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Industrial Design of Type-1 and Interval Type-2 Fuzzy Logic Control

Snejana Yordanova[1*]

[1]Department of Industrial Automation, Faculty of Automation, Technical University of Sofia, Sofia, Bulgaria
E-mail: sty@tu-sofia.bg

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***Abstract*—**This paper focuses on the design of type-1 and interval type-2 (IT2) PID fuzzy logic controllers (FLC) for ensuring - by a programmable logic controller (PLC) - a high-performance real-time liquid level control in a carbonization column (CCl) for soda production. Firstly, Takagi-Sugeno-Kang models - derived via genetic algorithms parameter optimizations, experimental data and simulations for the basic and the worst CCl loads - are studied at different operation points, and the worst Ziegler-Nichols (ZN) model is assessed. Next, two-input fuzzy units are designed - assuming various membership functions (MF) and uncertainties - and the greatest linearization gain is computed. Based on it and the ZN worst plant model, the parameters of the FLC input pre-processing differentiator and the PI post-processing are empirically tuned. Finally, a PLC oriented analytical description of the IT2 MF, fuzzy rules and type-reduction is suggested. The designed FLC systems are studied via simulation to determine the factors that have the greatest impact on the system performance improvement. The obtained results unveil that the tuned FLC outperforms the tuned linear PI. Better system performance is achieved by a small number of MF with large support ensuring economical PLC presentation.

***Keywords*—**Controller design; Programmable logic controller implementation; Simulations; Takagi-Sugeno-Kang plant models; Type-1 and interval type-2 PID FLC.

Nomenclature

CCl	Carbonisation column	PI(D)	Proportional-plus-integral (-plus derivative)
FL(C)	Fuzzy logic (control/ler)	PLC	Programmable logic controller
FU	Fuzzy unit	2I(SI) SO	Two-input (single-input) single-output
FOU	Footprint of uncertainty	TSK	Takagi-Sugeno-Kang model
GA	Genetic algorithms	T1 (IT2)	Type-1 (interval type-2)
MF	Membership function	ZN	Ziegler-Nichols model

1. INTRODUCTION

The fast development of technology and the market demand for an increased quantity and more sophisticated products of high quality urge the need to improve the control of processes by introducing intelligent techniques. The most widely applied approach is based on fuzzy logic controllers (FLC) for plants with no reliable and simple mathematical model. The FLC systems tackle plant nonlinearity and uncertainty ensuring also smooth and economical control action by simple design based mainly on expert knowledge and improved techniques via adaptation and optimisation using mainly genetic algorithms (GA) [1-5]. Different real time laboratory tests, hardware-in-the-loop simulations and industrial applications of FLC systems are reported in [1, 4, 6-9]. In [10] a single FLC for desired longitudinal acceleration is designed

* Corresponding author

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to improve and the adaptive cruise control that maintains a safe follow distance between the lead and the ego cars, the cruise control, and the automatic emergency braking. A FLC is suggested in [11] for the power flow between the loads and the multiple renewable energy sources in a microgrid by changing the charging rate of the battery to store the surplus energy from solar panels and wind turbines.

The parameters of FLC and of derived fuzzy logic plant models including the parameters of the membership functions (MF) can be GA optimised based on experimental data from real time operation and simulations [9-11]. The fitness function to be minimised is often the system or the modelling error (mean square error or integral square error) and for the FLC tuning – the control variance.

The uncertainty in the characteristics of the most industrial plants as well as the subjectivity and uncertainty in the expert knowledge and experience about the control of the plant needed in the FLC design give rise to the development of type-2 FLC. In type-2 FLC the MF are not fixed but change either in a Gaussian manner at each point or by an interval type-2 (IT2) FLC [12-19]. The main applications are based on laboratory tests, low-cost PIC microcontrollers and simulations for the control of inverted pendulum, robots, coupled tanks, DC motors, etc. [20, 21]. Despite the theoretical progress the great expectations for improvement of the system performance – dynamic accuracy and robustness, or for simplification of the control algorithm and the controller's design have not be experimentally confirmed. The tests of systems with IT2 FLC are few and mainly in simulations, often in controlling of simple plants with known mathematical model far from the industrial environment and applying a single step reference change – insufficient for nonlinear systems. Most of the performance improvements are too small to justify the complexity of the IT2 FLC. The improvement of the IT2 FLC system performance is assessed in comparison with systems with linear controllers and rarely with type-1 (T1) FLC systems designed to ensure the best possible performance. In [20] a Sugeno PD IT2 FLC with linear function of the inputs – the system error and its derivative each with 2 MF, in the rules' conclusions and adaptation of the function gains outperforms a little the linear PID controller for the inverted pendulum in the response to random noise, external disturbance and parameter changes in a single operation point. In [16] a PID IT2 FLC is suggested from parallel PI and PD Sugeno FLC with different MF and only three rules. Thus the fuzzyfication of the inputs passes through several MF. A linear and the type-1 (T1) FLC systems are also GA optimally tuned for fitness the integral square error and control energy to control three nonlinear plants for comparison. The IT2 PID FLC system outperforms these systems but in simulation of a single step response to disturbance and plant variations. The performance of the IT2 FLC is largely dependent on the footprint of uncertainty (FOU) which provides robustness and uncertainty handling when properly tuned. A larger FOU makes the fuzzy controller piecewise linear. Recommendations for the optimal FOU for different cases are derived in [17, 18] but they are rather complex to easily use and are not bound to system disturbances and plant changes. Other basic problems with the IT2 FLC are the complex structure and design which keep them far from industrial application as well as the computational cost of the type-reducer which has to be proper for real time online control and with positive impact on the system performance [22, 23]. The Nie-Tan method is assessed as one of the fastest and only 1.2-1.7 slower than T1 FLC.

Therefore, it is important to develop a FLC design approach common for T1 and IT2 and oriented to industrial implementation which ensures the best system performance via a simple

FLC design and algorithm based on data from the real time operation of an industrial plant and proper simulation experiments. The current investigation develops a methodology for a programmable logic controller (PLC) [24] oriented design of T1 and IT2 fuzzy logic controllers and to study via simulation their potential for high performance control of industrial plants for which no simple reliable mathematical models exist.

The industrial plant selected as an example is the level of the solution in a carbonisation column (CCl) in the production of soda ash [9, 25]. The investigation is based on MATLAB™ and its toolboxes Simulink and Fuzzy Logic [26].

The paper is organized as follows. Section 2 presents the plant, its TSK plant models and characteristics and the based on them empirical approach for tuning of the parameters of linear controllers. A procedure for the design of PID FLC is developed in Section 3. The PLC implementation of FLC is described in Section 4. A linear PI, different T1 and IT2 FLC systems are designed and investigated via simulation in Section 5. Section 6 contains a conclusion and outlines the future research.

2. CONTROL PLANT MODELLING AND STUDY

The plant to be controlled is the level H (y=H) of the solution in a carbonisation column for the production of soda ash in Solvay Soda – the town of Devnya in Bulgaria. The plant is difficult to be modelled because of its nonlinearity, variable parameters and disturbances mainly due to the alternation of the modes “operation” and “washing”, changes of the CCl load and of the operation points. The pressure in the common supply pipe of the several operating in parallel columns is considered as the main disturbance. Besides, H is influenced by the gases needed in the reversible exothermic chemical reaction which are let into the CCl from the middle and the bottom of the CCl.

In [27] two Takagi-Sugeno-Kang (TSK) plant models are derived and validated for the high (varied) and low (nominal) load of the CCl. Each of the nominal TSK and the varied TSKv plant models consists of a Sugeno model with outputs the three MF of belonging of the current H to the three defined by experts linearisation zones. Each TSK model dynamic part is expert defined to consist of three time lags $P_i(s)=K_i.(T_is+1)^{-1}$, $i=1\div3$, one for a linearisation zone, and one common for all zones time lag $P_4(s)=(T_4s+1)^{-1}$ as shown in Fig. 1.

The time lag $P_4(s)$ is added in series to increase the order of the local plant model. The time lags $P_i(s)$ operate in parallel with the control action U as a common input. The parameters of the time lags and the initial level $H(0)$ are computed via GA minimisation of a fitness function of the modelling error:

	K_1	T_1 [s]	K_2	T_2 [s]	K_3	T_3 [s]	T_4 [s]	$H(0)$ [%]
for TSK q_{TSK}	0.85	218	1	6.4	1.15	86.5	740	50
for TSKv q_{TSKv}	0.11	73	0.26	350	0.51	98	150	33

The fitness function is computed for various plant input $U(t)$-output $H(t)$ data recorded from the real time linear PI control of the level y=H in the operating CCl under two types of loads - nominal and varied and from the TSK model simulation. The TSK plant models enable to simulate the plant step responses in the different operation points for the two loads, shown in Fig. 2.

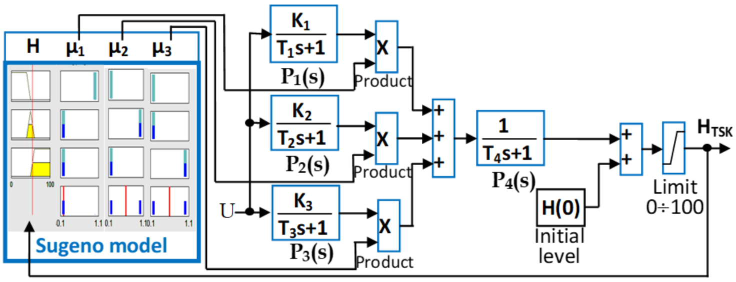


Fig.1. TSK plant model.

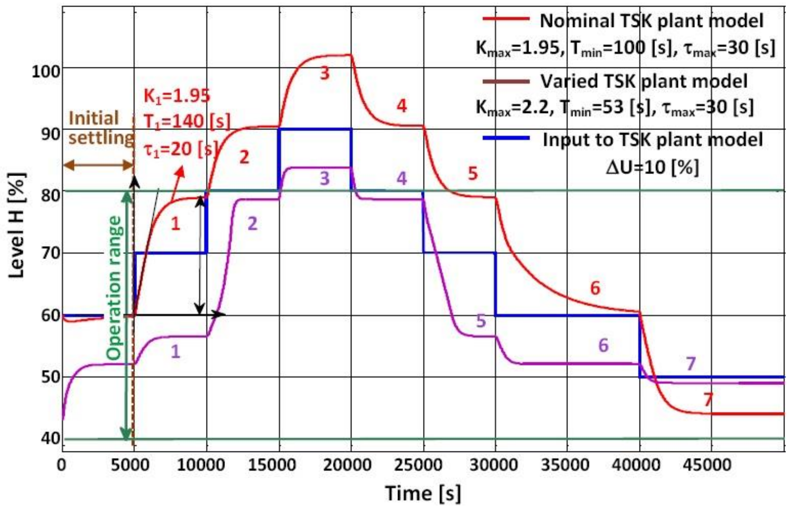


Fig. 2. Step responses of TSK and TSKv plant model and ZN models parameters.

The first process ensures settling of the initial conditions, i.e. reaching of initial equilibrium which is important for a nonlinear system. The graphically assessed parameters of the Ziegler-Nichols (ZN) plant models – gain K, time constant T and time delay τ (tau) are different for each operation point which shows that the plant is nonlinear.Linear PID and PI controllers $C_{PID}=K_p+K_is^{-1}+ K_ds$, $C_{PI}=K_p+K_is^{-1}$, $K_i=K_p/T_i$, $K_d=K_p/T_d$ are empirically tuned accounting for the heaviest with respect to the impact on the closed loop system stability parameters of the ZN plant models for all operation points and loads – the greatest gain K_{max}=2.2 and time delay τ_{max}=30[s] and the smallest time constant T_{min}=53[s] [28]:

$$K_p^*=\mathbf{A}.\, T_{min}/(K_{max}.\tau_{max}),$$
$$\text{for PID controllers } T_i^*=\mathbf{B}.\tau_{max},\ T_d^*=\mathbf{C}.\tau_{max},\ K_d^*=K_p^*.T_d^* \qquad (1)$$
$$\text{for PI controllers } T_i^*=\mathbf{B}.T_{min},\ K_i^*=K_p^*/T_i^*$$

where $\mathbf{A}$=0.3÷1; $\mathbf{B}$=0.9÷3 and $\mathbf{C}$=0.4÷1 take different values to shape the closed loop system step responses, ensuring desired performance indices such as overshoot σ, settling time t_s, etc.

A linear PI controller is empirically tuned using Eq.(1) to yield $\mathbf{q}_{PI}=[K_{p\,PI}=10, K_{i\,PI}=0.003]$ for the sake of comparison of the PI and FLC closed loop systems. A tuned linear PID cannot ensure a better system performance.

3. FUZZY LOGIC CONTROLLERS DESIGN

3.1. Structure Design

The structure of the most used in PLC implementations PID FLC is presented in the block diagram of the closed loop FLC system in Fig.3 where the measured level H_m is filtered from noise by the exponential filter with transfer function $W_f(s)$. The two-input fuzzy unit (2I FU) of the PID FLC has for inputs the normalized by K_e system error e in the range [-1, 1] and its normalized in [-1, 1] derivative $\dot{e}$ substituted here by the derivative of level $(-\dot{y})$, denoted by (-dy). The normalisation superscript "n" is further omitted to simplify the expressions. For constant reference y_r (-dy)= $\dot{e}$ but dy changes smoothly with the step reference changes. The derivative dy is computed by a first order noise insensitive differentiator $W_d(s)=K_d.T_d.(T_d s+1)^{-1}$ with $T_d=(2\div10)\Delta t$ and a sampling period Δt=1s. Small T_d and great K_d ensure almost ideal differentiation which step response is a high pulse for a short time. However, the differentiator output for too small T_d with respect to Δt may not influence an inertial plant. Therefore the practical trade-off is the accepted range for values for T_d. The post-processing is PI $W_{PI}(s)=K_p+K_i s^{-1}$. The 2I FU normalized output o in the range [-1, 1] is a nonlinear function of the inputs e and dy, $o=\Psi(e, dy)$. It can be considered an output of a nonlinear PD algorithm:

$$o=K_{p\text{-}PD}(e,dy).K_e.e+K_{d\text{-}PD}(e,dy).K_d.(-dy).$$

The PID FLC output is computed as a PI post-processing of o $u_{PID}=K_p.o+K_i.\int o.dt$, where $u_{PD}=K_p.o$ is the PD component and $u_{PI}=K_i.\int o.dt$ comprises a PI component via integration of nonlinear PD – integrating "P" yields the nonlinear integral of u_{PI} and integrating "D" results in the proportional part of u_{PI}. Thus, PID FLC = PD FLC+PI FLC. The PID FLC tuning parameters are $\mathbf{q}_{FLC}=[K_p\ K_i\ K_d]$.

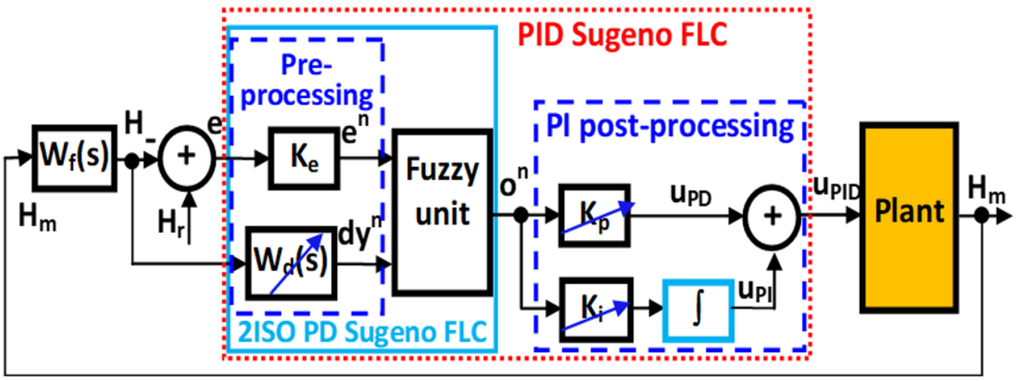
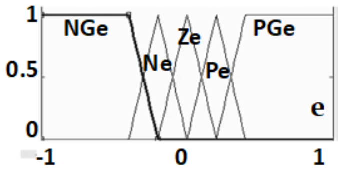
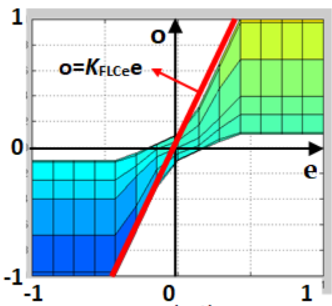
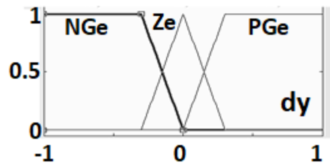
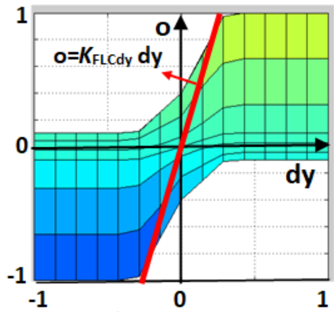
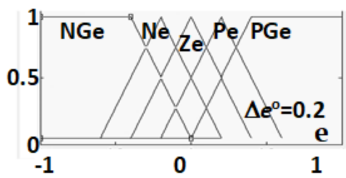
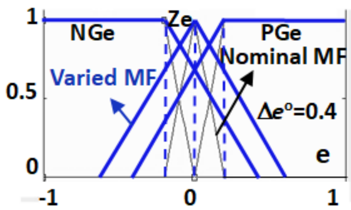
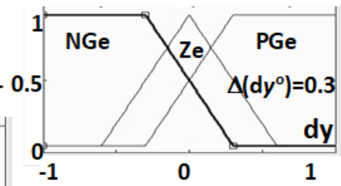
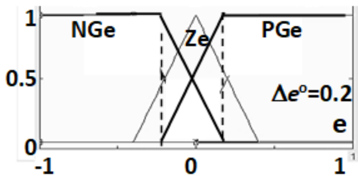
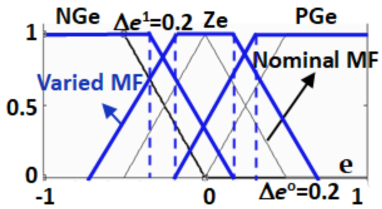


Fig. 3. Block diagram of closed loop system with PID FLC.

3.2. Fuzzy Unit Design

The 2I FU is designed based on the smallest possible number of MF and fuzzy rules respectively, triangular and trapezoidal shapes for the MF of the inputs and singletons for the output in order to facilitate the economical representation of the obtained Sugeno 2I FU in a PLC. Here 3 and 5 MF are accepted for e and 3 for dy, i.e the 2I FU is 3x3 or 5x3 respectively, and 7 singletons for the output S_o= [-1 -0.4 -0.1 0 0.1 0.4 1]. The various MF of the 2I FU used in the investigation are shown in Table 1 where (Ze5_02, Zdy03) denotes triangular symmetrical MF for the norm terms Z_e and Z_{dy}, Ze5_02 is the norm from total 5 MF for e with a range Z_e= [-0.2, 0.2]. The default number of MF is 3 and is not indicated, i.e. Zdy03 is the norm from total 3 MF for dy with a range dy= [-0.3, 0.3]. A soft rule base is used, e.g. for 2I FU with 3x3 MF the nine fuzzy rules are constructed for all combinations of the MF for the two FU inputs e and dy and the strongest outputs o=NG_o (negative great) and o=PG_o (positive great) appear once in the rules conclusions:

Table 1. Type of FU, MF and control surface projections.

Variant	FU and linearisation gains	MF μ_e with respect to e	MF μ_{dy} with respect to dy
1.	FU of 5x3 MF Nominal 1 (Ze5_02, Zdy03) orthogonal K_{FLCe}=2.5, K_{FLCdy}=3.3	  o-e projection	  o-dy projection
2.	- Varied from Nominal 1 (Ze5_04, Zdy06) non-orthogonal K_{FLCev}=2, K_{FLCdyv}=1.3		
3.	FU of 3x3 MF Nominal 2 (Ze02, Zdy03) orthogonal K_{FLCe}=5,K_{FLCdy}=3.3 - Varied 1 from Nominal 2 (Ze06,Zdy06) non-orthogonal K_{FLCev}=1.1, K_{FLCdyv}=1		
4.	- Varied 2 from Nominal 2 (Ze04, Zdy06) non-orthogonal K_{FLCev}=3.3, K_{FLCdyv}=1		
5.	FU of 3x3 MF Nominal 3 (Ze05, Zdy03), orthogonal K_{FLCe}=2.5, K_{FLCdy}=2 - Varied from Nominal 3 (Ze07,Zdy06) non-orthogonal K_{FLCev}=1.4, K_{FLCdyv}=1.1		

If e is N_e **and** dy is N_{dy} **Then** o is (NG_o=-1)
If e is N_e **and** dy is Z_{dy} **Then** o is (NS_o=-0.4)
If e is N_e **and** dy is P_{dy} **Then** o is (N_o=-0.1)

If e is Z_e **and** dy is N_{dy} **Then** o is (N_o=-0.1)
If e is Z_e **and** dy is Z_{dy} **Then** o is (Z_o=0) (2)
If e is Z_e **and** dy is P_{dy} **Then** o is (P_o=0.1)
If e is P_e **and** dy is N_{dy} **Then** o is (P_o=0.1)
If e is P_e **and** dy is Z_{dy} **Then** o is (PS_o=0.4)
If e is P_e **and** dy is P_{dy} **Then** o is (PG_o=1).

3.3. PID FLC Tuning

The parameters $\mathbf{q}_{FLC}$ of the PID FLC in the investigated systems are empirically tuned based on Eq. (1) after linearization of the 2I FU by the gains K_{FLCe} and K_{FLCdy} of the steepest lines of the sectors that enclose the o-e and o-dy projections of the control surface respectively, shown in Table 1 for Variant 1. The $\mathbf{q}_{FLC}$ tuning accounts for the greatest FU linearisation gain for the nominal and the corresponding varied MF for the two input variables, presented in Table 1, which is accepted as the worst case with respect to the impact on the linearised closed loop system stability. The linearised 2I FU output becomes $o=K_{FLCe}.K_e.e+K_{FLCdy}.K_d.(-dy)$. Considering also the pre- and post-processing linear dynamic elements the final control action u_{lin} of the linearised PID FLC becomes:

$$u_{lin}(t)=K_p^*.e(t)+K_i^*.\int e(t)dt+K_d^*.\dot{e}(t) \quad (3)$$

where the tuning parameters $\mathbf{q}_{linFLC}=[K_p^*, K_i^*, K_d^*]$ can be computed from eq. (1) to yield:

$$\mathbf{q}_{linFLC}=[K_p^*=(30\div90)[\%], K_i^*=(0.003\div0.01)\ [s^{-1}], K_d^*=(0.03\div0.1)[\%.s]].$$

The parameters of the original PID FLC $\mathbf{q}_{FLC}=[K_p, K_i, K_d]$ are determined from their relationship with the parameters of the linearised PID FLC:

$$K_p^*=(K_e.K_{FLCe}.K_p+K_i.K_d.K_{FLCdy}),\ K_i^*=K_{FLCe}.K_e.K_i,\ K_d^*= K_{FLCdy}.K_d.K_p. \quad (4)$$

Accepting the maximal FU gain for the nominal and varied MF of the corresponding variant in Table 1 $K_{FLCmax}=\max(K_{FLCe}, K_{FLCev}, K_{FLCdy}, K_{FLCdyv})$ eq. (4) becomes:

$$K_p^*=K_{FLCmax}.(K_e.K_p+K_i.K_d),\ K_i^*=K_{FLCmax}.K_e.K_i,\ K_d^*= K_{FLCmax}.K_d.K_p.$$

For a maximal possible $K_p=K_{pmax}=0.8$ (80[%]) that ensures closed loop system good robustness and dynamic accuracy the following final mean values of the tuning parameters are obtained:

$$\mathbf{q}_{FLC}=[K_p=80[\%], K_{imean}=0.012, K_{dmean}=0.014].$$

Further different FLC systems are simulated with the computed $\mathbf{q}_{FLC}$ and also with an optimised by GA $K_d=0.14$ that minimizes the sum of the mean square error and the control variance of the FLC closed loop system.

4. PLC IMPLEMENTATION OF TYPE-1 AND INTERVAL TYPE-2 SUGENO PID FLC

The design of both T1 and IT2 FLC is restricted by the requirement for economical representation of the MF with reduced memory demands, fast execution and easy programming in PLC for industrial implementation in real time control. Here for IT2 FLC the nominal orthogonal MF are designed to serve as a low boundary and the varied MF - an upper boundary for the MF of the FLC FU. The footprint of uncertainty (FOU) for IT2 MF is commonly defined as the area between the low and the upper MF [14].

4.1. Triangular and Trapezoidal Membership Functions Description

For triangular MF with respect to the input variable e FOU can be measured by the difference $\Delta e^{o}=e^{oU}-e^{oL}$ where e^{o} are the values for $\mu_e=0$ which are denoted as e^{oU} for the upper MF and as e^{oL} for the low MF. Most commonly Δe^{o} is accepted to be constant for all MF of a given variable. This measure is also accepted in regard to e^1 for $\mu_e=1$, $\Delta e^1=e^{1U}-e^{1L}$, and usually $\Delta e^1=0$ or $\Delta e^1=\Delta e^{o}$. With respect to the second input dy the FOU is defined in the same manner $\Delta(\mathrm{d}y^{o})=\mathrm{d}y^{oU}-\mathrm{d}y^{oL}$ and $\Delta(\mathrm{d}y^1)=\mathrm{d}y^{1U}-\mathrm{d}y^{1L}$. For different FOU measures four extra parameters Δe^{o}, Δe^1, $\Delta(\mathrm{d}y^{o})$, $\Delta(\mathrm{d}y^1)$ can be added to the PID FLC tuning parameters. The introduction of a proper FOU for the MF assumes preserving of the good system performance for a reduced number of MF used in the FLC or improvement of the system dynamic accuracy and robustness or reducing of the control span and oscillations.

The orthogonal MF can be mapped at the outputs of a specially designed Sugeno model on a single-input FU (SI FU) similarly to the Sugeno model of the TSK plant model in Fig. 1. [9]. E.g. for 3 MF for the input variable e the rule base consists of three rules with three outputs in the conclusion of each rule:

$$\begin{aligned}
&\mathbf{R^1: If}\ e \text{ is } \mathrm{N_e}\ \mathbf{Then}\ o_1^1 \text{ is } 1,\ o_2^1 \text{ is } 0,\ o_3^1 \text{ is } 0\\
&\mathbf{R^2: If}\ e \text{ is } \mathrm{Z_e}\ \mathbf{Then}\ o_1^2 \text{ is } 0,\ o_2^2 \text{ is } 1,\ o_3^2 \text{ is } 0\\
&\mathbf{R^3: If}\ e \text{ is } \mathrm{P_e}\ \mathbf{Then}\ o_1^3 \text{ is } 0,\ o_2^3 \text{ is } 0,\ o_3^3 \text{ is } 1,
\end{aligned} \tag{5}$$

where in rule $\mathbf{R}^j$ only the output for $i=j$ is equal to one, $o_i^j=1$, and the other two outputs are equal to zero, or $o_1^1=o_2^2=o_3^3=1$ and $o_1^2=o_1^3=o_2^1=o_2^3=o_3^1=o_3^2=0$. Each input MF is related to one fuzzy rule and one output in the fuzzy rule conclusion. The degree of belonging of the current measured input to the MF, computed in the rule condition, scales the corresponding outputs in the rule conclusion which are singletons at value 1 or 0. E.g. if the input e_k, measured at time t_k, belongs to the MF labelled $\mathrm{N_e}$ with a degree of $\mu_e(\mathrm{Ne})$, to the MF labelled $\mathrm{Z_e}$ with a degree of $\mu_e(\mathrm{Ze})$ and to the MF labelled $\mathrm{P_e}$ with a degree of $\mu_e(\mathrm{Pe})$, ($\mu_e(\mathrm{Ne})+\mu_e(\mathrm{Ze})+\mu_e(\mathrm{Pe})=1$ for orthogonal MF), then after accumulation of the three qualified outputs the Sugeno model outputs yield:

$$\begin{aligned}
&o_1=\boldsymbol{o_1^1}.\mu_e(\mathrm{Ne})+o_1^2.\mu_e(\mathrm{Ze})+o_1^3.\mu_e(\mathrm{Pe})=\mu_e(\mathrm{Ne})\\
&o_2=o_2^1.\mu_e(\mathrm{Ne})+\boldsymbol{o_2^2}.\mu_e(\mathrm{Ze})+o_2^3.\mu_e(\mathrm{Pe})=\mu_e(\mathrm{Ze})\\
&o_3=o_3^1.\mu_e(\mathrm{Ne})+o_3^2.\mu_e(\mathrm{Ze})+\boldsymbol{o_3^3}.\mu_e(\mathrm{Pe})=\mu_e(\mathrm{Pe}).
\end{aligned}$$

Thus, each output maps the respective input MF. For FLC with 2I FU with inputs the normalised error e and d$y=\dot{y}$ (d$y=-\dot{e}$) four SI FU are used to map four orthogonal nominal and varied MF with respect to e and dy. However, the varied MF most commonly are non-orthogonal as seen in Table 1.

Both orthogonal and non-orthogonal MF can be easily analytically described in a convenient form for programming and computation in industrial PLC. In Fig. 4 three MF of e and three MF of dy are presented, where in red are the nominal (low) MF. The varied MF are obtained for $\Delta e^1=\Delta(\mathrm{d}y^1)=0$ and $\Delta e^{o}=0.2$ and $\Delta e^{o}=0.3$ for e and $\Delta(\mathrm{d}y^{o})=0.3$ for dy.

The MF for each input variable can be described by two lines. E.g. for the leftmost trapezoidal MF for e, labeled $\mathrm{N_e}$, the lines are l=1 and l_1 which is based on (e^{o}_1, e^1_1), and for the rightmost trapezoidal MF for e, labeled P_e, the lines are l_4, based on ($e^o{}_4$, $e^1{}_4$), and l=1. For the triangular MF for e, labeled Z_e, the lines are l_2, based on ($e^o{}_2$, $e^1{}_2$), and l_3, based on ($e^o{}_3$, $e^1{}_3$). In a similar way the MF with respect to dy can be described as well as other triangular and trapezoidal MF in the middle of the universe of discourse.

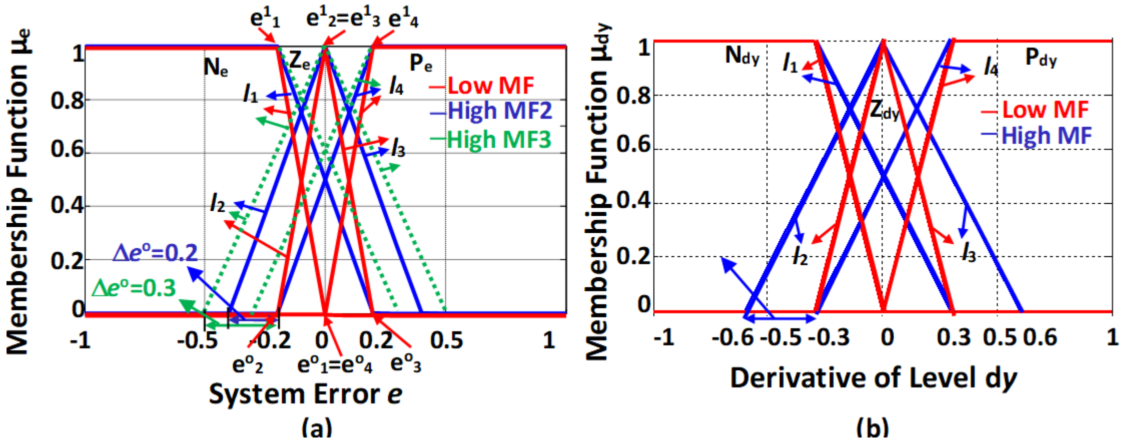


Fig. 4. Nominal and varied MF described by lines with respect to: a) the error e; b) the derivative dy.

The computation of the analytically described MF for different ranges of the FLC input variables e and dy is illustrated in Table 2.

Table 2. Computation of membership functions from Fig. 4.

Membership functions with respect to e											
Nominal				Δe^o=0.2, Δe^1=0				Δe^o=0.3, Δe^1=0			
Ranges for e	μ_e(N)	μ_e(Z)	μ_e(P)	Ranges for e	μ_e(N)	μ_e(Z)	μ_e(P)	Ranges for e	μ_e(N)	μ_e(Z)	μ_e(P)
[-1,-0.2)	1	0	0	[-1, -0.4)	1	0	0	[-1, -0.5)	1	0	0
[-0.2,0)	l_1	l_2	0	[-0.4,-0.2)	1	l_2	0	[-0.5,-0.3)	1	l_2	0
[0, 0.2)	0	l_3	l_4	[-0.2, 0)	l_1	l_2	l_4	[-0.3,-0.2)	1	l_2	l_4
[0.2, 1]	0	0	1	[0, 0.2)	l_1	l_3	l_4	[-0.2, 0)	l_1	l_2	l_4
				[0.2, 0.4)	0	l_3	1	[0, 0.2)	l_1	l_3	l_4
				[0.4, 1]	0	0	1	[0.2, 0.3)	l_1	l_3	1
								[0.3, 0.5)	0	l_3	1
								[0.5, 1]	0	0	1

Membership functions with respect to dy							
Nominal				Δ(dy^o)=0.3, Δ(dy^1)=0			
Ranges for dy	μ_{dy}(N)	μ_{dy}(Z)	μ_{dy}(P)	Ranges for dy	μ_{dy}(N)	μ_{dy}(Z)	μ_{dy}(P)
[-1, -0.3)	1	0	0	[-1, -0.6)	1	0	0
[-0.3, 0)	l_1	l_2	0	[-0.6,-0.3)	1	l_2	0
[0, 0.3)	0	l_3	l_4	[-0.3, 0)	l_1	l_2	l_4
[0.3, 1]	0	0	1	[0, 0.3)	l_1	l_3	l_4
				[0.3, 0.6)	0	l_3	1
				[0.6, 1]	0	0	1

The equations of the lines l_i are based on the parameters of the nominal MF e^o, e^1, dy^o, dy^1 and the FOU measures Δe^o, Δe^1, Δ(dy^o), Δ(dy^1):

$$l_i = a_i.e + b_i,\ l_i = c_i.\mathrm{d}y + d_i, \quad (6)$$

where

$$a_i = \{[e_i^1 + \text{sign}(e_i^0 - e_i^1)\Delta e^1] - [e_i^o + \text{sign}(e_i^o - e_i^1).\Delta e^o]\}^{-1}$$
$$b_i = -a_i[e_i^o + \text{sign}(e_i^o - e_i^1).\Delta e^o]$$
$$c_i = \{[\mathrm{d}y_i^1 + \text{sign}(\mathrm{d}y_i^o - \mathrm{d}y_i^1).\Delta(\mathrm{d}y^1)] - [\mathrm{d}y_i^o + \text{sign}(\mathrm{d}y_i^o - \mathrm{d}y_i^1).\Delta(\mathrm{d}y^o)]\}^{-1}$$
$$d_i = -c_i[\mathrm{d}y_i^o + \text{sign}(\mathrm{d}y_i^o - \mathrm{d}y_i^1).\Delta(\mathrm{d}y^o)]$$

The lines of the nominal MF are computed from eq. (6) for $\Delta e^o=\Delta e^1=\Delta(\mathrm{d}y^o)=\Delta(\mathrm{d}y^1)=0$. The parameters e^o_i, e^1_i and $\mathrm{d}y^o_i$, $\mathrm{d}y^1_i$ of the i-th nominal MF from Fig.4 and the computed gains a_i, b_i, c_i and d_i of the corresponding lines for $\Delta(\mathrm{d}y^o)=0.3$ and $\Delta e^o=0.2$ and $\Delta e^o=0.3$ are presented in Table 3. The computation of the MF for e and the parameters of their describing lines for variant 5 in Table 1 where $\Delta e^1=0.2\neq0$ and the triangular nominal MF labelled Z_e is changed to trapezoidal varied MF, are shown in Table 4.

Table 3. Parameters of membership functions from Fig. 4 and their shaping lines.

MF	i	e_i^o	e_i^1	Nominal		$\Delta e^o=0.2$		$\Delta e^o=0.3$	
				a_i	b_i	a_i	b_i	a_i	b_i
μ_{ei}	1	0	-0.2	-5	0	-2.5	0.5	-2	0.6
	2a	-0.2	0	5	1	2.5	1	2	1
	2b	0.2	0	-5	1	-2.5	1	-2	1
	3	0	0.2	5	0	2.5	0.5	2	0.6
	i	$\mathrm{d}y_i^o$	$\mathrm{d}y_i^1$	Nominal		$\Delta(\mathrm{d}y^o)=0.3$			
				c_i	d_i	c_i	d_i		
μ_{dyi}	1	0	-0.3	-3.33	0	-1.67	0.5		
	2a	-0.3	0	3.33	1	1.67	1		
	2b	0.3	0	-3.33	1	-1.67	1		
	3	0	0.3	3.33	0	1.67	0.5		

Table 4. Computation of membership functions and parameters of their shaping lines for e from variant 5 in Table 1.

Nominal				$\Delta e^o=0.2$, $\Delta e^1=0.2$			
Ranges for e	$\mu_e(N)$	$\mu_e(Z)$	$\mu_e(P)$	Ranges for e	$\mu_e(N)$	$\mu_e(Z)$	$\mu_e(P)$
[-1, -0.5)	1	0	0	[-1, -0.7)	1	0	0
[-0.5,0)	l_1	l_2	0	[-0.7,-0.3)	1	l_2	0
[0, 0.5)	0	l_3	l_4	[-0.3,-0.2)	l_1	l_2	0
[0.5, 1]	0	0	1	[-0.2, 0.2)	l_1	1	l_4
				[0.2, 0.3)	0	l_3	l_4
				[0.3, 0.7)	0	l_3	1
				[0.7, 1]	0	0	1
e_i^o	e_i^1	a_i	b_i	a_i	b_i		
0	-0.5	-2	0	-2	0.4		
-0.5	0	2	1	2	1.4		
0.5	0	-2	1	-2	1.4		
0	0.5	2	0	2	0.4		

4.2. Fuzzy Rules Description and Solution

The fuzzy rules of T1 FLC, described for 3x3 FU in eq.(2), are solved for each measured level H_k at time t_k by fuzzyfication for obtaining the degrees of matching of e_k to all MF for e and the degrees of matching of dy_k to all MF for dy.

This is achieved using Table 2 and Table 3 for the computation of the MF with respect to e $\mu_{ek}=[\mu_{ek}(N_e)\ \mu_{ek}(Z_e)\ \mu_{ek}(P_e)]$ and with respect to $\mu_{dyk}=[\mu_{dyk}(N_{dy})\ \mu_{dyk}(Z_{dy})\ \mu_{dyk}(P_{dy})]$ for the corresponding ranges where e_k and dy_k fall. Next, the two connected by "**and**" conditions in each rule j in eq.(2) are aggregated by applying some of the t-norm operators**,** usually **MIN** or **PROD**. The result is the degree of activation (firing) of the rule w_{jk}=**MIN**(μ_{ejk}, μ_{dyjk}). The singleton of the rule conclusion is qualified yielding $o_{jk}=w_{jk}.S_{ojk}$. The accumulation and defuzzyfication using weighted average are united and easy to compute by a PLC. The final FU crisp output becomes $o_k^o=So_{kj}/Sw_{kj}$.

For IT2 FLC each normalised by K_e error e_k for a given measured level H_k at time t_k is transformed into the degrees of belonging to the three defined nominal MF $\mu_{ek}^n=[\mu_{ek}(N^n_e),\mu_{ek}(Z^n_e),\mu_{ek}(P^n_e)]$ and to the three defined varied MF $\mu_{ek}^v=[\mu_{ek}(N^v_e),\mu_{ek}(Z^v_e),\mu_{ek}(P^v_e)]$. The couples (μ_{ek}^n, μ_{ek}^v) for each of the MF negative N_e, zero Z_e and positive P_e are compared to define the low $L\mu_{ek}$ and upper $U\mu_{ek}$ boundary of the interval values for the MF $\mu_{ek}=[(L\mu_{ek}(N_e), U\mu_{ek}(N_e)); (L\mu_{ek}(Z_e), U\mu_{ek}(Z_e)); (L\mu_{ek}(P_e), U\mu_{ek}(P_e))]$. In the same manner the computed derivative dy_k of H_k is processed to obtain the interval values for the MF $\mu_{dyk}=[(L\mu_{dyk}(N_{dy}), U\mu_{dyk}(N_{dy})); (L\mu_{dyk}(Z_{dy}), U\mu_{dyk}(Z_{dy})); (L\mu_{dyk}(P_{dy}), U\mu_{dyk}(P_{dy}))]$.

Each condition in the rule is satisfied to a certain degree μ which is an interval number. The aggregation of the two conditions connected by "**and**" in the rule is based on the **MIN** operator applied to the interval numbers $(L\mu_e, U\mu_e)$ and $(L\mu_{dy}, U\mu_{dy})$ and results in an interval value for the degree of activation of the rule (Lw, Uw) where $Lw=MIN(L\mu_e, L\mu_{dy})$, $Uw=MIN(U\mu_e, U\mu_{dy})$. Then the rule conclusion is qualified by (Lw, Uw) to yield (Lo, Uo) with $Lo=o.Lw$ and $Uo=o.Uw$. All qualified conclusions are united by summing of the interval numbers for the nine rules, j=1÷9, according to the interval numbers arithmetics:

$$\Sigma(Lo_j, Uo_j)=[\Sigma(Lo_j), \Sigma(Uo_j)].$$

The rule activation degrees are also summed as interval numbers:

$$\Sigma(Lw_j, Uw_j)=[\Sigma(Lw_j), \Sigma(Uw_j)].$$

Here it is suggested the weighted average to be applied by dividing the two interval numbers that correspond to the two sums above:

$$(Lo^o, Uo^o)=\Sigma(Lo_j, Uo_j)/ \Sigma(Lw_j, Uw_j),$$

which according to the rules for operations with interval numbers requires first the computation of $1/[\Sigma(Lw_j), \Sigma(Uw_j)]=[1/\Sigma(Uw_j), 1/\Sigma(Lw_j)]$. Then the resulting interval number is multiplied with the interval number $[\Sigma(Lo_j), \Sigma(Uo_j)]$ to yield the final defuzzyfied interval conclusion:

$$Lo^o=\min\{[\Sigma(Lo_j)/\Sigma(Uw_j)], [\Sigma(Lo_j)/\Sigma(Lw_j)], [\Sigma(Uo_j)/\Sigma(Uw_j)], [\Sigma(Uo_j)/\Sigma(Lw_j)]\}$$
$$Uo^o=\max\{[\Sigma(Lo_j)/\Sigma(Uw_j)], [\Sigma(Lo_j)/\Sigma(Lw_j)], [\Sigma(Uo_j)/\Sigma(Uw_j)], [\Sigma(Uo_j)/\Sigma(Lw_j)]\}.$$

Finally, the weighted average interval number (Lo^o, Uo^o) is reduced to a real crisp number $o^o=(Lo^o+Uo^o)/2$ which is the defuzzified FLC output.

Nie and Tan suggest in [23] another simple direct approach for type-reduction and defuzzyfication which is suitable for PLC computation. The crisp FLC output is obtained in the following way:

$$o^{o} = [\sum_{j=1}^{N} o_j^{o}(\mathrm{Lw_j} + \mathrm{Uw_j})]/[\sum_{j=1}^{N}(\mathrm{Lw_j} + \mathrm{Uw_j})].$$

The Nie and Tan approach is further used for comparison.

5. SIMULATION OF DESIGNED SYSTEMS, PERFORMANCE ASSESSMENT AND DISCUSSION OF RESULTS

The investigation via simulation of the designed linear PI, type-1 FLC (FLC-1) and IT2 FLC (FLC-2) closed loop systems for the control of the level in an operating CCl for the production of soda ash aims to study the impact of the following factors on the closed loop systems performance:

- The type of control algorithm applied - linear PI, FLC-1 and FLC-2;
- The number (5 or 3) and the parameters of the MF for the error as a FU main input variable;
- The tuning parameters of the controllers;
- The type and magnitude of the MF FOU;
- The method for type-reduction used.

Besides, all systems designed for nominal plant are investigated for the control of both the nominal and the varied plant in order to study the impact of the variation of the plant characteristics as a result of disturbances and changes in the CCl load and the operation point.

The investigation is based on the simulation of the systems step responses with respect to the level H and the control U for successive reference H_r changes (50-60-50-40-50)[%] that determine the most often used operation points and enable the study of the impact of the system nonlinearity.

The comparison of the systems is based on the following performance indicators assessed as the worst from all step response from one simulation experiment:

- Dynamic accuracy, measured by overshoot and settling times.
- Control action smoothness, expressed by the control span and oscillations magnitudes.
- System robustness, measured by the deflection of the performance indicators for dynamic accuracy and control action smoothness of the system with varied plant from the indicators of the system with nominal plant.

The study aims at selection for tuning of the factors with great impact on the improvement of the performance of the systems for the control in industrial environment and real time via PLC of a nonlinear plant with no plant model.

The simulation of the designed systems is based on the block diagram of the closed loop system in Fig. 3. The PD Sugeno FLC with 2I FU and nominal MF is used to study T1 PID FLC systems. For a linear PI control the PID FLC is substituted by a PI controller. For IT2 PID FLC system the Fuzzy Unit in Fig. 3 is substituted by two SIFU and three computation blocks as shown in Fig. 5.

The orthogonal low boundary (nominal) MF with regards to e and dy are computed by SIFUe and SIFUdy respectively. The non-orthogonal upper boundary (varied) MF with respect to e and dy are computed by the corresponding computation blocks $\mathrm{MFe^v}$ and $\mathrm{MFdy^v}$. The aggregation of conditions, the individual rules output qualifications followed by their accumulation, the type-reduction and the defuzzyfication are computed in the Rules Computation block to yield the final crisp output o^o.

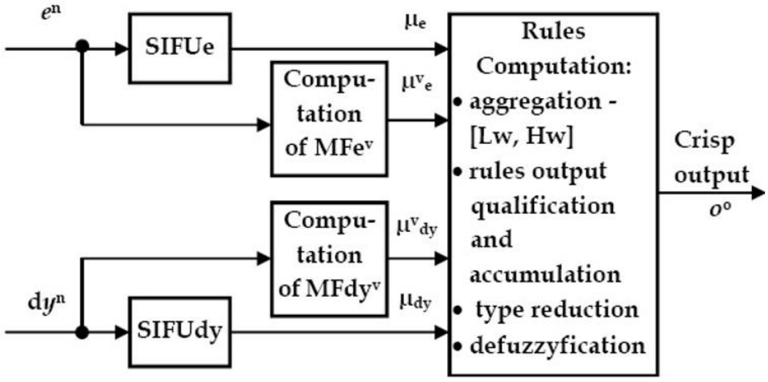


Fig. 5. FU of IT2 PID FLC.

The plan of the simulation experiments is the following.

a) Simulation of the step responses (Fig.6) of the following systems for K_d=0.14:

- System 1 with PI linear controller with step responses for the nominal plant (H_{PI}, U_{PI}) and for the varied plant ($H^v{}_{PI}$, $U^v{}_{PI}$);
- System 2 with PID FLC-1 with the Nominal 1 FU MF of variant 1 in Table 1, i.e. T1 2I FU of 5x3 orthogonal MF (5 MF for e and 3 MF for dy) with step responses for the nominal plant (H_{5x3}, U_{5x3}) and for the varied plant ($H^v{}_{5x3}$, $U^v{}_{5x3}$);
- System 3 with PID FLC-1 with the Nominal 2 FU MF of variant 3 in Table 1 and shown also in Fig.4, i.e. T1 2I FU of 3x3 orthogonal MF (Ze02, Zdy03) with step responses for the nominal plant ($H_{3x3Ze02}$, $U_{3x3Ze02}$) and for the varied plant ($H^v{}_{3x3Ze02}$, $U^v{}_{3x3Ze02}$);
- System 4 with PID FLC-1 with the Nominal 3 FU MF of variant 5 in Table 1, i.e. T1 2I FU of 3x3 orthogonal MF (Ze05, Zdy03) with step responses for the nominal plant ($H_{3x3Ze05}$, $U_{3x3Ze05}$) and for the varied plant ($H^v{}_{3x3Ze05}$, $U^v{}_{3x3Ze05}$).

The analysis of the step responses shows that the FLC systems outperform the linear PI System 1 in all performance indicators, i.e. the FLC algorithm improves the system performance. System 2 and System 4 have identical performance indicators. System 3 has a similar performance but with a greater control span and oscillations magnitudes which cause intensive wearing of the actuator. System 2 with 5 MF with respect to e and Ze02 and System 4 with 3 MF and Ze05 have the best performance. The greater number of MF for e reflects better the plant nonlinearity but leads to more fuzzy rules. System 4 achieves the same good performance with 3 MF and increased support Ze05 for the middle term Z_e. The less number of MF and fuzzy rules makes more economical the presentation and the computation of the FU.

b) Simulation of the step responses (shown only for nominal plant in Fig. 7) of the 2I FU based PID FLC-1 systems from simulation experiment b for K_d=0.014.

The increased K_d from 0.014 to 0.14 reduces significantly the settling time, the overshoot and the control span of all systems and improves the system robustness. Thus, System 4 with K_d=0.14 and FU with nominal MF is considered the best for comparison in the further investigation. Besides, its PID FLC T1 2I FU is simpler in structure with less number of MF and fuzzy rules which facilitate the PLC implementation for real time control of an industrial plant. The 3x3 MF with respect to *e* with Ze05 and d*y* are further accepted as nominal (low boundaries) for the MF of IT2 FU.

c) Simulation of the step responses of the following couples of PID FLC systems with 2I FU with various nominal and varied 3x3 MF, shown in Fig. 8 in solid lines for nominal plant and in dotted lines for varied plant:

- System 4 with the Nominal 3 MF (Ze05; Zdy03) and System 5 with MF (trapezoidal Ze07;Zdy06) varied from the MF of System 4 by ($\Delta e^{o}=\Delta e^{1}=0.2$;$\Delta(dy^{o})=0.3$)-variant 5 in Table 1;
- System 3 with the Nominal 2 MF (Ze02; Zdy03) and two systems with different varied MF, i.e. different types of FOU:
  - System 6 with MF (Ze04; Zdy06), varied from the MF of System 3 by ($\Delta e^{o}=0.2$; $\Delta(dy^{o})=0.3$) - variant 4 in Table 1;
  - System 7 with MF (Ze06; Zdy06), varied from the MF of System 3 by ($\Delta e^{o}=0.4$; $\Delta(dy^{o})=\Delta(dy^{1})=0.3$) - variant 3 in Table 1.

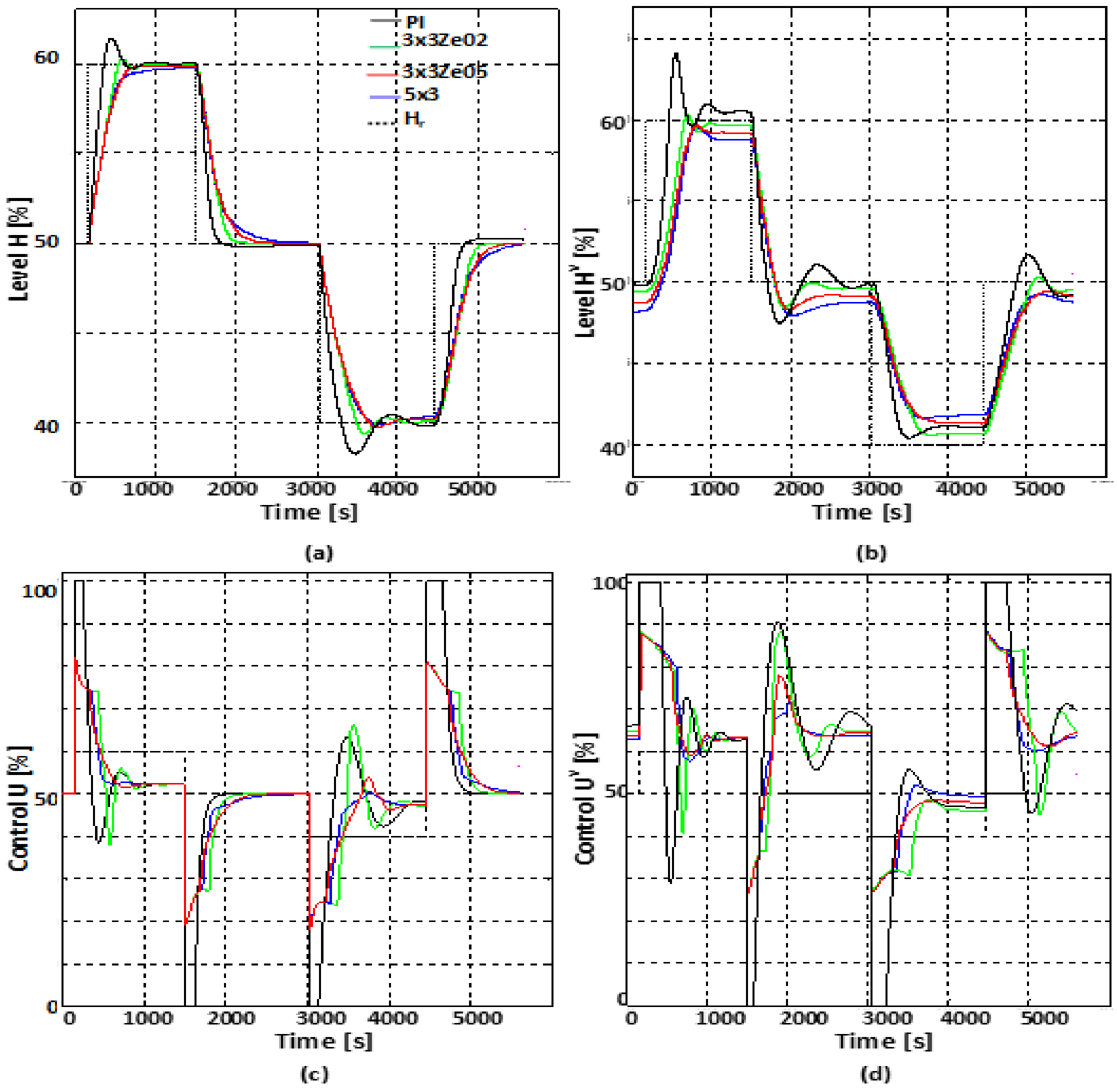


Fig. 6. Step responses of systems with linear PI controller (System 1) and T1 PID FLC with 2I FU 5x3 MF (System 2), 3x3 MF Ze02 (System 3) and 3x3 MF Ze05 (System 4) for control of: a) nominal plant with respect to level *H*; b) varied plant with respect to level H^{V}; c) nominal plant with respect to control *U*; d) varied plant with respect to control U^{V}.

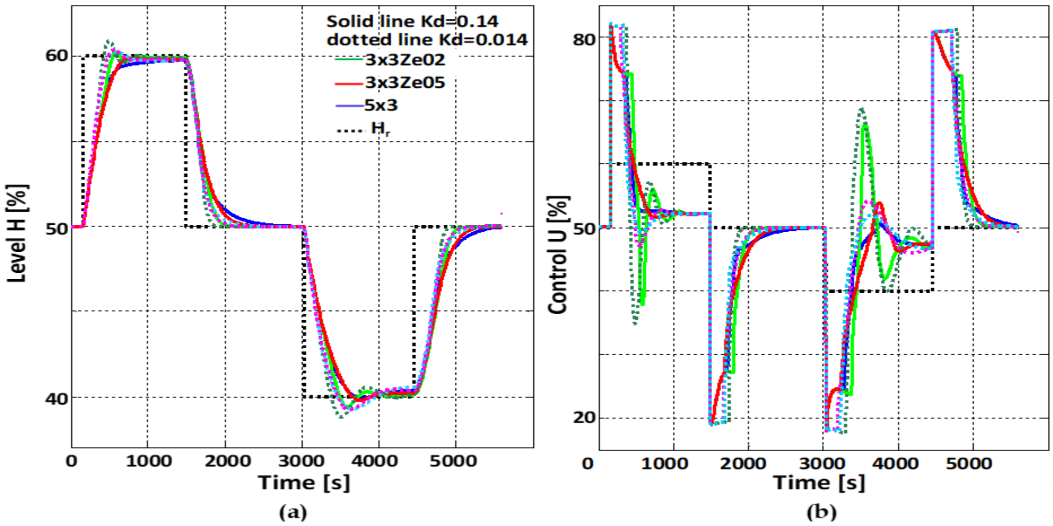


Fig. 7. Step responses of systems for control of nominal plant with T1 PID FLC on 2I FU with 5x3 MF (System 2), 3x3 MF Ze02 (System 3) and 3x3 MF Ze05 (System 4) for K_d=0.14 and K_d=0.014 with respect to: a) level H; b) control U.

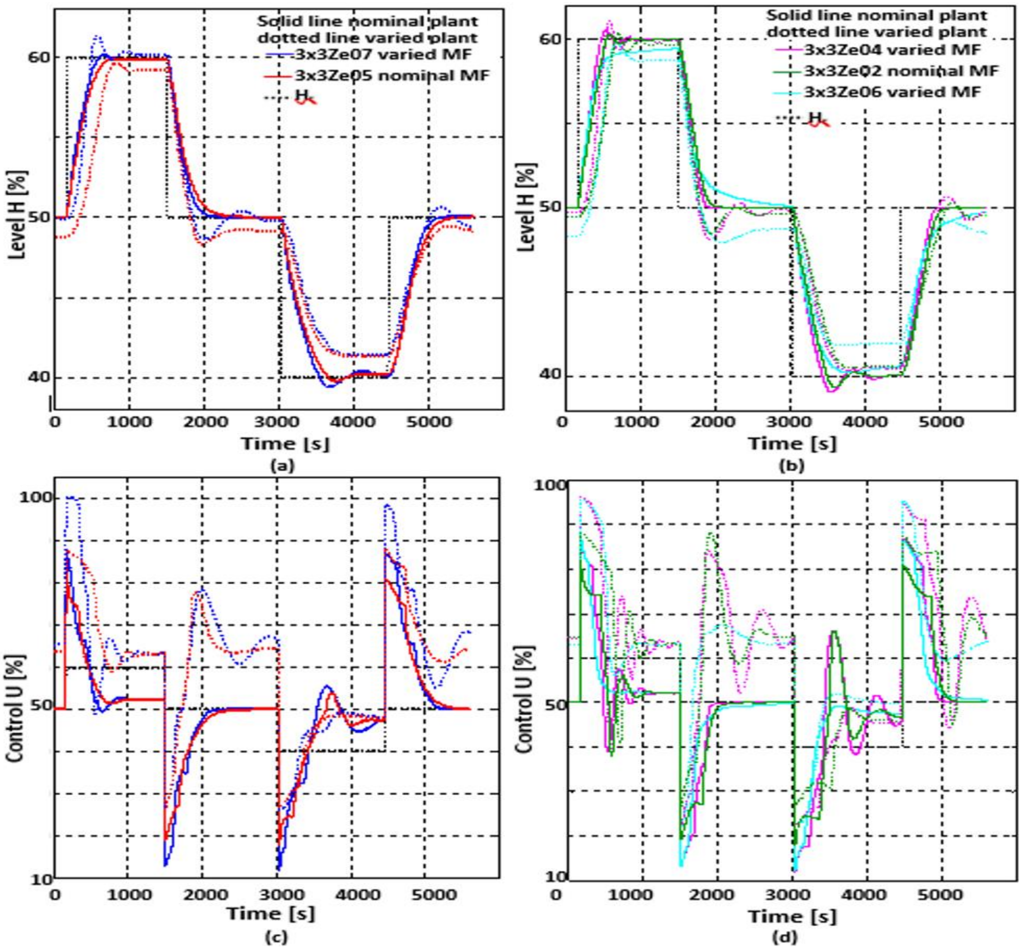


Fig. 8. Step responses of systems for control of nominal and varied plants using T1 PID FLC on 2I FU with 3x3 MF with respect to level H for: a) nominal Ze05 and varied Ze07 MF; b) nominal Ze02 and varied Ze04 and Ze06 MF and with respect to control U for: c) MF (from a); d) MF (from b).

System 3 and System 4 with different nominal MF have a better performance in controlling of the nominal plant - System 4 has no overshoot and the smallest control span. The deviations from the nominal system performance due to changes in the plant are smaller for System 3, i.e. System 3 is more robust.

The variations of the MF lead to small changes in the performance of all systems controlling the nominal plant and more significant changes in controlling the varied plant. System 6 is the most robust to MF changes. The variation of the plant has the greatest impact on the control action of System 5 with varied MF. It also leads to steady state errors which are significant for System 4 and System 7. The greater MF FOU results in a greater deflection in the system performance from the performance of the system with nominal MF. The deviation in the performance of System 7 is greater than of System 6.

d) Simulation of the step responses (Fig. 9) of:

- T1 PID FLC System 4 on 2I FU with 3x3 nominal Ze05 and System 5 with varied Ze07 MF;
- System 8 with IT2 PID FLC based on the same MF, the orthogonal ones computed by SIFU and the non-orthogonal - by analytical description, and the Nie-Tan type-reduction [23]
- System 9 – like System 8 but with type-reduction on interval numbers mathematics.

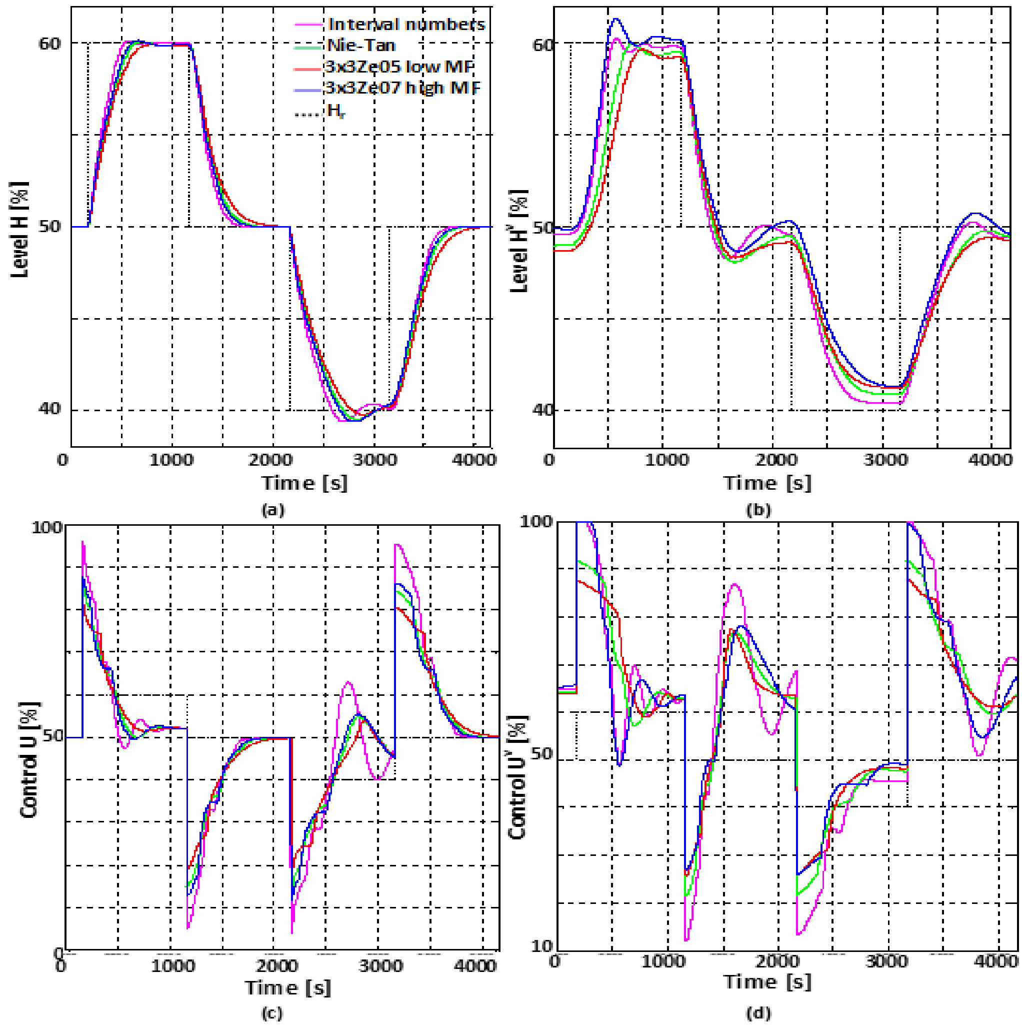


Fig. 9. Step responses of systems using T1 and IT2 PID FLC with Nie-Tan and interval numbers mathematics type-reduction for control of: a) nominal plant with respect to level H; b) varied plant with respect to level H^v; c) nominal plant with respect to control U; d) varied plant with respect to control U^v.

The step responses of Systems 4 and System 5 from the previous investigation with T1 PID FLC are used for comparison and assessment of the system performance improvement due to IT2 PID FLC. System 9 outperforms System 4 and System 5 as seen from Table 5. System 8 based on Nie-Tan type-reduction has almost identical performance with System 5 (shown in blue bold font) in controlling of the nominal plant. It has smaller control action span and oscillations magnitude than System 9. System 9 based on interval numbers mathematics type-reduction has the shortest settling time and the smallest steady state error but at the expense of an increased control span and oscillations magnitude. FLC-1 and FLC-2 have identical overshoot. The best performance indicators for all reference step responses when comparing of the two systems are grey highlighted in Table 5.

Table 5. Performance indicators of FLC-2 systems.

System	Performance indicators	H_r=(50-60)[%]		H_r=(60-50)[%]		H_r=(50-40)[%]		H_r=(40-50)[%]	
		Nom. plant	Varied plant	Nom. plant	Varied plant	Nom. plant	Varied plant	Nom. plant	Varied plant
8	σ [%]	2	4	0	15	13	0	0	3
	t_s [s]	700	800	700	1100	1100	700	800	1000
	E (∞) [%]	0	2	0	1.5	0	2	0	2
	U span [%]	40	35	35	55	40	25	35	32
	U oscillations magnitude [%]	2	7	0	10	5	0	0	3
9	σ [%]	2	3	0	13	15	0	0	2
	t_s [s]	500	600	500	1000	1000	600	600	900
	E (∞) [%]	0	1.5	0	1	0	1	0	1
	U span [%]	50	50	43	75	55	33	45	50
	U oscillations magnitude [%]	3	13	0	15	13	0	0	10
4	σ [%]	0	3	0	12	4	0	0	0
	t_s [s]	500	800	800	1000	1000	600	800	900
	E (∞) [%]	0	1.5	0	2	0	2	0	1
	U span [%]	30	30	30	50	35	20	30	28
	U oscillations magnitude [%]	0	6	0	13	6	0	0	3
5	σ [%]	2	8	0	10	8	0	0	8
	t_s [s]	600	1000	500	1100	1100	900	600	1200
	E (∞) [%]	0	1.5	0	1.5	0	2	0	1
	U span [%]	35	50	35	50	45	22	35	45
	U oscillations magnitude [%]	4	13	0	15	8	0	0	10

The worst (maximal) values of the performance indicators from all step responses for the nine investigated systems are systemized in Table 6 to ease comparison. The best values of the corresponding indicator for a nominal and a varied plant are circulated to point to the best system. System 4 has the smallest maximal overshoot, the shortest maximal settling time and the best robustness assessed by the minimal deviations of the maximal overshoot and settling time for all step responses of the system with varied plant from the same performance indicators of the system with nominal plant $|\Delta\sigma_{max}|$=min and $|\Delta t_{smax}|$=min. System 2 is the second best with respect to the same indicators. It has the smallest control action span, oscillations magnitude and the best robustness with respect to these indicators $|\Delta U_{max}\text{span}|$=min and $|\Delta U_{max}\text{Os.mag}|$=min. The grey highlighted areas show that System 2 and System 4 have nearly identical performance indicators. The best values in each column are in red. System 3 is the most sensitive to the decrease of K_d.

Table 6. Performance indicators of the investigated systems.

Experiment FLC	K_d	No	System	Overshoot σ_{max} $(H_{max}-H_r)/\Delta H_r$ [%] Nom. plant	Overshoot Varied plant	Settling time t_{smax} [s] Nom. plant	Settling time Varied plant	Steady state error E_{max} (∞)[%] Nom. plant	Steady state error Varied plant	U_{max} span [%] Nom. plant	U_{max} span Varied plant	Oscillations magnitude $U_{maxOs.mag}$ [%] Nom. plant	Oscillations magnitude Varied plant
PI			1	18	35	1800	2500	0	1.5	60	90	12	35
FLC-1 Nominal MF	0.14	1	2	2	10	1100	1000	0	2	35	40	3	5
			3	15	20	1100	1300	0	1	40	35	3	7
			4	5	8	900	850	0	1.5	35	50	7	12
	0.014	2	2	3	-	1100	-	0	-	35	-	8	-
			3	22	-	1400	-	0	-	45	-	22	-
			4	15	-	1100	-	0	-	35	-	10	-
FLC-1 Varied MF	0.14	3	5	12	20	900	1200	0	1.7	40	50	7	10
			6	15	20	1000	1300	0	0.8	50	56	20	23
			7	0	15	1500	1000	0	2.2	40	45	2	8
FLC-2	0.14	4	8	12	25	950	900	0	2	40	54	7	10
			9	12	22	900	1000	0	1	60	72	15	20
Best system robustness				$\lvert\Delta\sigma_{max}\rvert_{min}=3$		$\lvert\Delta t_{smax}\rvert_{min}=50$		-		$\Delta U_{max}Span\rvert_{min}=5$		$\lvert\Delta U_{max}Os.mag\rvert_{min}=2$	

The impact of the MF variations is assessed by the deviation of the performance indicators of the system with varied MF from the indicators of the system with nominal MF. E.g. System 5 uses varied MF with respect to the MF of System 4 and considering the indicator σ of the system with nominal plant the impact of MF variation is assessed as $|\Delta\sigma_{max45}|=|\sigma_{max4}-\sigma_{max5}|=7$ and for the system with varied plant $|\sigma_{max}{}^{v}{}_{4}-\sigma_{max}{}^{v}{}_{5}|=12$. System 6 and System 7 have MF with different variations with respect to the MF of System 3 and hence $|\Delta\sigma_{max36}|=|\sigma_{max3}-\sigma_{max6}|$ and $|\Delta\sigma_{max37}|=|\sigma_{max3}-\sigma_{max7}|$ are assessed for the system with nominal and varied plant respectively. The deviations of all performance indicators as a result of MF variations classify System 6 as the most robust with respect to changes in overshoot and settling time, and System 5 as the most robust to changes in control action span and oscillations magnitude. Most often the system with varied plant is more sensitive to MF variations.

6. CONCLUSIONS AND FUTURE RESEARCH

A methodology is developed for the design of T1 and IT2 PID FLC which is oriented to PLC industrial implementation. It is illustrated for the solution level control in a carbonization column for the production of soda ash. The plant is nonlinear and changes its characteristics for different loads and operation points. The derived and validated in previous research TSK models for the boundary loads from experimental data enable the study via simulation and assessment of different designed FLC systems.

A novel approach for the design of IT2 FLC is developed on the basis of interval numbers mathematics and various IT2 MF and FOU representations. Novelty is the empirical approach for tuning the FLC pre- and post-processing parameters based on linearization of the FLC control surface. An analytical method is suggested to describe the membership functions, the FOU and the fuzzy rules with the aim to facilitate the FLC economical presentation in industrial PLC and the fast computation in real time control.

Various designed FLC systems are investigated via simulation to study the impact of different factors on the system performance. The analysis showed that the tuned FLC outperforms the tuned linear PI. Good system performance can be achieved by a small number of MF with large support; also ensuring economical PLC presentation. The FLC tuning parameters have a significant influence on the system performance. The use of a type-reduction based on interval number mathematics leads to a better IT2 FLC system performance than the Nie-Tan direct approach. The performance improvement of IT2 FLC systems often is not so great as to justify the increased complexity of IT2 FLC design and computation.

The future research will be focused on the off-line simulation-based GA optimization of: i) the FOU type and size of the IT2 FLC system and ii) the nominal MF of the T1 2I FU PID FLC system. The aim is to see which system has a better performance - the T1 FLC system with optimized MF and simple algorithm and design or the more complex IT2 FLC system with optimized FOU.

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