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Performance Evaluation of Multi-Hop Wireless Network with Point-to-Point Traffic Model and Fuzzy System

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Abstract— In wired communication, the point-to-point traffic model is used to determine the blocking probability for four different types of network topology: i) links are in series, ii) links are in parallel, iii) links are in a combination of series and parallel and iv) complex topology where links are neither series nor parallel like the delta-star network. In this paper, the concept of point-to-point traffic model is first applied in the multi-hop wireless link under small scale fading environment to determine the outage probability of four different configurations of the source to the destination link. The variation of the outage probability against signal to noise ratio (SNR) is shown analytically and verified with simulation for four topologies under both Rayleigh and Nakagami-*m* fading cases. The same concept is further applied in the fuzzy system to determine the probability of success of all the four topologies of multi-hop wireless networks. The analytical, simulation, and fuzzy outputs of this investigation are compared with those of previous reported works and close results are achieved. The results also reveal that the performance of the network under the proposed model depends on its topology unlike the series model of the conventional dual-hop wireless link of previous works.

Keywords— Signal to noise ratio; Cumulative distribution function; Outage probability; Fuzzy system; Point-to-point traffic; Fading channel; Muti-hub wirelesss network.

1. INTRODUCTION

In wireless networks when the separation between source and destination is large and fails to retain threshold signal to noise ratio (SNR) at the receiving end, the link is assisted by one or more relays. In dual-hop cases, one relay is used with some amplification. The concept of the dual-hop wireless link is found in [1], where decode-and-forward (DF) relaying is used in the cognitive radio network. Here a secondary user (SU) communicates with a secondary destination with the assistance of a secondary relay. The profile of the outage probability of primary user and SU are shown against different link parameters under SNR and signal to noise plus interference ratio. In [2], a relay selection scheme is proposed when several relays in a multi-hop network are present. Here, the authors mainly emphasize on symbol error rate for the relay selection. Another analysis is found in [3] where a full-duplex two-hop relay network is considered. The network consists of a base station (BS), amplify-and-forward (AF) relay, and user equipment; provided BS is equipped with massive multi-input multi-output (MIMO) i.e. the BS transmits and receives signal by multiple antenna elements. The main objective of the paper is to select an appropriate antenna so that outage probability or bit error rate is minimum. The secrecy performance of multi-hop wireless link is found in [4] where eavesdroppers are randomly distributed within the network. Here the feedback from the receiver is used to estimate the SNR of the communication system and hence, the sender decides to participate in communication if the received SNR is above the threshold. Similar

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work of secrecy outage performance is found in [5] for the cognitive dual-hop relay system. The direct and relayed signals are added at both receiver and eavesdroppers. Both selection combining and maximal ratio combining (MRC) are used at receiver and eavesdroppers; the next threshold-based relaying of [4] is tested to achieve the best performance. The final outcome of the paper is that secrecy performance is more sensitive to direct link between source and eavesdropper compared to that of the source to destination. Recently the concept of energy harvesting is applied in the dual-hop wireless link, where - among several relays - the best relay is selected to have the largest harvesting energy. Two parameters - namely the time splitting ratio and energy conversion efficiency - play a vital role to maximize system capacity or throughput discussed in [6, 7]. In [8], dual-hop decode-to-forward cooperative system is suggested. The relay harvests energy from the destination under Nakagami-*m* fading condition for both single-antenna source and multiple-antenna source with transmit antenna selection. The profile of outage probability and system throughput is shown against the channel gain (source to relay and relay to destination) under different link parameters.

In this paper, we use the fuzzy system to evaluate the ''probability of successful communication" of the wireless network under the fading channel. Some state-of-art regarding the application of the fuzzy system in the wireless networks is first discussed to find out the research gap. Application of fuzzy logic in wireless link selection is found in [9] for wireless sensor networks. Here, battery level of cluster head, the distance between cluster heads, and node density are considered as the input parameters with triangular membership function (MF) to maximize the total number of packets. A similar analysis is found in [10] where fuzzy inputs are remaining energy of the node and distance to sink while relay cost is the fuzzy output. Each input fuzzy variable has three Gaussian MF: low, medium, and high. The output fuzzy variable has five MFs: very low, low, medium, high, and very high. The MFs of input and output fuzzy variables are used to compromise energy-efficient and shortest path of routes. The extension of the work is found in [11] where an additional input fuzzy variable namely "confidence factor" is used with seven MFs. Finally, the variation of the total number of transmitted packets, network lifetime and energy dissipation against the number of rounds are shown for wireless sensor network (WSN).

Despite the work done in the aforementioned papers, none of them is relevant to the multi-hop wireless link under the concept of point-to-point traffic model to evaluate the outage probability or probability of successful communication. In real life, the topology of a wireless network can take any one of four different shapes as will be discussed in section 2. Previous works deal only with the first two types of networks. For a complicated network like the third and fourth category, we have to apply the point-to-point traffic model which is the first research gap that we found studying the recent works. Next, using SNR of a wireless link as the parameter of cumulative distribution function (CDF) of the fading channel, we derive the expression of outage probability for all types of topologies of the wireless network in a generalized form. We also found a research gap pertinent to the application of the fuzzy system in determining outage probability (or inversely probability of successful communication) of a multi-hop wireless network. We evaluate the probability of successful communication of multi-hop wireless networks using the fuzzy system with the same range of SNR in analytical or theoretical results. Finally, comparing the theoretical (or analytical) and simulation results with fuzzy output (probability of successful communication), we get a closer result.

The rest of the paper is organized as follows: section 2 gives the basic concept of point-to-point traffic model to derive blocking probability, section 3 deals with the multi-hop wireless link under four cases of the circuit where we derive outage probability in terms of SNR, section 4 shows the application of fuzzy system in the multi-hop wireless link, section 5 provides results based on the analysis of the previous two sections and, finally, section 6 concludes the entire analysis.

2. BASIC THEORY OF POINT-TO-POINT TRAFFIC

In a large network, several routers and switches are interconnected to route traffic along appropriate direction. In this section, three cases of connections are considered and will be applied in a multi-hop wireless link in the next section. The SNR of a wireless link is related to its channel capacity and if the SNR of a wireless link falls below a threshold value, the link is considered a block. The SNR of a link is directly related to both carried traffic and blocking probability. Hence, we can apply the point-to-point traffic model to evaluate the blocking probability of multi-hop wireless network, discussed in the next section. In the point-to-point-blocking model, carried traffic of one link is considered as the offered traffic on the end node but the lost traffic of one node is carried by the parallel alternate link. The concept of point-to-point blocking is applied in [12] for cognitive radio ad-hoc networks under *M/M/n* traffic.

2.1. Links are in Series

When several links are connected in series, the carried traffic of the 1st link becomes the offered traffic on the second link; and that of the second link becomes the offered traffic of the third link and so on. Let us consider two links that are in series and having the blocking probability B_1 and B_2 as shown in Fig. 1, provided the offered traffic is A.

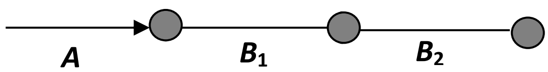

Fig. 1. Two links are in series.

Carried and lost traffic of the individual combined link is shown in Table 1.

Table 1. The traffic of the series link.

Link	Offered traffic	Carried traffic	Lost traffic
1	A	$A(1-B_1)$	AB_1
2	$A(1-B_1)$	$A(1-B_1)(1-B_2)$	$A(1-B_1)B_2$
Combined	A	$A(1-B_1)(1-B_2)$	$AB_1+ A(1-B_1)B_2$

Overall call blocking probability based on [13, 14] is:

B = Overall lost traffic/overall offered traffic

= 1- Overall carried traffic/overall offered traffic

= $1- A(1-B_1)(1-B_2)/A=1- (1-B_1)(1-B_2)$

If L links are in series then,

$$B = 1\text{-}(1\text{-}B_1)(1\text{-}B_2)\ldots\ \ldots\ \ldots(1\text{-}B_L) = 1\text{-} \prod_{i=1}^{L}(1-B_i) \tag{1}$$

The series link is applicable in multi-hop wireless link.

2.2. Links are in Parallel

When links are in parallel, lost/overflow traffic of the first link is carried by the second link and that of the second link is carried by the third link and so on. Two parallel links are shown in Fig. 2.

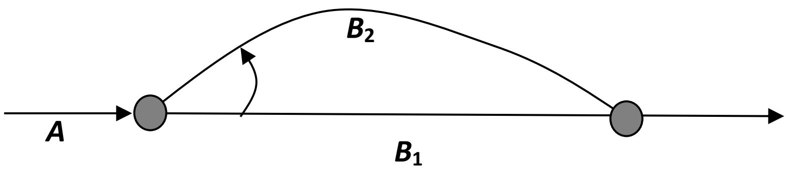

Fig. 2. Two links are in parallel.

The carried and lost traffic of individual links are shown in Table 2.

Table 2. The traffic of the parallel link.

Link	Offered traffic	Carried traffic	Lost traffic
1	A	$A(1\text{-}B_1)$	AB_1
2	AB_1	$AB_1(1\text{-}B_2)$	AB_1B_2
Combined	A	$A(1\text{-}B_1)+AB_1(1\text{-}B_2)$	AB_1B_2

The overall call blocking probability is:

B = Overall lost traffic/overall offered traffic

= $AB_1B_2/A=B_1B_2$

If L links are in parallel then,

$$B = \prod_{i=1}^{L} B_i \tag{2}$$

The parallel link is applicable in the fusion center of a cooperative cognitive radio network. The third category of the network - which is a combination of series and parallel links - can be solved by Eqs. (1) and (2).

2.3. Links are Neither in Series Nor in Parallel

The links of Fig. 3 are neither in series nor in parallel like Δ-Y network. The nodes of the figure can be considered as a part of a big network. First of all, we have to determine the blocking probability of the available paths: A-C-B, A-D-B, A-D-C-B, and A-C-D-B.

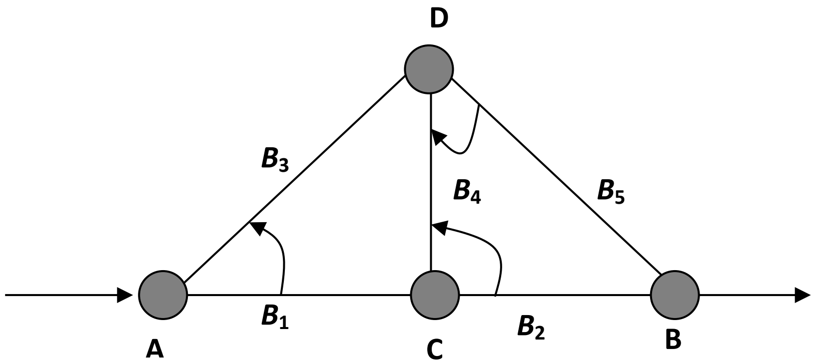

Fig. 3. Part of a network, the links of which are neither in series nor in parallel.

Path	Carried traffic
A-C-B	$A(1\text{-}B_1)(1\text{-}B_2)$
A-D-B	$AB_1(1\text{-}B_3)(1\text{-}B_5)$
A-D-C-B	$AB_1(1\text{-}B_3)B_5(1\text{-}B_4)(1\text{-}B_2)$
A-C-D-B	$A(1\text{-}B_1)B_2(1\text{-}B_4)(1\text{-}B_5)$

Let the total carried traffic is $\overline{X}$ then, the overall blocking probability will be:

$$B=1- \overline{X} / A \tag{3}$$

3. MODELING OF POINT-TO-POINT TRAFFIC IN A WIRELESS NETWORK

In this section, we apply the concept of point-to-point traffic model on multi-hop wireless link under four cases. Section 2 deals with the point-to-point traffic model where each link is characterized by its blocking probability or its quality of service (QoS). In this paper, we adopt the concept of traffic engineering of trunk/link for multi-hop wireless network for the first time, where we introduce SNR as the link parameter instead of blocking probability or QoS. In this section, we derive the 'outage probability' of four types of wireless networks using CDF of Rayleigh and Nakagami-*m* cases, taking SNR indicated by γ as the random variable as in [15]. Here the CDF of the *i*th link is indicated as $F_{\Gamma_i}(\gamma)$ in a generalized form, which can be the CDF of Rayleigh and Nakagami-*m*. The expression of probability density function (PDF) and CDF of Nakagami-*m* distribution is available in [16], the Rayleigh PDF or CDF is found from that of Nakagami-*m* distribution taking $m = 1$.

3.1. Case 1: Relays are in Series

First of all, we consider the simplest model of n relays that are in series as shown in Fig. 4. The SNR of the source (S) to the first relay (R_1) is γ_1; SNR of first relay (R_1) to second relay (R_2) is γ_2 and so on. Let the threshold SNR - to maintain a link with minimum capacity - is γ_{th}.

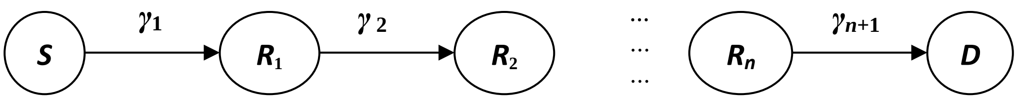

Fig. 4. Relays are in series.

For simplicity of analysis, we consider two in series links. The probability of successful communication is only possible if the SNR of the individual link is greater than the threshold SNR. Therefore the probability of successful communication is,

$$P_{success} = P_{S1} \cdot P_{S2} = P_r\left\{\gamma_1 > \gamma_{th}\right\} \cdot P_r\left\{\gamma_2 \geq \gamma_{th}\right\}$$

$$= \left\{1 - F_{\Gamma_1}(\gamma_{th})\right\}\left\{1 - F_{\Gamma_2}(\gamma_{th})\right\}$$; where $F_{\Gamma_i}(\gamma)$ is the CDF of SNR of ith link

The outage probability is,

$$P_{out} = 1 - P_{success} = 1 - \prod_{i=1}^{2}\left(1 - F_{\Gamma_i}(\gamma_{th})\right)$$

For n links in series, the generalized outage probability will be,

$$P_{out} = 1 - \prod_{i=1}^{n}\left[1 - F_{\Gamma_i}(\gamma_{th})\right] \tag{4}$$

3.2. Case 2: Relays are in Parallel

When n relays are in parallel, successful communication occurs when anyone of a 2-hop link makes possible communication between S and D. In this case we can ignore other links of Fig. 5. The operation of the network follows 'OR' logic instead of 'AND' logic of the previously discussed case 1. Now the probability of successful transmission is,

$$P_{success} = P_{success-link\_1} + P_{success-link\_2} + \ldots \quad \ldots \quad \ldots + P_{success-link-n}$$

$$= P_r\left\{\gamma_{SR_1} \geq \gamma_{th}\right\} \cdot P\left\{\gamma_{R_1D} \geq \gamma_{th}\right\} + P_r\left\{\gamma_{SR_2} \geq \gamma_{th}\right\} P\left\{\gamma_{R_2D} \geq \gamma_{th}\right\} + \cdots\cdots\cdots\cdots\cdots + P_r\left\{\gamma_{SR_n} \geq \gamma_{th}\right\} P\left\{\gamma_{R_nD} \geq \gamma_{th}\right\}$$

$$P_{success} = \sum_{i=1}^{n}\left\{P_r\left(\gamma_{SR_i} \geq \gamma_{th}\right) . p_r\left(\gamma_{R_iD} \geq \gamma_{th}\right)\right\} \tag{5}$$

The outage probability,

$$P_{out} = 1 - P_{success} = 1 - \sum_{i=1}^{n}\left[P_r(\gamma_{SR_i} \geq \gamma_{th}) . P_r\left\{\gamma_{R_iD} \geq \gamma_{th}\right\}\right]$$

$$= 1 - \sum_{i=1}^{n}\left[\left\{1 - F_{\Gamma_{SR_i}}(\gamma_{th})\right\}\left\{1 - F_{R_iD}(\gamma_{th})\right\}\right] \tag{6}$$

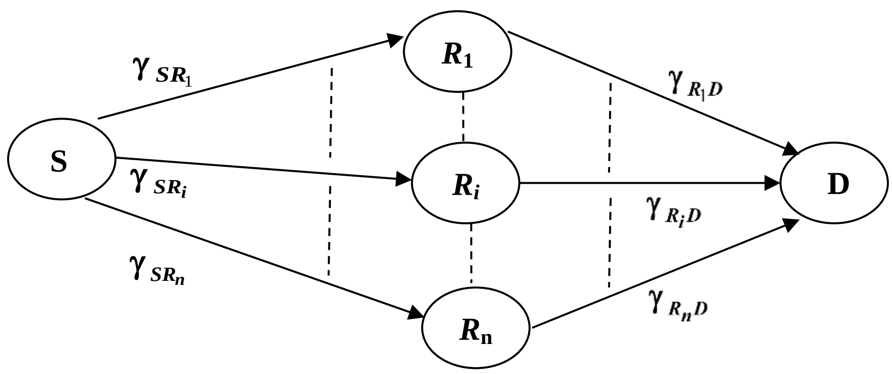

Fig. 5. Relays are in parallel.

3.3. Case 3: Relays are in a Series-Parallel Combination

When links are in series-parallel combination, we can solve it using a combination of 'AND' and 'OR' logic of the previously discussed case 1 and case 2. Fig. 6 shows such a network where four links are in series and a 2-hop link is in parallel with the combined series link.

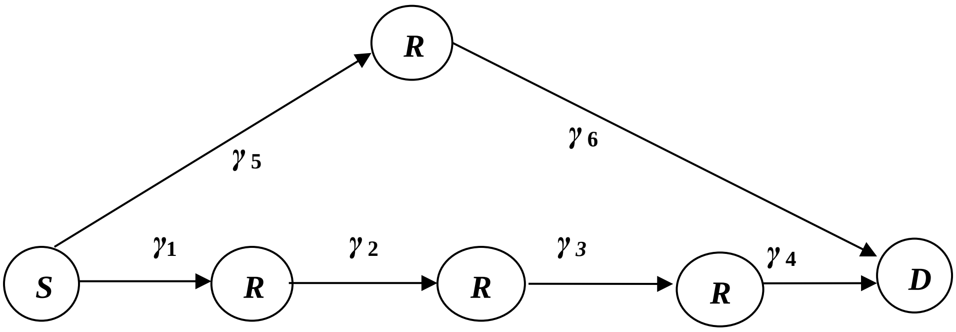

Fig. 6. Relays are in a series-parallel combination.

The outage probability of four series links is, $P_{out\_series1} = 1 - \prod_{i=1}^{4}\left(1 - F_{\Gamma_i}(\gamma_{th})\right)$ and that of a 2-hop link is, $P_{out\_series2} = 1 - \prod_{j=5}^{6}\left(1 - F_{\Gamma j}(\gamma_{th})\right)$.

The outage probability of the combined network is,

$$P_{out} = 1 - \left(P_{S1} + P_{S2}\right) = 1 - \left[\prod_{i=1}^{4}\left(1 - F_{\Gamma_i}(\gamma_{th})\right) + \prod_{j=5}^{6}\left(1 - F_{\Gamma_j}(\gamma_{th})\right)\right] \tag{7}$$

3.4. Case 4: Relays are neither in Series nor in Parallel

Now we consider a network where the links are neither in series nor in parallel as discussed in [17] and shown in Fig. 7. In this case, we have to solve the network for all possible paths from source to destination.

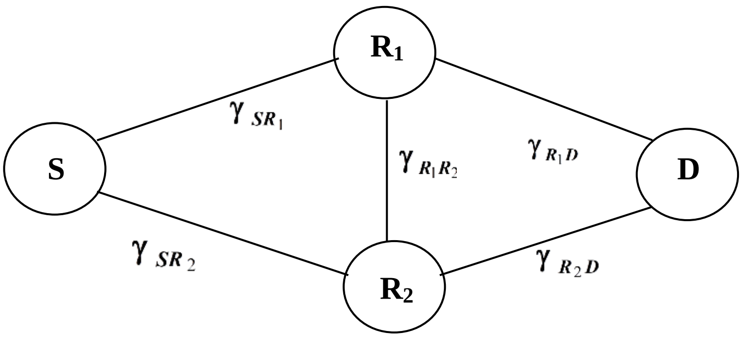

Fig. 7. Relays are neither in series nor in parallel.

For path $S - R_1 - D$, the probability of successful transmission is:

$$P_{success\_1} = \left[1 - F_{\Gamma_{SR_1}}(\gamma_{th})\right]\left[1 - F_{\Gamma_{R_1D}}(\gamma_{th})\right]$$

For path $S - R_2 - D$,

$$P_{success\_2} = \left\{1 - F_{\Gamma_{SR_2}}(\gamma_{th})\right\}\left\{1 - F_{\Gamma_{R_2D}}(\gamma_{th})\right\}$$

For path $S - R_1 - R_2 - D$,

$$P_{success\_3} = \left[1 - F_{\Gamma_{SR_1}}(\gamma_{th})\right]\left[1 - F_{\Gamma_{R_1R_2}}(\gamma_{th})\right]\left[1 - F_{\Gamma_{R_2D}}(\gamma_{th})\right]$$

For path $S - R_2 - R_1 - D$,

$$P_{success\_4} = \left[1 - F_{\Gamma_{SR_2}}(\gamma_{th})\right]\left[1 - F_{\Gamma_{R_2R_1}}(\gamma_{th})\right]\left[1 - F_{\Gamma_{R_1D}}(\gamma_{th})\right]$$

Now the outage probability will be,

$$P_{out} = 1 - \sum_{i=1}^{4} P_{success\_i} \tag{8}$$

We can further extend the concept of the source to sink data flow using the Ford-Fulkerson algorithm of [18]. The algorithm determines the maximum flow from source to sink taking possible augmenting paths, which is beyond the scope of the paper since we only concentrate on the point-to-point traffic model.

4. FUZZY SYSTEM IN WIRELESS LINK

Although few applications of fuzzy inference system (FIS) is found for WSN, but still no application of FIS is found in the multi-hop wireless network. The outage probability and probability of successful communication is derived - in section 3 - based on CDF of SNR under small scale fading. It is also possible to evaluate both probabilities from FIS taking the idea of [19, 20]. We apply two input fuzzy variables: 'SNR of link-1' and 'SNR of link-2' of a 2-hop wireless link in a FIS as shown in Fig. 8. Here the profile of MFs is Gaussian in shape and each input has three MFs designated as Low (*L*), Moderate (*M*), and High (*H*). The fuzzy system has one output provided with two MFs of 'Success' and 'Fail'.

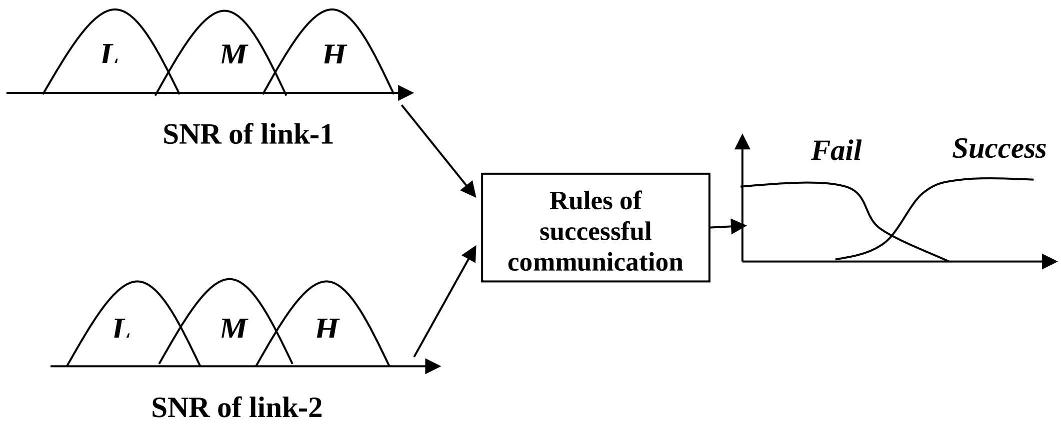

Fig. 8. Fuzzy wireless link of 2-hop.

According to Fig. 8, there are six possible fuzzy rules:

Rule-1: If (SNR of 1st hop is *L*) and (SNR of 2nd hop is *L*) then (communication fails)
Rule-2: If (SNR of 1st hop is *M*) and (SNR of 2nd hop is *L*) then (communication fails)
Rule-3: If (SNR of 1st hop is *M*) and (SNR of 2nd hop is *M*) then (communication succeds)
Rule-4: If (SNR of 1st hop is *M*) and (SNR of 2nd hop is *H*) then (communication succeds)
Rule-5: If (SNR of 1st hop is *H*) and (SNR of 2nd hop is *M*) then (communication succeds)
Rule-6: If (SNR of 1st hop is *H*) and (SNR of 2nd hop is *H*) then (communication succeds)

In the case of the parallel link, we have to use the 'OR' logic in fuzzy rules instead of the 'AND' logic of the series circuit. When there is a combination of series and parallel links, the fuzzy rules will be combinations of 'AND' and 'OR' logic.

For a multi-hop wireless link of several relays (neither series nor parallel), the solution with the fuzzy system will be complicated because of the large number of rules. Let us consider a wireless network of multiple relays as in Fig. 9. The numerical value of SNR of links can be converted to fuzzy data like in Table 3 for four different range where ε is an infinitesimal value. The Gaussian MFs used in this paper are shown in Fig. 10 taking standard deviation and means of: σ = 0.85, μ_1= -2, μ_2= 0, μ_3= 2 and μ_4= 4. The numerical to fuzzy conversion of Fig. 9 is shown in Table 4 as an example. Next, we have to construct a graph equivalent matrix of size 6×6 (number of nodes is M = 6) of Fig. 9 as shown in Table 5. Any element of the matrix $A(i, j)$ (i = 1, 2, … , 6 and j = 1, 2, 3, ... , 6) has the fuzzy values as $A(i, j)$ ε {V, L, M, H, 0, NC} where 0 is for the case of no loop and NC for no connection.

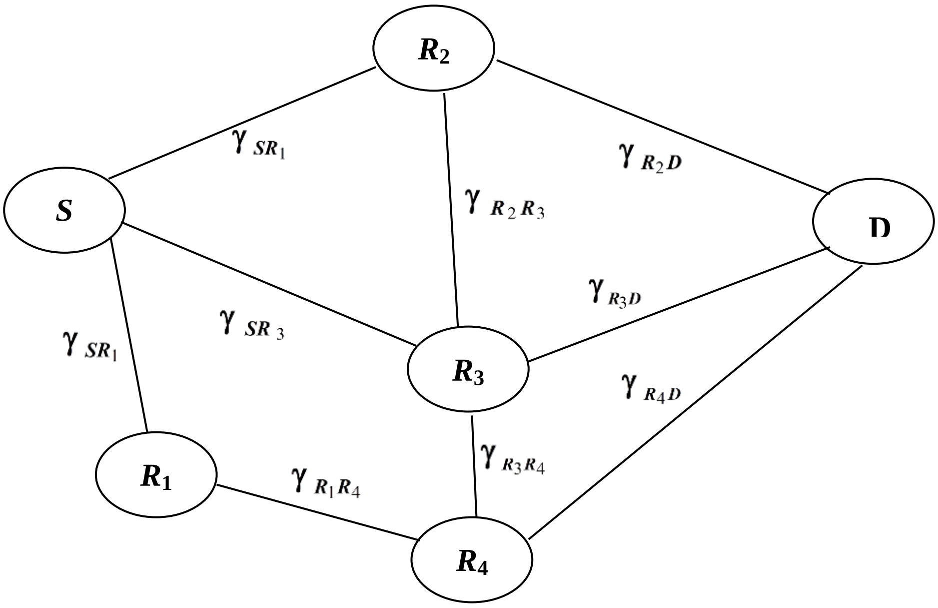

Fig. 9. Multi-hop wireless network.

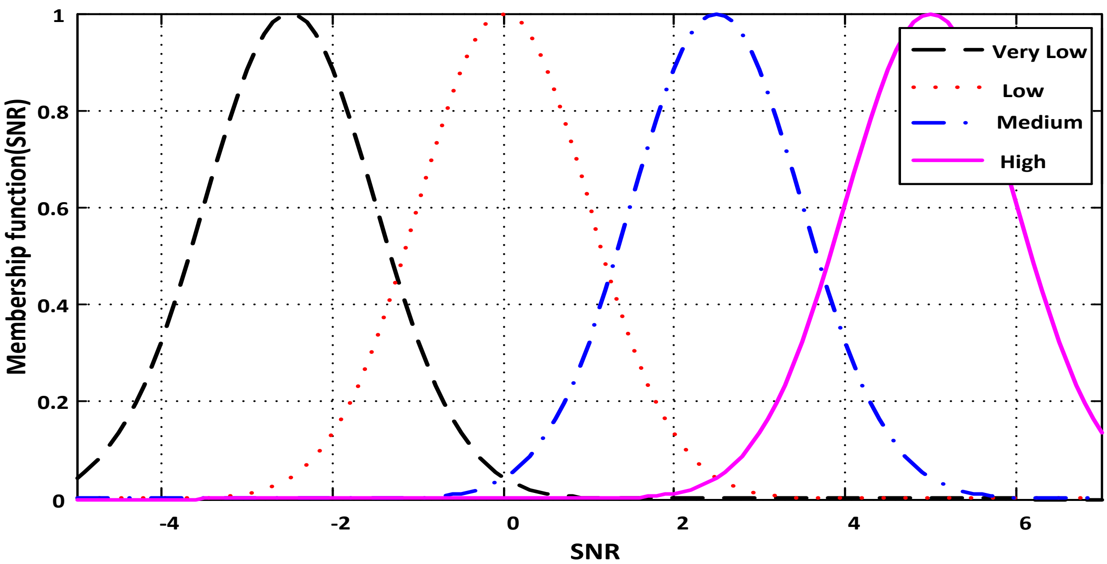

Fig. 10. Fuzzy MFs of SNR.

Table 3. Range of SNR and fuzzy symbols.

Range of SNR in dB	-05-01	-01+ε to 01	1+ε to 03	03+ε or above
Category	Very low	Low	Moderate	High
Symbol	*V*	*L*	*M*	*H*

Table 4. Numerical and fuzzy data.

Link	S-R_1	S-R_2	S-R_3	R_1-R_4	R_2-R_3	R_2-D	R_3-R_4	R_3-D	R_4-D
SNR in dB	5.56	4.2	4.6	0.85	-0.5	2.1	-1.6	2.2	2.0
Fuzzy symbol	*H*	*H*	*H*	*L*	*L*	*M*	*V*	*M*	*M*

Table 5. Graph equivalent matrix.

	S	R_1	R_2	R_3	R_4	D
S	0	H	H	H	NC	NC
R_1	H	0	NC	NC	L	NC
R_2	NC	NC	0	L	NC	M
R_3	L	NC	L	0	N	M
R_4	V	L	NC	V	0	M
D	NC	NC	M	M	M	0

To construct a path from source to destination, we can use the following algorithm where SNR of each link must be at least *H* or *M* along the path. The algorithm to construct a path from the source, *S* (node 1) to the destination, *D* (node *M* = 6) with the constraint of SNR of any link to be *M* or *H* is given below.

Algorithm

for i=1: M
for j= 1: M
{
Scan all the elements A(i, j)
If A(i, j) = H or A(i, j) = M
S(i, j) = 'There is a path between node i and j'
}
Based on the index (i, j) of array S determine all possible path from node 1 to node M using rules:
S(i, k)^S(k, l) → 'There is a path between node i and l'
; where ^ indicates 'AND' rule i.e. finding S(i, k) and S(k, l) from the algorithm

5. RESULTS AND DISCUSSION

First, we determine the outage probability of a multi-hop wireless link for the above four cases under Rayleigh fading case, taking average SNR of each link as γ_{av} = 2 dB. For series or parallel relaying (case 1 and case 2), the number of the relay n = 3; in series-parallel combination 2 relays are in series and the third one is in parallel (case 3); and for the fourth case, we take the circuit like Fig. 7. The variation of outage probability P_{out} against threshold SNR (in dB) γ_{th} is done as in [21, 22] only for the series link but in this paper, we do the job for three additional cases as shown in Fig. 11 under Rayleigh fading environment. In Section 4, the input of the FIS is shown in generalized form but the input SNR of links follows Rayleigh and Nakagami-*m* fading separately. The random variables against SNR are taken from Matlab-16 using the corresponding PDF. The P_{out} is found maximum under series network and minimum for a parallel network designated as case 1 and case 2 in Fig. 11. The performance of case 3 (series-parallel combination) and case 4 (neither series nor parallel) depends on the shape of the network. The analytical results are verified by simulation for all the four cases and found a confidence level above 95%. We run the simulation generating 10,000 random numbers in Matlab-18 with the following steps and parameters:

1. Generate N = 10 random number following Rayleigh PDF with $\gamma_{av} = (\gamma_{max} + \gamma_{min})/2$ and take the mean of 10 numbers and assign it by T where γ_{max} = 6 dB and γ_{min} = -5 dB.
2. Repeat step 1 for M = 10000 times and store the means as an array $T(i)$, i = 1 to 10000.
3. Take SNR, γ = -5 dB to 6 dB with interval of $(\gamma_{max} - \gamma_{min})/M$.
4. Find the number of elements of array $T > \gamma(i)$ and store the number $b(i)$ for i=1 to (length of γ) and the corresponding probability of success $P_{success1}(i) = b(i)/M$.
5. Repeat steps 1 to 4 for link 2 and the corresponding probability of success $P_{success2}(i) = b(i)/M$.
6. The outage probability of two-hop series link, $P_{out}(i) = 1 - P_{success1}(i) * P_{success2}(i)$ as in Eq.(1).

The above algorithm reveals the results for the two-hop series link but other topologies of the network with more links are included and step 6 is modified accordingly. For the case of Nakagami-m fading, the generated random number of step-1 will follow the corresponding PDF.

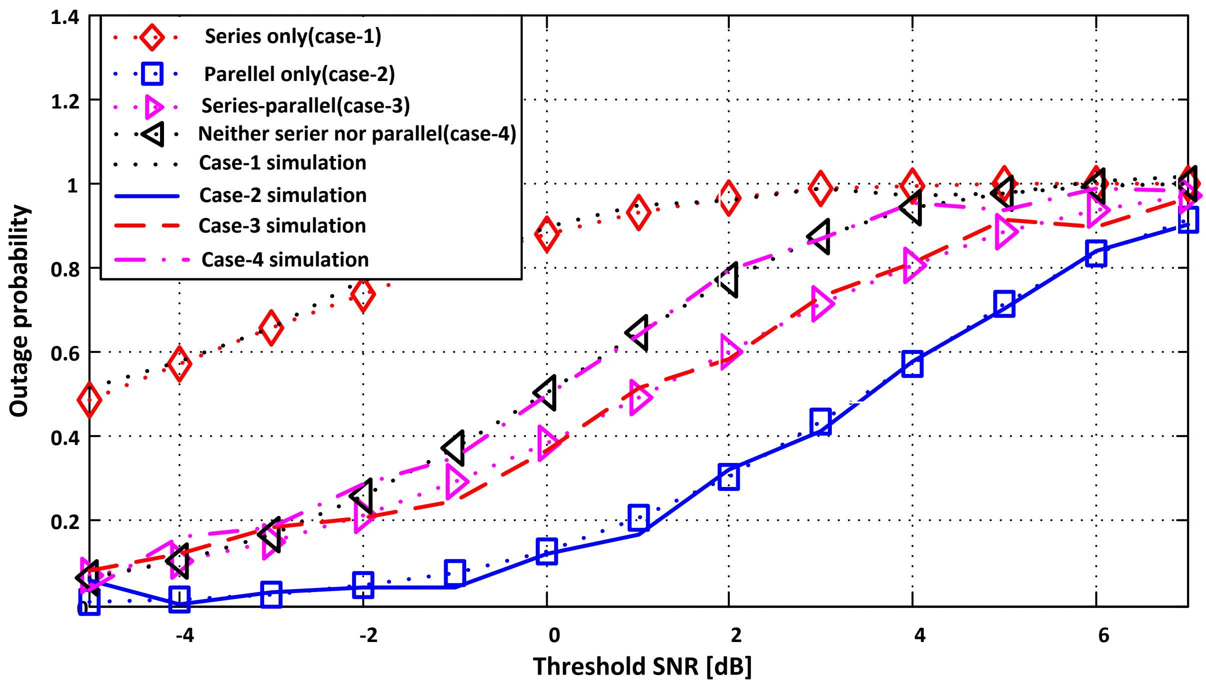

Fig. 11. Variation of outage probability against SNR under Rayleigh fading.

Next, we determine the similar results under the Nakagami-m fading case, taking m = 4. Because of m weak links between adjacent nodes, the outage probability of Nakagami-m fading is found a little bit smaller than that of Rayleigh fading depicted in Fig. 12 for both analytical and simulation cases.. In Figs. 11 and 12, the y-axis is made linear to grasp the outage probability at a glance.

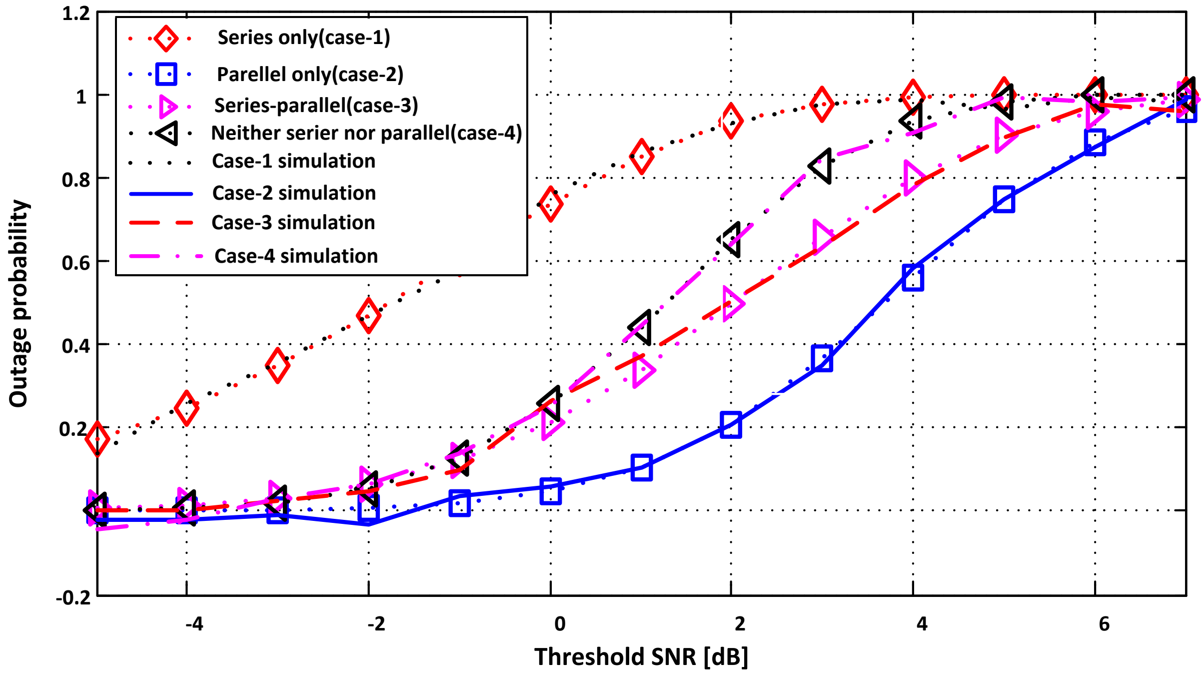

Fig. 12. Variation of outage probability against SNR under Nakagami-*m* fading.

For the fuzzy system of the wireless link, we - first of all - consider the simple two-hop wireless link (case 1) where the input of the fuzzy system consists of two SNRs of two links. The first input is the SNR of first-hop with four Gaussian MFs: very low (VL), low (L), moderate (M), and high (H); whereas the MFs of second input reveals the SNR of the second hop. Few rules of the system are given in Fig. 13 for fuzzy output of success/failure of transmission. The profile of MFs is shown in Fig. 14(a) and the surface plots of input versus output are shown in Fig. 14(b) and 14(c) for the cases of before and after normalization of SNR. We first apply SNR in the FIS of Fig. 8 in the range of -5 dB to 6 dB with an interval of 0.1 dB. The fuzzy output using the centroid method is found in the range of -2 to 4 as shown in Fig. 14(b). To avoid negative results, we normalized input SNRs in the range of 0 to 1 and we get the output in the range of 0 to 1 as shown in Fig. 14(a). The fuzzy output is now the probability of success.

1. If (SNR-of -Link-1 is L) and (SNR-of-link-2 is V) Then (Fuzzy-Output is F) (1)
2. If (SNR-of -Link-1 is L) and (SNR-of-link-2 is M) Then (Fuzzy-Output is F) (1)
3. If (SNR-of -Link-1 is L) and (SNR-of-link-2 is H) Then (Fuzzy-Output is F) (1)
4. If (SNR-of -Link-1 is L) and (SNR-of-link-2 is L) Then (Fuzzy-Output is F) (1)
5. If (SNR-of -Link-1 is V) and (SNR-of-link-2 is L) Then (Fuzzy-Output is F) (1)
6. If (SNR-of -Link-1 is V) and (SNR-of-link-2 is H) Then (Fuzzy-Output is F) (1)
7. If (SNR-of -Link-1 is V) and (SNR-of-link-2 is M) Then (Fuzzy-Output is F) (1)
8. If (SNR-of -Link-1 is V) and (SNR-of-link-2 is V) Then (Fuzzy-Output is F) (1)
9. If (SNR-of -Link-1 is M) and (SNR-of-link-2 is M) Then (Fuzzy-Output is S) (1)
10. If (SNR-of -Link-1 is M) and (SNR-of-link-2 is H) Then (Fuzzy-Output is S) (1)
11. If (SNR-of -Link-1 is H) and (SNR-of-link-2 is H) Then (Fuzzy-Output is S) (1)
12. If (SNR-of -Link-1 is M) and (SNR-of-link-2 is V) Then (Fuzzy-Output is F) (1)
13. If (SNR-of -Link-1 is M) and (SNR-of-link-2 is L) Then (Fuzzy-Output is F) (1)
14. If (SNR-of -Link-1 is H) and (SNR-of-link-2 is L) Then (Fuzzy-Output is F) (1)
15. If (SNR-of -Link-1 is H) and (SNR-of-link-2 is V) Then (Fuzzy-Output is F) (1)

Fig. 13. Fuzzy rules of case 1.

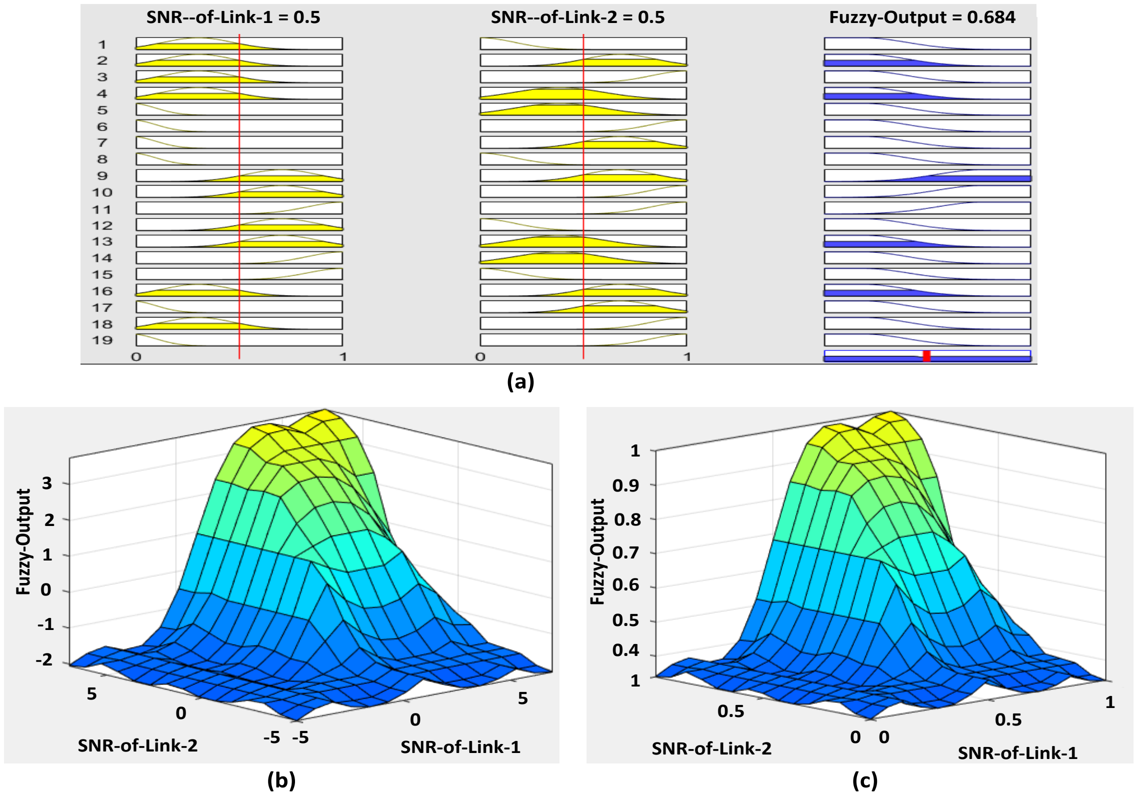

Fig. 14. Profile of MFs of case 1 with the surface plot: a) MF SNR; b) surface plot before normalization; c) surface plot after normalization.

Next, we consider the fuzzy system of parallel transmission of the two-hop wireless link of case 2, using two relays; where the number of possible rules is 26. Only 12 of them are shown in Fig. 15.

1. If (SNR-of-1st-hop-link-1 is Low) and (SNR-oflink-2nd-hop-link-1 is Low) and (SNR-of-1st-hop-link-2 is Low) and (SNR-of-2nd -hop-link-2 is Low) then (output is Fail) (1)
2. If (SNR-of-1st-hop-link-1 is Moderate) and (SNR-of-2nd-hop-link-1 is Low) and (SNR-of-1st-hop-link-2 is Low) and (SNR-of-2nd -hop-link-2 is Low) then (output is Fail) (1)
3. If (SNR-of-1st-hop-link-1 is High) and (SNR-of-2nd-hop-link-1 is Low) and (SNR-of-1st-hop-link-2 is Low) and (SNR-of-2nd -hop-link-2 is Low) then (output is Success) (1)
4. If (SNR-of-1st-hop-link-1 is Moderate) and (SNR-of-2nd-hop-link-1 is Low) and (SNR-of-1st-hop-link-2 is Low) and (SNR-of-2nd -hop-link-2 is Low) then (output is Fail) (1)
5. If (SNR-of-1st-hop-link-1 is Moderate) and (SNR-of-2nd-hop-link-1 is Moderate) and (SNR-of-1st-hop-link-2 is Moderate) and (SNR-of-2nd -hop-link-2 is Low) then (output is Success) (1)
6. If (SNR-of-1st-hop-link-1 is High) and (SNR-of-2nd-hop-link-1 is Moderate) and (SNR-of-1st-hop-link-2 is Moderate) and (SNR-of-2nd -hop-link-2 is Low) then (output is Success) (1)
7. If (SNR-of-1st-hop-link-1 is High) and (SNR-of-2nd-hop-link-1 is High) and (SNR-of-1st-hop-link-2 is Moderate) and (SNR-of-2nd -hop-link-2 is Low) then (output is Success) (1)
8. If (SNR-of-1st-hop-link-1 is High) and (SNR-of-2nd-hop-link-1 is High) and (SNR-of-1st-hop-link-2 is High) and (SNR-of-2nd -hop-link-2 is Low) then (output is Success) (1)
9. If (SNR-of-1st-hop-link-1 is High) and (SNR-of-2nd-hop-link-1 is High) and (SNR-of-1st-hop-link-2 is High) and (SNR-of-2nd -hop-link-2 is Moderate) then (output is Success) (1)
10. If (SNR-of-1st-hop-link-1 is High) and (SNR-of-2nd-hop-link-1 is High) and (SNR-of-1st-hop-link-2 is High) and (SNR-of-2nd -hop-link-2 is High) then (output is Success) (1)
11. If (SNR-of-1st-hop-link-1 is High) and (SNR-of-2nd-hop-link-1 is High) and (SNR-of-1st-hop-link-2 is Moderate) and (SNR-of-2nd -hop-link-2 is High) then (output is Success) (1)
12. If (SNR-of-1st-hop-link-1 is High) and (SNR-of-2nd-hop-link-1 is Moderate) and (SNR-of-1st-hop-link-2 is Moderate) and (SNR-of-2nd -hop-link-2 is High) then (output is Success) (1)

Fig. 15. Few fuzzy rules of case 2.

The corresponding profiles of MFs for case 2 are shown in Fig. 16(a). The normalized outputs against SNR of link-1 and against SNR of link-2 are shown in Fig. 16(b) and Fig. 16(c), respectively. The fuzzy output of both Figs. 16(b) and (c) is the probability of success.

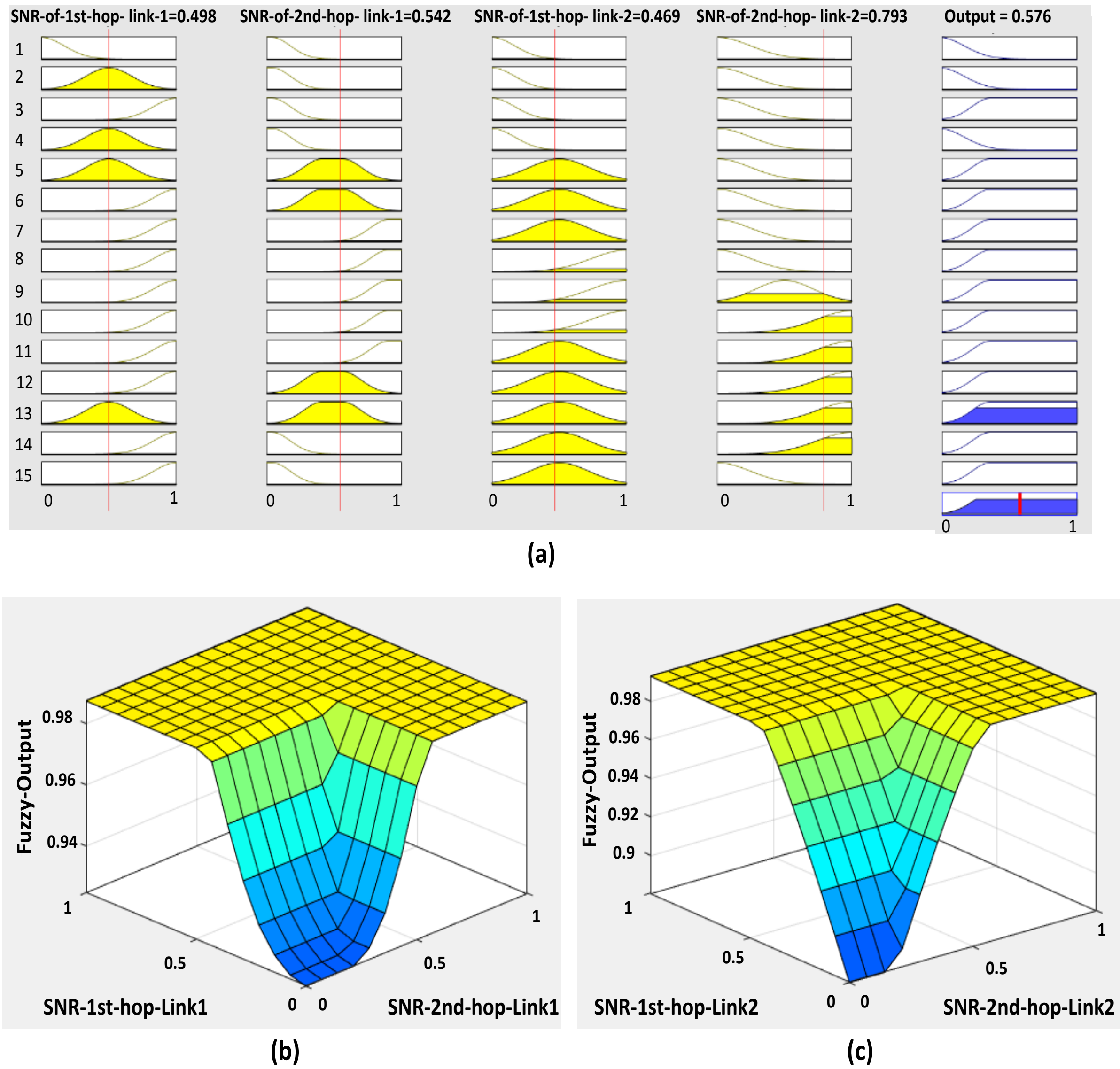

Fig. 16. Fuzzy results of parallel links: a) MF SNR; b) normalized output against SNR of link-1; c) normalized output against SNR of link-2.

Finally, we apply a fuzzy system for the network of Fig. 7 where each link is considered individually for possible success/failure of communication, hence we need 84 rules. Among them, few rules are shown in Fig. 17. Profile of MFs and two surface plot under relay 1 and relay 2 are shown in Figs. 18(a), (b) and (c) respectively.

For case 4, there are a lot of rules against failure as well as success in communication compared to cases 1 and 2. Therefore, the surface plot of case 4 has a lot of variations compared to the previous two cases visualized from Figs. 18 (b) and (c). In this paper, we ignore case 3 for fuzzy analysis, since it is the only combination of cases 1 and 2 that provides some intermediate results.

1. If (SNR-S-to-R1 is Low) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is Low) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is Low) then (output1 is Fail) (1)
2. If (SNR-S-to-R1 is Low) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is Low) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is Moderate) then (output1 is Fail) (1)
3. If (SNR-S-to-R1 is Low) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is Low) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is High) then (output1 is Fail) (1)
4. If (SNR-S-to-R1 is Low) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is Moderate) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is Low) then (output1 is Fail) (1)
5. If (SNR-S-to-R1 is Low) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is High) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is Low) then (output1 is Fail) (1)
6. If (SNR-S-to-R1 is Moderate) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is High) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is Low) then (output1 is Fail) (1)
7. If (SNR-S-to-R1 is High) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is High) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is Low) then (output1 is Fail) (1)
8. If (SNR-S-to-R1 is High) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is High) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is Moderate) then (output1 is Fail) (1)
9. If (SNR-S-to-R1 is High) and (SNR-R1-to-D is Low) and (SNR-S-to-R2 is High) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is High) then (output1 is Fail) (1)
10. If (SNR-S-to-R1 is Moderate) and (SNR-R1-to-D is Moderate) and (SNR-S-to-R2 is High) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is High) then (output1 is Success) (1)
11. If (SNR-S-to-R1 is Moderate) and (SNR-R1-to-D is Moderate) and (SNR-S-to-R2 is Low) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is High) then (output1 is Success) (1)
12. If (SNR-S-to-R1 is Moderate) and (SNR-R1-to-D is Moderate) and (SNR-S-to-R2 is High) and (SNR-R2-to-D is Low) and (SNR-R1-to-R2 is High) then (output1 is Success) (1)

Fig. 17. Few fuzzy rules of case 4.

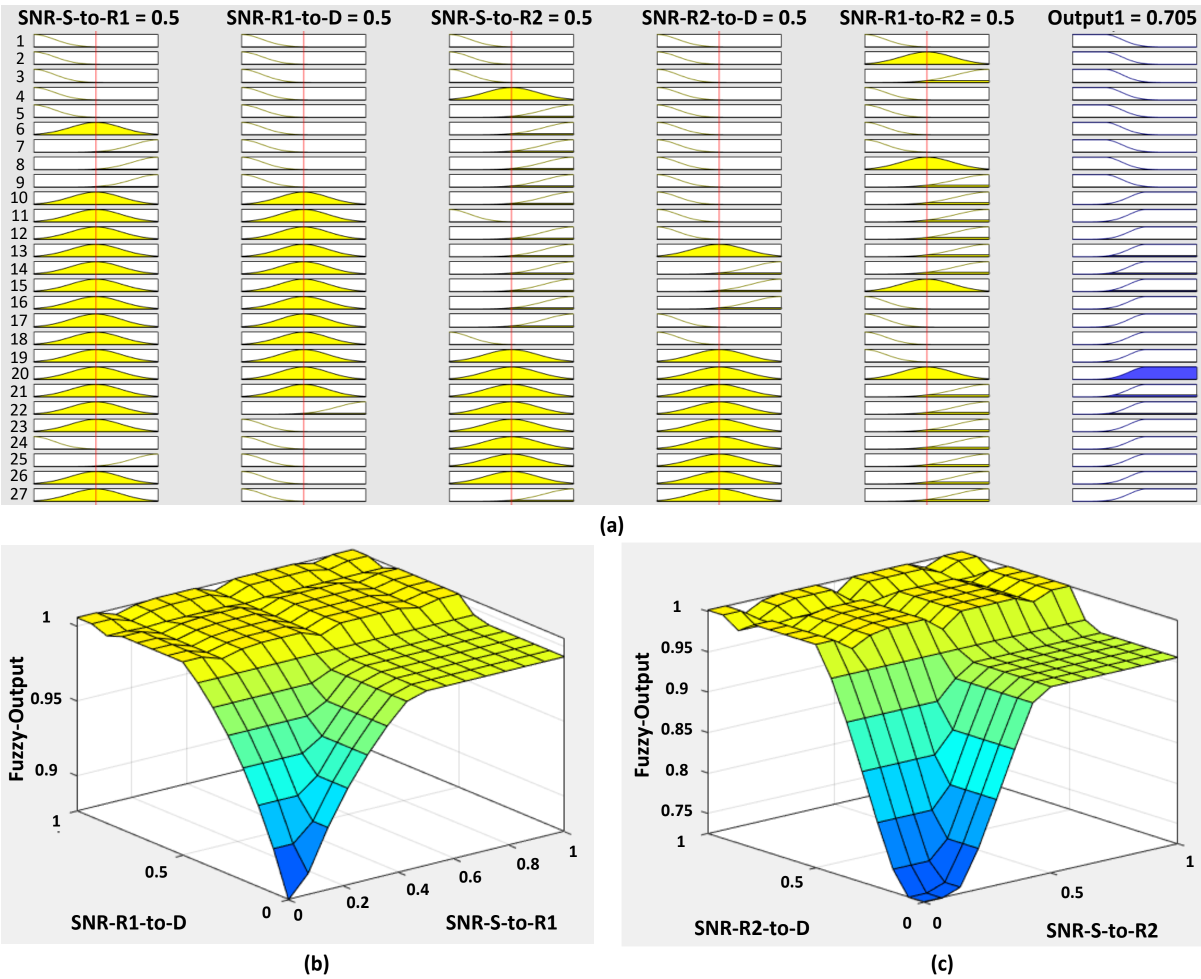

Fig. 18. Fuzzy results of case 4: a) MF SNR; b) surface plot under relay 1; c) surface plot under relay 2.

Finally, we concentrate on Figs. 9 and 10, Table 3, Table 4, and the algorithm of section 4 to determine the probability of successful communication of the network. The objective is to determine the accuracy of fuzzy output compared to the analytical (theoretical) value of $P_{success}$. For the complete path from source S to destination D, the links are combined with AND logic. If all the links have SNR of H, the weight of the success is 100% and if only one link has SNR of M, then; the weight of the link will be 0.8. Similarly for a successful path of two links with M, the weight will be 0.6. If any link of the path from S to D has SNR of L or V, it will be avoided. If two successful paths have a common link, then the path with higher weight will be selected. To compare the probability of successful communication - of the two systems, fuzzy system and the analytical results using Eqs. (4) to (8) - we provide a numerical example based on Fig. 9. We assume numerical values of SNR of different branches of Fig. 9 as: S-R_1 = 3.3 dB, S-R_2 = 4.2 dB, S-R_3 = 4.6 dB, R_2-D = 2.1 dB, R_3-D = 2.8 dB, R_4-D = 2.6 dB, R_1-R_4 = -0.6 dB, R_3-R_4 = -2.6 dB and R_2-R_3 = -0.5 dB.

The five possible shortest paths and corresponding fuzzy outputs are:

1) S-R_2-D, H^M → Success;
2) S-R_2-R_3-D, H^L^M → Fail;
3) S-R_3-D, H^M → Success;
4) S-R_3-R_4-D, H^V^M→ Fail;
5) S-R_1-R_4-D, H^L^M→ Fail.

Here the operator ^ indicates AND logic and using rules of Fig. 17 (paths consist of SNRs of M or H that provides successful communication), we get the possible links with success and failure. Next, considering possible links, the numerical assumed values of SNR of links in dB and numerical values to fuzzy symbols conversion, the above mentioned rules are combined and the output of the fuzzy system is de-fuzzified using centroid method. The normalized de-fuzzified output is an estimated probability of success, P_s = 1-P_{out} = 0.862 for path 1 and 0.838 for path 3. The corresponding analytical results (under the concept of Eq. (8)) are found as 0.899 and 0.885. Finally, simulated SNR using algorithm-1 under Rayleigh PDF taking γ_{av} =1.5 dB is applied in FIS and the normalized de-fuzzified output is evaluated for the path S-R_2-D and S-R_3-D. Similar work is done under Nakagami-m PDF. Both results give the probability of successful communication. Again, using the CDF of Rayleigh and Nakagami-m distribution used in Eqs. (4) to (8), we evaluated the theoretical (analytical) probability of success commination P_s. The theoretical /analytical result is compared with de-fuzzified P_s in Table 6. There is a small variation between analytical and fuzzy results of Table 6 for both γ_{av} =1.5 dB and γ_{av} = 2.5 dB. If we increase the number of MFs for SNR of links, then, the difference between the two results will be reduced.

Table 6. Possible path selection.

Average SNR [dB]	Selected shorted path	Normalized de-fuzzified output (Rayleigh)	Normalized de-fuzzified output (Nakagami-m)	Analytical (Rayleigh)	Analytical (Nakagami-m)
1.5	S-R_2-D	0.813	0.822	0.851	0.884
	S-R_3-D	0.804	0.811	0.829	0.838
2.5	S-R_2-D	0.851	0.856	0.889	0.916
	S-R_3-D	0.837	0.866	0.875	0.930

6. CONCLUSIONS

In this paper, we adopted the concept of the point-to-point traffic model of teletraffic engineering for a multi-hop wireless network of all possible topologies. The probability of successful communication is derived for all the topologies analytically and by simulation. Taking the concept of applying the fuzzy system in WSN of previous works, we used the fuzzy system to evaluate the probability of success of a multi-hop wireless network of all possible topologies and obtained almost the same results. Still, we have the scope of analyzing multi-hop wireless links under the maximum flow algorithm with minimum cut theorem like in [23, 24]. In the future, we will use the energy harvesting scheme in the point-to-point traffic modelpresented in this paper to observe the improvement of the throughput of the network compared to the conventional dual-hop model. The concept of this paper is applicable in CRN, WSN, MANET, and WLAN.

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