

## Speed Synchronization of Single-Phase Induction Motors by Electrical Shaft System

Ali S. Akayleh

Department of Electrical Power Engineering and Mechatronics, Faculty of Engineering, Tafila Technical University, Tafila, Jordan  
e-mail: akayleh\_em@yahoo.com

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**Abstract**— A new connection of two medium-power capacitor-run single-phase induction motors is proposed. Also, an adjustable three-phase electrical shaft speed synchronization system with the single-phase induction motors is described. Synchronous capability and recovery time variations are analyzed and obtained by adjusting the stator and the rotor additional parameters. The analysis includes the determination of the relationship between additional parameters, load difference and time response of the mechanical torque and rotational speed. The proposed system is mathematically modeled and simulated using MATLAB. Simulation results are provided and discussed.

**Keywords**— Electrical shaft, induction motor, speed synchronization.

### I. INTRODUCTION

Usually in industrial applications, single-phase induction motors are not widespread; however, due to the greater availability of single-phase power, these motors can be used in some applications. Single-phase induction motors include: split-phase, capacitor-start and capacitor-run, permanent-magnet and shaded-pole motors [1, 2]. Capacitor-run single-phase induction motor is very widespread in different drives such as: pumps, fans and mowers. Speed synchronization systems that use traditional single-phase induction motors require some inefficient and complex mechanical or electrical methods to complete direct synchronization. To simplify the solution in our case, we will drive the system using the electrical shaft system [3]-[5], where squirrel-cage rotors are replaced by slip-ring rotors as shown in Fig. 1.

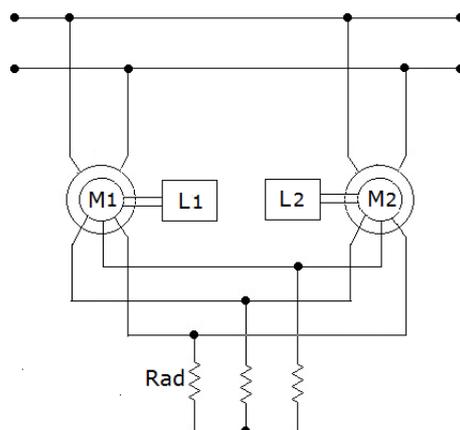


Fig. 1. Electrical shaft system

Investigation and control ability of starting torque of single-phase capacitor-run induction motors makes it possible to use them in medium-power synchronization systems.

Electromagnetic torque and rotational speed in both motors can be controlled by adjusting the additional resistance and capacitor. In this paper, we describe adjustable speed synchronization between two identical capacitor-run single-phase induction motors based on the electrical shaft system, where rotor circuits are connected together to the common additional resistor. The additional resistor in the rotor common circuit plays the most important role in the determination of the synchronous capability and recovery time of the system. The additional capacitor in the stator circuit plays the main role in adjusting and controlling the electromagnetic torque [6]-[8]. The system was designed, modeled and tested with various loads and additional parameters.

## II. EQUIVALENT CIRCUIT OF THE SYSTEM

In order to derive the equivalent circuit, a machine consisting of forward and backward sequence circuits is considered [2]-[9]. Under this condition, equivalent circuits can be presented as shown in Fig. 2.

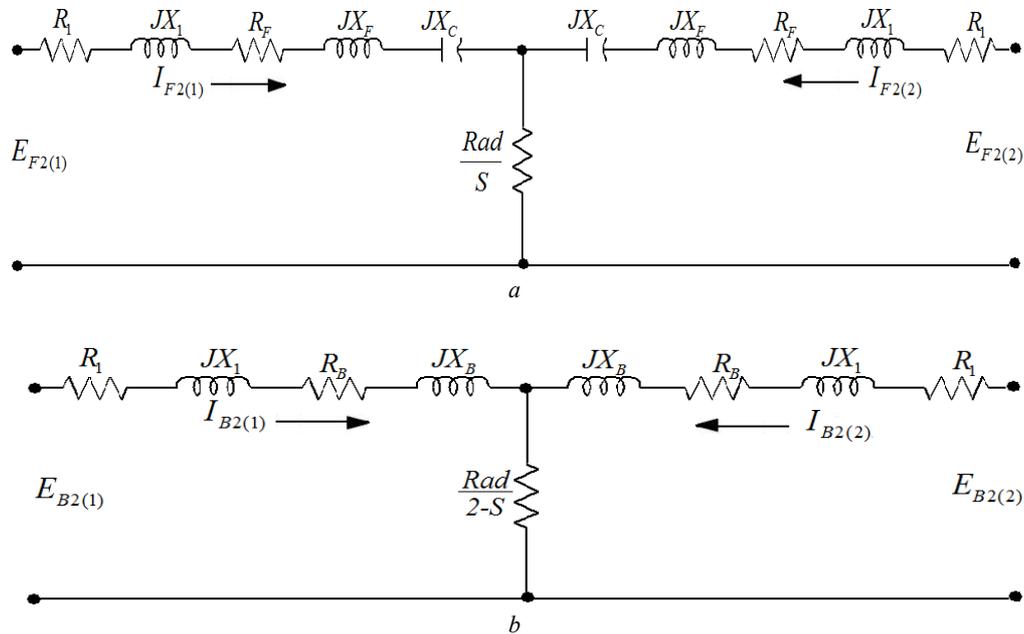


Fig. 2. Forward (a) and backward (b) sequence equivalent circuits

Where the forward and backward impedances of the rotor circuit ( $Z_F$  and  $Z_B$ ) are calculated as follows:

$$Z_F = \frac{1}{2} \left( \frac{\left( \frac{R_2' X_\mu (X_2 + X_\mu)}{S} \right) - \left( \frac{R_2' X_2 X_\mu}{S} \right)}{\left( \frac{R_2'}{S} \right)^2 + (X_2 + X_\mu')^2} \right) + j \left( \frac{\left( \frac{R_2'^2 X_\mu}{S^2} \right) + (R_2' X_2 (X_2 + X_\mu))}{\left( \frac{R_2'}{S} \right)^2 + (X_2 + X_\mu')^2} \right)$$

$$Z_B = \frac{1}{2} \left( \frac{\left( \frac{R_2' X_\mu (X_2 + X_\mu)}{(2-S)} \right) - \left( \frac{R_2' X_2 X_\mu}{(2-S)} \right)}{\left( \frac{R_2'}{2-S} \right)^2 + (X_2 + X_\mu')^2} \right) + j \left( \frac{\left( \frac{R_2'^2 X_\mu}{(2-S)^2} \right) + (R_2' X_2 (X_2 + X_\mu))}{\left( \frac{R_2'}{2-S} \right)^2 + (X_2 + X_\mu')^2} \right)$$

where  $R_1$ ,  $X_1$  are the stator resistance and inductive reactance;  $R_2$ ,  $X_2$  are the rotor resistance and inductive reactance;  $E_{F2(1)}$ ,  $E_{F2(2)}$  are the forward induced voltages in the rotors;  $E_{B2(1)}$ ,  $E_{B2(2)}$  are the backward induced voltages in the rotors;  $I_{F2(1)}$ ,  $I_{F2(2)}$  are the forward currents in the rotors;  $I_{B2(1)}$ ,  $I_{B2(2)}$  are the backward currents in the rotors;  $X_\mu$  is the inductive reactance of the magnetization circuit;  $X_C$  is the capacitive reactance, and  $S$  is the slip of the motor.

Difference in motors' speeds causes difference in phase angles between the stator and rotor windings ( $\alpha_1$ ,  $\alpha_2$ ). Therefore, it is possible to represent the difference in loads by the differences in phase angles. At equal loads, phase angles will be equal too ( $\alpha_1=\alpha_2$ ), and  $\Delta\alpha=0$  [10, 11]. Using the equivalent circuit of Fig. 2, the rotor balance voltage can be calculated as follows:

$$E'_{F2(1)} = I'_{F2(1)} [(R_1 + R_F) + j(X_1 + X_F - X_C)] + I'_{F2(2)} [R_{ad} / S] \quad (1)$$

$$E'_{F2(2)} = I'_{F2(2)} [(R_1 + R_F) + j(X_1 + X_F - X_C)] + I'_{F2(1)} [R_{ad} / S] \quad (2)$$

$$E'_{B2(1)} = I'_{B2(1)} [(R_1 + R_B) + j(X_1 + X_B - X_C)] + I'_{B2(2)} [R_{ad} / (2 - S)] \quad (3)$$

$$E'_{B2(2)} = I'_{B2(2)} [(R_1 + R_B) + j(X_1 + X_B - X_C)] + I'_{B2(1)} [R_{ad} / (2 - S)] \quad (4)$$

From (1)-(4), the rotor current will be:

$$I'_{F2(1)} = \frac{1}{2} \left[ \frac{E'_{F2(1)} + E'_{F2(2)}}{(R_1 + R_F) + j(X_1 + X_F - X_C) + 2R_{ad} / S} + \frac{E'_{F2(1)} - E'_{F2(2)}}{(R_1 + R_F) + j(X_1 + X_F - X_C)} \right] \quad (5)$$

$$I'_{F2(2)} = \frac{1}{2} \left[ \frac{E'_{F2(1)} + E'_{F2(2)}}{(R_1 + R_F) + j(X_1 + X_F - X_C) + 2R_{ad} / S} + \frac{E'_{F2(2)} - E'_{F2(1)}}{(R_1 + R_F) + j(X_1 + X_F - X_C)} \right] \quad (6)$$

$$I'_{B2(1)} = \frac{1}{2} \left[ \frac{E'_{B2(1)} + E'_{B2(2)}}{(R_1 + R_B) + j(X_1 + X_B - X_C) + 2R_{ad} / (2 - S)} + \frac{E'_{B2(1)} - E'_{B2(2)}}{(R_1 + R_B) + j(X_1 + X_B - X_C)} \right] \quad (7)$$

$$I'_{B2(2)} = \frac{1}{2} \left[ \frac{E'_{B2(1)} + E'_{B2(2)}}{(R_1 + R_B) + j(X_1 + X_B - X_C) + 2R_{ad} / (2 - S)} + \frac{E'_{B2(2)} - E'_{B2(1)}}{(R_1 + R_B) + j(X_1 + X_B - X_C)} \right] \quad (8)$$

If the first motor is considered as a reference motor of the system, then  $E'_{F2(1)} = E'_2$ ,  $E'_{F2(2)} = E'_2 e^{j\Delta\alpha}$ ,  $S = S_1$  and  $\Delta\alpha = \alpha_1 - \alpha_2$ . If the second motor is considered as a reference motor of the system, then  $E'_{F2(2)} = E'_2$ ,  $E'_{F2(1)} = E'_2 e^{j\Delta\alpha}$ ,  $S = S_2$  and  $\Delta\alpha = \alpha_2 - \alpha_1$ ; and consequently, the second motor torque can be determined in a similar way to the first motor. Since the calculations of the first and the second motors are identical, then it is enough to perform calculations and investigations for the first motor only. The torque of the first motor may be calculated by the following formula [2], [12, 13]:

$$T_{F(1)} = \frac{E_2'^2}{2\omega_o} [I'_{F2(1)} + \dot{I}'_{F2(1)}] \quad (9)$$

$$T_{B(1)} = \frac{E_2'^2}{2\omega_o} [I'_{B2(1)} + \dot{I}'_{B2(1)}] \quad (10)$$

where  $\omega_o$  is the no-load rotational speed in rad/s.

Having added the value of the rotor current ( $I'_{F2(1)}$ ) and its conjugate ( $\dot{I}'_{F2(1)}$ ) and performed some mathematical manipulations, the torque represented by (9) and (10) becomes:

$$T_{F(1)} = \frac{E_2'^2}{2\omega_o} \left[ \left( \frac{(R_1 + R_F)(1 - \cos\Delta\alpha)}{(R_1 + R_F)^2 + (X_1 + X_F - X_C)^2} + \frac{\left(R_1 + R_F + \frac{R_{ad}}{S}\right)(1 + \cos\Delta\alpha)}{\left(R_1 + R_F + \frac{R_{ad}}{S}\right)^2 + (X_1 + X_F - X_C)^2} \right) + \left( \frac{(X_1 + X_F - X_C)\sin\Delta\alpha}{\left(R_1 + R_F + \frac{R_{ad}}{S}\right)^2 + (X_1 + X_F - X_C)^2} - \frac{(X_1 + X_F - X_C)\sin\Delta\alpha}{(R_1 + R_F)^2 + (X_1 + X_F - X_C)^2} \right) \right] \quad (11)$$

$$T_{B(1)} = \frac{E_2'^2}{2\omega_o} \left[ \left( \frac{(R_1 + R_B)(1 - \cos\Delta\alpha)}{(R_1 + R_B)^2 + (X_1 + X_B - X_C)^2} + \frac{\left(R_1 + R_B + \frac{R_{ad}}{2-S}\right)(1 + \cos\Delta\alpha)}{\left(R_1 + R_B + \frac{R_{ad}}{2-S}\right)^2 + (X_1 + X_B - X_C)^2} \right) + \left( \frac{(X_1 + X_B - X_C)\sin\Delta\alpha}{\left(R_1 + R_B + \frac{R_{ad}}{2-S}\right)^2 + (X_1 + X_B - X_C)^2} - \frac{(X_1 + X_B - X_C)\sin\Delta\alpha}{(R_1 + R_B)^2 + (X_1 + X_B - X_C)^2} \right) \right] \quad (12)$$

The total torque of the first motor becomes:

$$T_{(1)} = T_{F(1)} - T_{B(1)} \quad (13)$$

### III. MODELING AND TESTING

The mathematical model of the system has been built as a subsystem block of the main equivalent circuit, dynamic torque, angular speed, and angular position equations as shown in Fig. 3. This model consists of two blocks. Each of them has a specific function. Simulation and modeling characteristics of single-phase machine using MATLAB applications are increasingly implemented [15]-[17]. The investigation and control of the system use two identical single-phase induction motors with the following parameters: 0.25-hp, 50-Hz, 4-Pole,  $V_{ph}=220\text{V}$ ,  $I_o=3.56\text{A}$ ,  $X_\mu=66.89\Omega$ ,  $R_l=2.02\Omega$ ,  $R_2=4.12\Omega$ ,  $X_l=2.793\Omega$  and  $X_2=4.12\Omega$ .

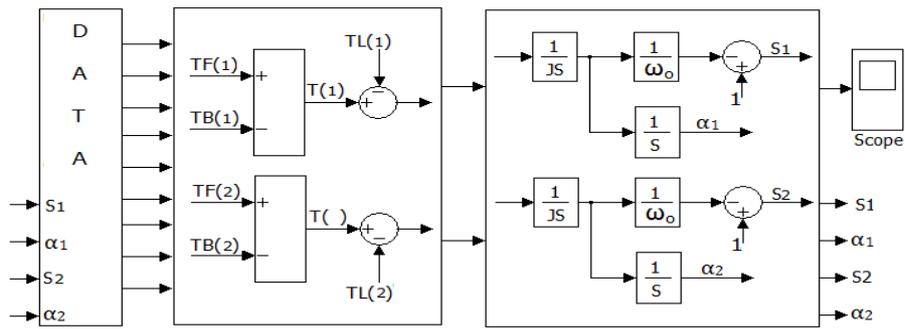


Fig. 3. Mathematical model of the study system

The main effect of additional parameters ( $X_c, R_{ad}$ ) on the system behavior can be found using the factor  $K=X_c/R_{ad}$ . Fig. 4 shows the speed response, in per unit, at various loads. It shows that increasing the load difference decreases the synchronous capability of the system. Consequently, the motors operate as individual induction motors.

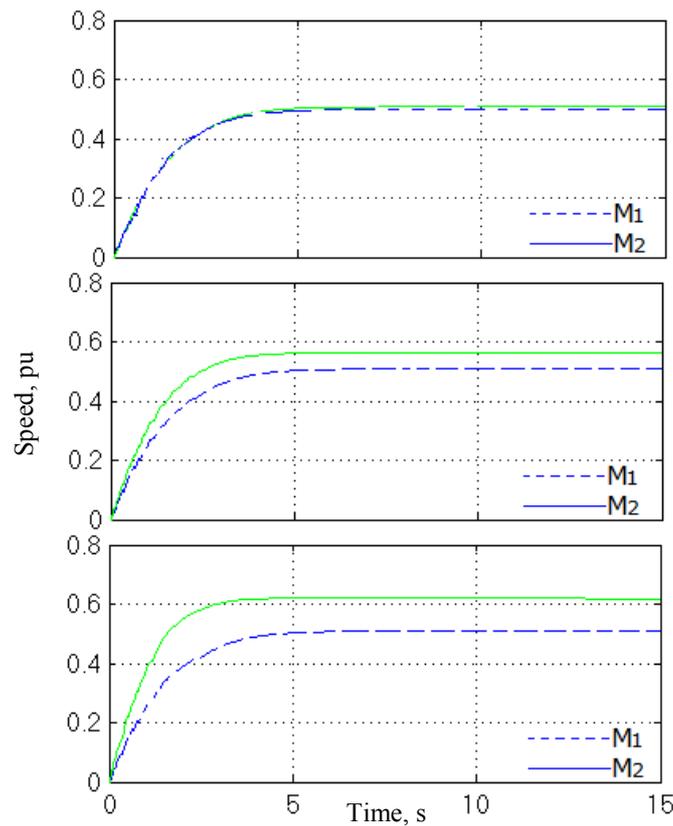


Fig. 4. Speed response at  $K=5$  and  $L_1=(1, 1.2 \text{ and } 1.6)L_2$

The effect of factor  $K$  on starting torques and speed response at  $L_1=1.6L_2$  can be found in Figs. 5 and 6. Fig. 5 shows that increasing factor  $K$  increases the starting torque value especially at the first moment of starting.

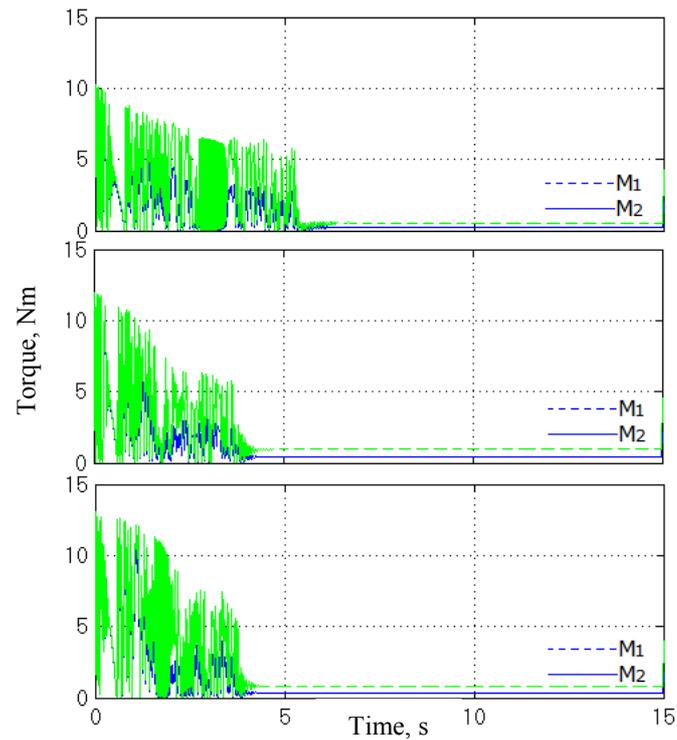


Fig. 5. Torque response at  $L_1=1.6L_2$ , and  $K=5, 10$  and  $25$

Fig. 6 shows that increasing factor  $K$  leads to increase the synchronous capability and decrease the recovery time of the system; and consequently, it leads to an increase in system performance specifications.

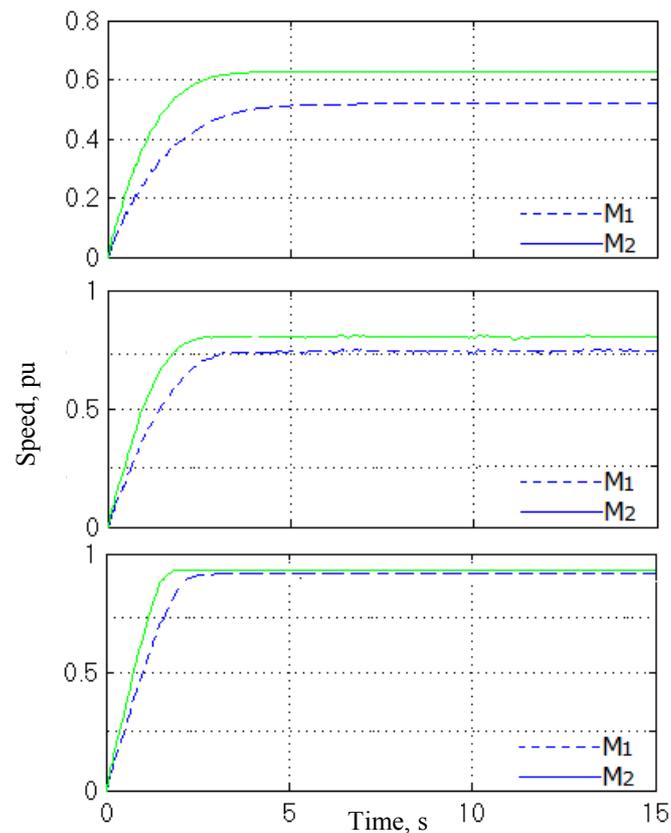


Fig. 6. Speed response at  $L_1=1.6L_2$ , and  $K=5, 10$  and  $25$

#### IV. CONCLUSIONS

Speed synchronization of multiple induction motors can work with capacitor-run single-phase induction motor based on simplified circuit electrical shaft system. By selecting the stator and rotor's additional parameters (adjusting the  $K$  factor value), we can find the maximum limit of the synchronous capability and recovery time of the system. The main disadvantage of the system is the reduction of system efficiency which is associated with the slip ring rotor and backward torque. Increasing factor  $K$  leads to improved system performance and reduction of this drawback. Practically, performance specifications of the synchronization system that uses single-phase induction motors do not coincide with modulation results due to the following:

- 1) Calculations were made with some simplification, such as linearization of the magnetic circuit.
- 2) The actual speed of synchronization systems will be done through a particular transmission, where the actual speed will be reduced to a tenth of the corrected or controlled speed.

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