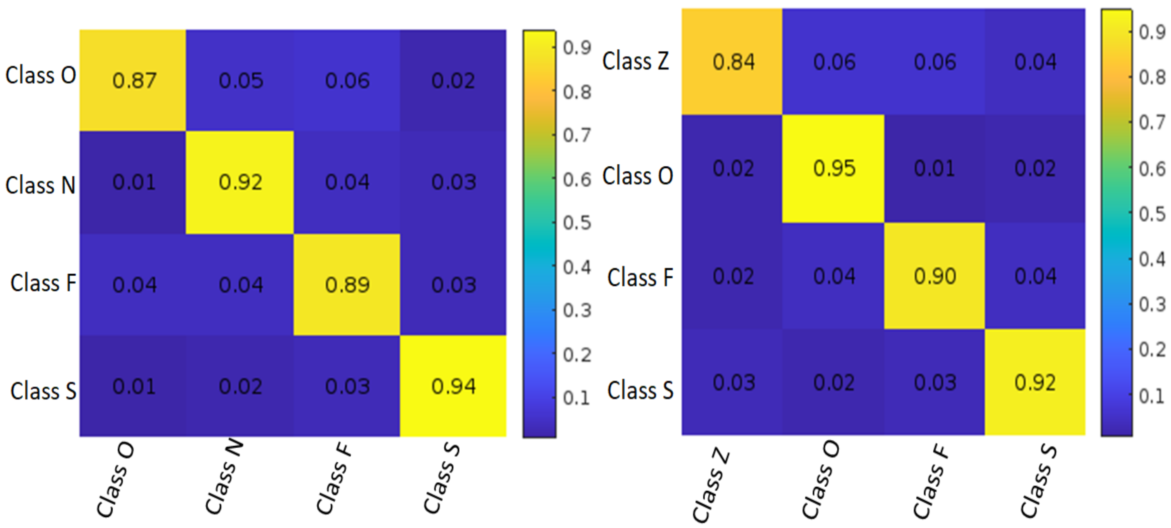

Fig. 6. Confusion matrix for four-class combinations of Bonn dataset using FBSE-FTFAWT-SAE-SVM model.

4.6. Performance of Proposed Models Based on Receiver Operating Characteristics Using FBSE-FTFAWT Hybrid Feature Extraction Technique

Another performance metric was deployed to further validate the models developed in this work. Receiver Operating Characteristics (ROC) curve analysis was used to compute the area Under the Curve (AUC) for a five-class, multi-classification problem. Figure 7 shows how the detection methods developed in this work distinguished the epileptic seizure characteristics efficiently.

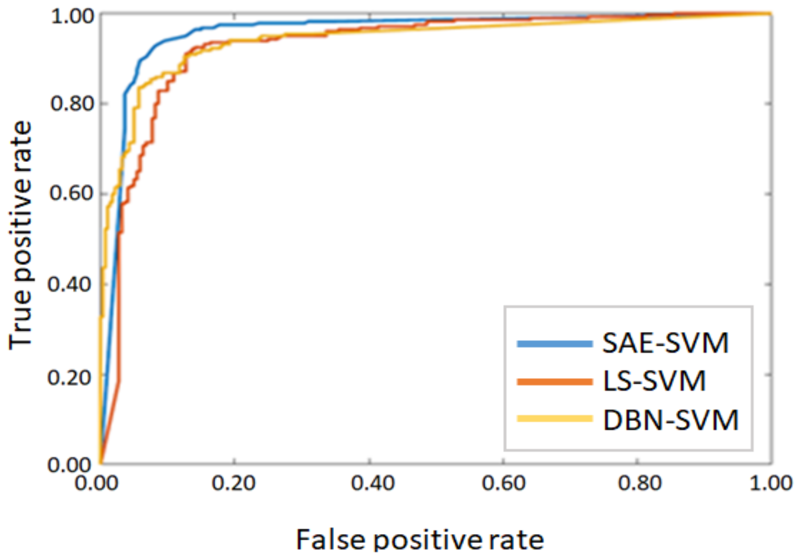

Fig. 7. ROC curve of the developed methods using the FBSE-FTFAWT hybrid feature.

In this section, seizure recognition and prediction models were proposed using the popularly employed Bonn dataset from the University of Bonn. The model developed in this section of this chapter involves the proposed robust feature extraction technique that characterizes the embedded features hidden in the EEG signal. The epileptic EEG signal has been decomposed using the proposed FBSE-FTFAWT transform. The performance of this method was compared with another popularly modified version of DWT known as DTCWT, which proved to be effective in decomposing non-linear and non-stationary signals such as epileptic EEG signals. Hand-crafted features were extracted by employing the advanced non-linear, entropy, energy, and higher statistical parameters to explore further the characteristics of epileptic EEG signals as well as to decrease the dimension of the features to decrease the computational complexity of the machine learning classifier.

The detection and recognition stage of the proposed model consists of deep learning architectures and conventional machine learning classifiers. This new hybrid system aims to optimize the classifier so that the most relevant and significant features are fed to the classifier. This was achieved with the feature selection and feature ranking using the computation of statistically significant features. The hybridization of SAE and DBN architectures with the SVM classifier proved to bring a cutting-edge solution to some of the limitations of using a standalone machine learning classifier by utilization of the benefits of deep learning algorithms in extracting useful features that can be fed to the traditional machine learning classifier that significantly reduced the computational complexity of training and testing the models.

The performance of each energy, entropy, non-linear, and hybrid feature was based on binary and multi-class classification problems. For the binary classification problem, we only considered the most challenging case of class combination, as many researchers have proposed several methods for binary classification of ictal and non-ictal EEG signals. This detection problem is classifying the ictal state (class S) and the combination of the remaining classes (class Z, O, N, F). Our model shows improved performance due to the robust features we proposed in this section, using the improved version of FBSE-FTFAWT. The FBSE-FTFAWT-LS-SVM, FBSE-FTFAWT-SAE-SVM, and FBSE-FTFAW-DBN-SVM yield average accuracy, sensitivity, specificity, precision, and F1-score are 98.2%, 98.3%, 99.8%, 99.1%, and 99.2%, respectively. For the multi-class classification problem, we have experiments three and five multi-class classification problems. Also, several combinations of classes of the three, four, and five classes were evaluated with the proposed models.

A total of 11 combinations of three classes were also investigated and experimented with to validate our FBSE-FTFAWT-LS-SVM, FBSE-FTFAWT-SAE-SVM, and FBSE-FTFAWT-DBN-SVM models. 5 combinations of four classes were also experimented with to validate the proposed model. At the same time, the most challenging problem in this dataset of five class combinations was also evaluated with our model. For all the class combinations, FBSE-FTFAWT-SAE-SVM with hybrid features performed very well on detecting ictal, inter-ictal, and normal classification sets.

4.7. Performance of Proposed Models Based for Five-Class Classification Problem Under Noisy EEG Signal

Our proposed model of epileptic seizure recognition is evaluated under a noisy scenario to handle the detection of five class classification problems. The signals are mixed with synthetic generated artifacts and noise, and the performance was evaluated at various SNRs. It is evident from Fig. 8 that the proposed approach shows a better performance in which a high recognition accuracy was obtained in this intractable recognition problem for generated eye blinking artifacts induced in the EEG signals, achieving the recognition accuracy of 100% at the SNR of 10dB, 15dB, and 20dB, respectively. The degraded recognition accuracies were obtained in the case of white noise, with accuracy decreased to 74.2%, 75.9%, and 77.0% at SNR of -20dB, -15dB, and -10dB, respectively. The muscle artifacts show more significant improvements compared to white noise but poor performance compared to eye blinks; this due to the wide range of frequency spectrum that muscle artifact dwells in, which results in distortions observed in the EEG waveform shapes. The proposed method witnesses better recognition accuracy as the SNR is greater than 0dB (SNR>0).

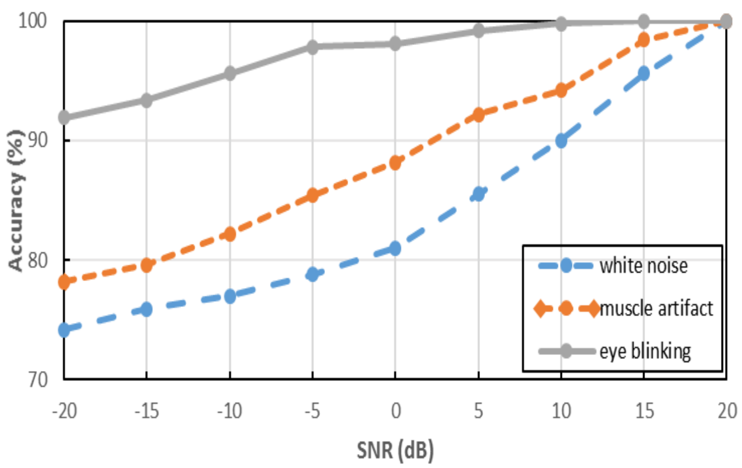

Fig. 8. Performance of DCNN-LSTM model for the five-class multi-classification problem under the corrupted EEG signal.

4.8. Comparison of the Proposed Models with Recently Reported Works in Literature

This section compares the performance of the proposed models with the state-of-the-art method reported recently in the literature. Three-class multi-classification with different combinations was selected to compare with those reported works that performed similarly to this work. Also, four-class and five-class multi-classification were selected and compared. This work limits the comparison to three, four, and five-class multi-classification because these problems are the most challenging in the detection system, as most researchers have investigated and reported their results for the binary classification approach. Tables 9, 10, and 11 provide the performance values for three, four, and five classifications.

Table 9. Comparison of the proposed models with the state-of-the-art methods for three-class multi-classification problem.

Class	Author	Year	Method	Related works Acc [%]	FBSE-FTFAWT-DBN-SVM Acc [%]	FBSE-FTFAWT-SAE-SVM Acc [%]
Z vs. O vs. N	Omer et al. [67]	2019	CWT/CNN	95.0	96.2	98.0
Z vs. O vs. F	Omer et al. [67]	2019	CWT/CNN	96.7	95.2	98.9
Z vs O vs S	Ilakiyasel et al. [68]	2020	CNN	93.3	97.7	99.4
Z vs. O vs. S	Omer et al. [67]	2019	CWT/CNN	95.7	97.7	99.4
Z vs. N vs. F	Omer et al. [67]	2019	CWT/CNN	88.0	97.8	93.2
O vs. N vs. F	Omer et al. [67]	2019	CWT/CNN	91.33	97.1	95.6
Z vs N vs S	Ilakiyasel et al. [68]	2020	CNN	94.8	98.7	98.6
Z vs N vs S	Sahani et al. [69]	2021	OVMD/DCNN	100	98.7	98.6
Z vs F vs S	Ilakiyasel et al. [68]	2020	CNN	95.9	99.2	99.7
Z vs F vs S	Sahani et al. [69]	2021	OVMD/DCNN	100	99.2	99.7
O vs N vs S	Ilakiyasel et al. [68]	2020	CNN	97.0	99.7	98.5
O vs N vs S	Sahani et al. [69]	2021	CWT/CNN	100	99.7	98.5
	Acharya et al. [70]	2018	CNN	88.7	99.7	98.5
	Omer et al. [67]	2019	CWT/CNN	98.0	99.7	98.5
O vs F vs S	Ilakiyasel et al. [68]	2020	CNN	95.6	98.8	99.4
O vs F vs S	Sahani et al. [69]	2019	OVMD/DCNN	100	98.8	99.4
ZO vs FN vs S	Ilakiyasel et al. [68]	2020	CNN	96.0	97.8	98.6
ZO vs FN vs S	Sahani et al. [69]	2021	OVMD/DCNN	100	97.8	98.6
	Zhang et al. [70]	2017	LMD/GASVM	98.4	97.8	98.6
	Ullah et al. [72]	2018	CNN/M-V	99.1	97.8	98.6
N vs. F vs. S	Omer et al. [67]	2019	CWT/CNN	89.0	99.1	99.4

Based on the results presented in the tables, it was clearly shown that our proposed models are robust and improve the classification accuracy compared to the state-of-the-art method reported. However, some researchers presented performance accuracy higher than our methods in a few combinations of datasets, such as [34], which reported an accuracy of 100%. This is due to the reduced number of datasets employed in that work, against the full dataset employed in our methods.

Table 10. Comparison of the proposed models with the state-of-the-art methods for combination of four class multi-classification.

Class	Author	Year	Method	Acc [%]
Z vs. N vs. F vs. S	Omer et al. [67]	2019	CWT/CNN	90.5
	Sharma et al. [57]	2019	CWT/CNN	100
	This study	2024	FBSE-FTFAWT /LS-SVM	89.8
	This study	2024	FBSE-FTFAWT /DBN-SVM	96.9
	This study	2024	FBSE-FTFAWT /SAE-SVM	99.2
O vs. N vs. F vs. S	Omer et al. [67]	2019	CWT/CNN	93.6
	Omer et al. [67]	2019	CWT/CNN	91.5
	This study	2024	FBSE-FTFAWT /LS-SVM	92.1
	This study	2024	FBSE-FTFAWT /DBN-SVM	95.5
	This study	2024	FBSE-FTFAWT /SAE-SVM	99.5

Table 11. Comparison of the proposed models with the state-of-the-art methods for combination of five class multi-classification.

Class	Author	Year	Method	Acc [%]
Z vs O vs N vs F vs S	Zhao et al. [74]	2020	DNN	93.55
	Sharma et al. [56]	2020	TOC/DNN	97.2
	Sahani et al. [69]	2021	OVMD/DCNN	99.88
	Zahra [75]	2017	MEMD/ANN	87.2
	Omer et al. [67]	2019	CWT/CNN	93.6
	This study	2024	FBSE-FTFAWT /LS-SVM	96.5
	This study	2024	FBSE-FTFAWT /DBN-SVM	98.2
	This study	2024	FBSE-FTFAWT /SAE-SVM	99.7

4.9. Discussions

Tables 9, 10, and 11 depict the performance results of the models developed in this paper compared to other reported works in the literature that utilized the same database and performed the same classification problems with different combinations of Bonn dataset classes. From the results presented, it is observed that our proposed models performed very well on the University of Bonn EEG dataset. The tables presented the comparison of our proposed models with other works in three, four, and five multi-class classifications, respectively. We found only one related work [67] that investigated the classification of four multi-class classification problems with a combination of Z vs. N vs. F vs. S and O vs. N vs. F vs. S with an accuracy of 90.5% and 91.5%, respectively. Our model of FBSE-FTFAWT /LS-SVM, FBSE-FTFAWT-DBN-SVM, and FBSE-FTFAWT-SAE-SVM achieves accuracy of 89.8%, 96.9%, 99.2%, and 92.1%, 95.5%, 99.5% for Z vs. N vs. F vs. S and O vs. N vs. F vs. S multi-class combination, respectively.

For the combination of three-class classification problems, few researchers investigated this multi-classification problem. Ilakiyaselvan et al. [68] reported an accuracy of 99.0% for the Z vs. O vs. S combination. Acharya et al. [68] presented an accuracy of 99.0 for the O vs. N vs. S multi-class combination. ZO vs. NF vs. S multi-class combination was investigated and presented an accuracy of 99.0%. In all the cases, our models show superior performance compared to the reported accuracy. The combination of ictal, interictal, and normal involves Z vs. N vs. S, O vs. F vs. S, and ZO vs. NF vs. S, achieving better performance than the other combinations. The combination that involves N and F in the same classification provides a lower performance because the sets N and F are from the epileptogenic zone.

The hardest and most complicated classification problem is five-class multi-classification. This is because the differentiation of the set from the same patients, such as class Z and O or class N and F, needs to be properly identified and recognized by the model. The method proposed by the optimized SVM classifier and the FBSE-FTFAWT /SAE-SVM shows better detection accuracy, sensitivity, specificity, precision, and F1-score of 99.7%, 99.6%, 99.6%, 99.7%, and 99.6%, respectively.

To the best of our knowledge, this is the first work that investigated the efficacy of the hybrid models of deep learning and conventional classifiers in detecting EEG epileptic seizure signals. Also, the work analyzed the importance of flexible frequency analysis in EEG seizure classification by decomposing the signals with a flexible nature of partitioned time-frequency by adjusting some parameters such as quality factor, dilation control factor, and redundancy control factor. The features computed from the decomposed signals using a feature selection and ranking help further distinguish the EEG signal's characteristics and discrimination. This translates that the model, apart from its high detection accuracy, will also be suitable for the deployment in real-world practices. The proposed method could be developed on a standalone and cloud-based diagnostics system. Also, the flexibility of FAWT, higher-order statistics, energy, and entropy features improves the understanding and interpretation of epileptic EEG signals by clinicians.

The performance of the models was evaluated using the average accuracy obtained during the test set of the model. However, the detection of the multi-class event cannot be described by the model's accuracy alone; sensitivity and F1-score are also suitable performance metrics to evaluate the classifier's effectiveness in distinguishing each class appropriately. Therefore, quantitative and qualitative performance analysis has been employed to assess the robustness and sensitivity of the model properly.

5. CONCLUSIONS

High complexity algorithms that have been developed recently pose a great challenge in the IoMT devices due to the devices’ requirements of low-cost and low complexity. The work proposed in this paper aims to develop a hybrid method that characterizes the epileptic EEG seizure components with high precision and accuracy. Robust feature extraction technique that utilizes the advantage of the Fourier Basel series Expansion based Flexible Time-Frequency Analytic Wavelet Transform to effectively characterize and identify the embedded EEG patterns in a challenging artifact-induced environment. The proposed model improved the detection and classification accuracy in the complex multi-class classification scenario. The multi-classification of intracranial EEG epileptic seizures was performed with the proposed novel hybrid classifiers known as Stacked Auto Encoder based Support Vector Machine (SAE-SVM) and Deep Belief Network based Support Vector Machine (DBN-SVM). Various combination of multi-class seizure classes was experimented with in this work to validate the developed model. More advanced feature selection techniques, such as Mutual Information (MI), Recursive Feature Elimination (RFE), LASSO (L1 regularization), and Random Forest Feature Importance that might offer more robust in terms of feature interdependencies and classifier relevance will be explored in future work to enhance the model's generalization capability and performance robustness. Future work should also be focused on exploring deep learning and machine learning approaches that are suitable for the recent IoMT device. Validating the proposed model on the time complexity and real-time scenarios should also be investigated.

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